SUSY gauge theories

SUSY QCD

Consider a SUSY $SU(N)$ with F "flavors" of "quarks" and squarks

$$\mathcal{Q}_i = (\phi_i, Q_i, \mathcal{F}_i), i = 1, \ldots, F \ ,$$

where ϕ is the squark and Q is the quark.

$$\overline{\mathcal{Q}}_i = (\overline{\phi_i}, \overline{Q_i}, \overline{\mathcal{F}_i}) \ ,$$

in the antifundamental representation. Note the the bar ($\overline{}$) is part of the name not a conjugation, the conjugate fields are

$$\mathcal{Q}_i^\dagger = (\phi_i^*, Q_i^\dagger, \mathcal{F}_i^*), \ \ \overline{\mathcal{Q}}_i^\dagger = (\overline{\phi}_i^*, \overline{Q}_i^\dagger, \overline{\mathcal{F}_i^*}).$$

SUSY QCD

matter content is:

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_B$	$U(1)_R$
$\mathcal{Q}$	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\mathcal{Q}}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

$$W = 0$$

R-charge

$$[R, Q_\alpha] = -Q_\alpha .$$

chiral supermultiplet:

$$R_\psi \quad = \quad R_\phi - 1 \ ,$$

normalize the R-charge by

$$R\lambda^a = \lambda^a \ ,$$

R-charge of the gluino is 1, and the R-charge of the gluon is 0.

Group Theory: Bird Tracks

Identify the group generator with a vertex as in Fig. **??**.

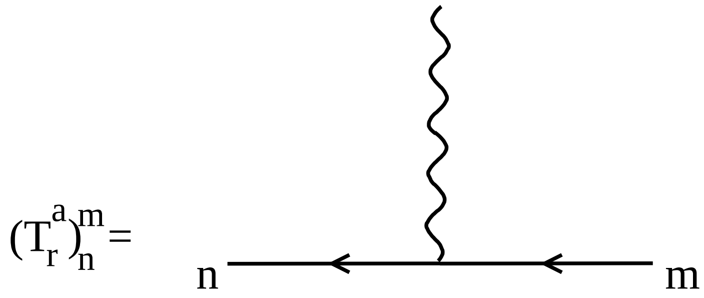


Figure 1: Bird-track notation for the group generator T^a.

Bird Tracks

quadratic Casimir $C_2(\mathbf{r})$ and the index$T(\mathbf{r})$ of the representation $\mathbf{r}$,

$$(T^a_{\mathbf{r}})_l^m (T^a_{\mathbf{r}})_n^l = C_2(\mathbf{r})\delta_n^m \ ,$$
$$(T^a_{\mathbf{r}})_n^m (T^b_{\mathbf{r}})_m^n = T(\mathbf{r})\delta^{ab} \ ,$$

are given diagrammaticly as

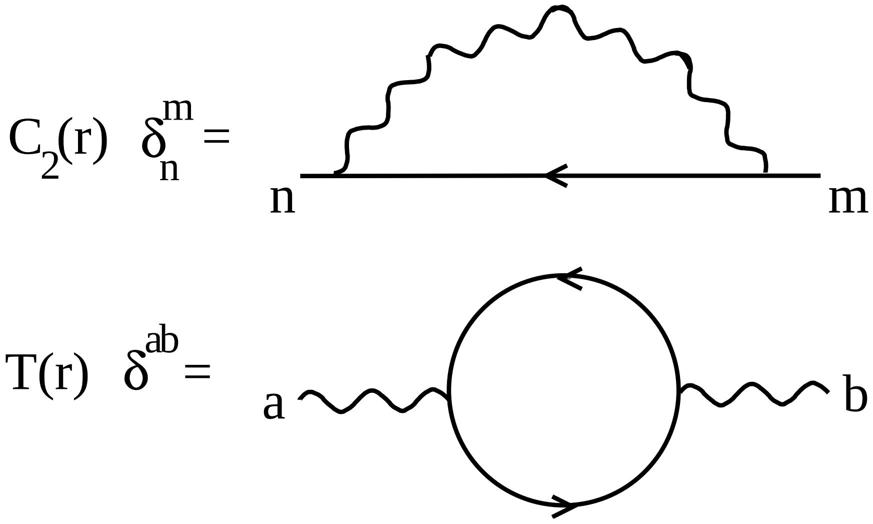


Figure 2:

Bird Tracks

Contracting the external legs: In the first diagram setting m equal

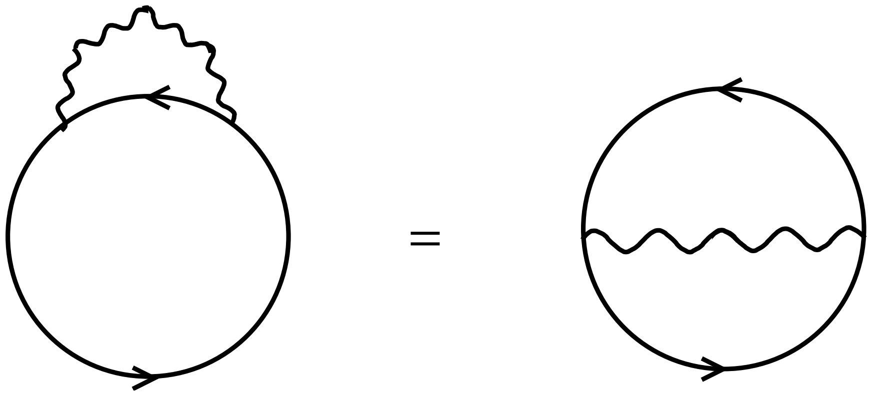


Figure 3:

to n and summing over n yields a factor of $d(\mathbf{r})$. In the second diagram setting a equal to b and summing yields a factor $d(\mathbf{Ad})$.

$$d(\mathbf{r})C_2(\mathbf{r}) = d(\mathbf{Ad})T(\mathbf{r}) \ .$$

Casimirs

$$\begin{aligned} d(\Box) &= N,\ d(\mathbf{Ad}) = & N^2 - 1 \\ T(\Box) &= \tfrac{1}{2},\ T(\mathbf{Ad}) = & N \end{aligned}$$

so

$$C_2(\Box) = \tfrac{N^2-1}{2N},\ C_2(\mathbf{Ad}) = N \ .$$

Sum over Generators

For the fundamental representation $\Box$:

$$(T^a)^l_p (T^a)^m_n = \frac{1}{2}(\delta^l_n \delta^m_p - \frac{1}{N}\delta^l_p \delta^m_n) \ .$$

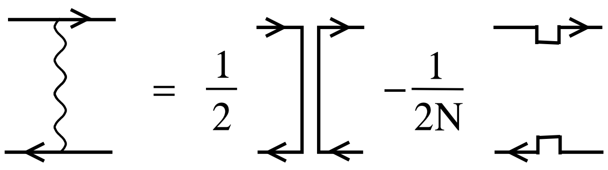


Figure 4:

We can reduce the sums over multiple generators to an essentially topological exercise

Anomalies

Since we can define an R-charge by taking arbitrary linear combinations of the $U(1)_R$ and $U(1)_B$ charges we can choose Q_i and $\overline{Q}_i$ to have the same R-charge. For a $U(1)$ not be to broken by instanton effects the $SU(N)^2U(1)_R$ anomaly diagram vanishes

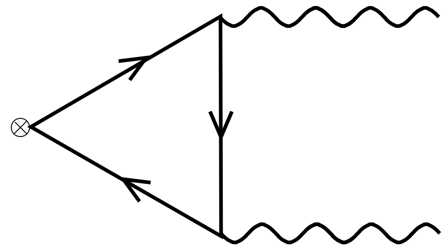


Figure 5:

fermion contributes its R-charge times $T(r)$. Sum over gluino, quarks:

$$1 \cdot T(\mathbf{Ad}) + (R-1)T(\Box)\,2F = 0\ ,$$

$$\text{so}\ \ R = \tfrac{F-N}{F}$$

Renormalization group

tree-level SUSY: $Y = \sqrt{2}g$, $\lambda = g^2$. For SUSY to be a consistent quantum symmetry these relations must be preserved under RG running.

the β function for the gauge coupling at one-loop is

$$\beta_g = \mu\frac{dg}{d\mu} = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}T(\mathrm{Ad}) - \frac{2}{3}T(F) - \frac{1}{3}T(S)\right) \equiv -\frac{g^3\,b}{16\pi^2}\ ,$$

For SUSY QCD:

$$b = (3N - F)$$

Renormalization group

the β function for the Yukawa coupling is :

$$\begin{aligned}(4\pi)^2\beta_Y^j &= \frac{1}{2}\left[Y_2^\dagger(F)Y^j + Y^jY_2(F)\right] + 2Y^kY^{j\dagger}Y^k \\ &+Y^k\,\mathrm{Tr}\,Y^{k\dagger}Y^j - 3g_m^2\{C_2^m(F), Y^j\}\ ,\end{aligned}$$

where

$$Y_2(F) \equiv Y^{j\dagger}Y^j$$

$Y_2^\dagger(F)Y^j$ represents the scalar loop corrections to the fermion legs
$2Y^kY^{j\dagger}Y^k$ contains the 1PI vertex corrections
$Y^k\,\mathrm{Tr}\,Y^{k\dagger}Y^j$ represents fermion loop corrections to the scalar leg
$C_2^m(F)$ is the quadratic Casimir of the fermion fields in the mth gauge group, and represents gauge loop corrections to the fermion legs

SUSY QCD RG

For SUSY QCD the Yukawa coupling of quark i with color index m, gluino a, and antisquark j with color index n is given by

$$Y^{jn}_{im,a} = \sqrt{2} g (T^a)^n_m \delta^j_i \ .$$

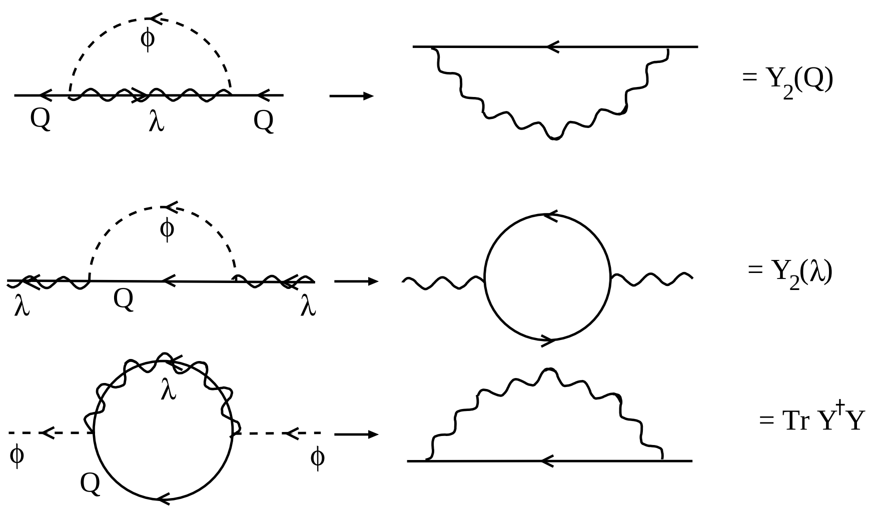


Figure 6: Feynman diagrams and associated bird-track diagrams.

$$Y_2(Q) = 2g^2 C_2(\square), \; Y_2(\lambda) = 2g^2 \, 2F \, T(\square)$$

SUSY QCD RG

no scalar corrections corresponding to $Y^k Y^{\dagger j} Y^k$. As for the fermion loop correction it always has a quark (antisquark) and gluino for the internal lines so we have

$$Y^k \,\mathrm{Tr}\, Y^{k\dagger} Y^j = Y^{kq}_{im,a} \, (Y^{kq}_{fp,b})^\dagger Y^{jn}_{fp,b} = 2g^2 C_2(\Box)(T^a)^n_m \delta^j_i \ ,$$

gauge loop corrections are

$$\{C_2^m(F), Y^j\} = (C_2(\Box) + C_2(\mathbf{Ad}))Y^j \ .$$

all the terms in β^j_Y proportional to $C_2(\Box)$ cancel:

$$\begin{aligned} (4\pi)^2 \beta^j_Y &= \sqrt{2} g^3 (C_2(\Box) + F + 2C_2(\Box) - 3C_2(\Box) - 3N) \\ &= -\sqrt{2} g^3 (3N - F) \\ &= \sqrt{2} (4\pi)^2 \beta_g \end{aligned}$$

so the relation between the Yukawa and gauge couplings is preserved under RG running

SUSY QCD Quartic RG

SUSY also requires the D-term quartic coupling $\lambda = g^2$. The auxiliary D^a field is given by

$$D^a = g(\phi^{*in}(T^a)^m_n \phi_{mi} - \overline{\phi}^{in}(T^a)^m_n \overline{\phi}^*_{mi})$$

and the D-term potential is

$$V = \frac{1}{2} D^a D^a$$

The β function for a quartic scalar coupling at one-loop is

$$(4\pi)^2 \beta_\lambda = \Lambda^{(2)} - 4H + 3A + \Lambda^Y - 3\Lambda^S,$$

$\Lambda^{(2)}$ corresponds to the 1PI contribution from the quartic interactions
H corresponds to the fermion box graphs
A to the two gauge boson exchange graphs
Λ^Y to the Yukawa leg corrections
Λ^S corresponds to the gauge leg corrections

SUSY QCD Quartic RG

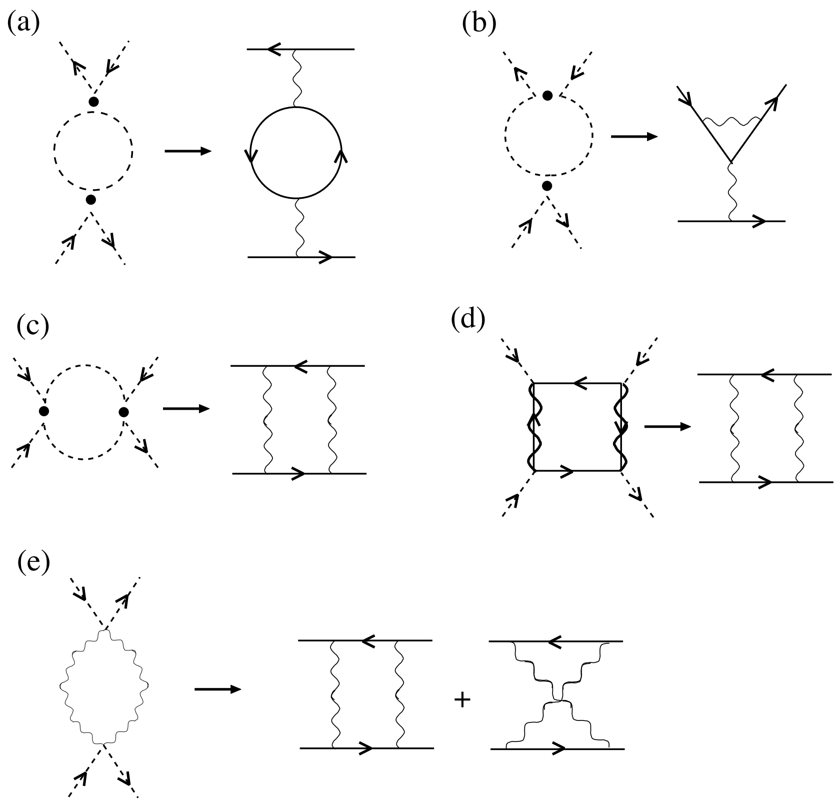


Figure 7:

SUSY QCD Quartic RG

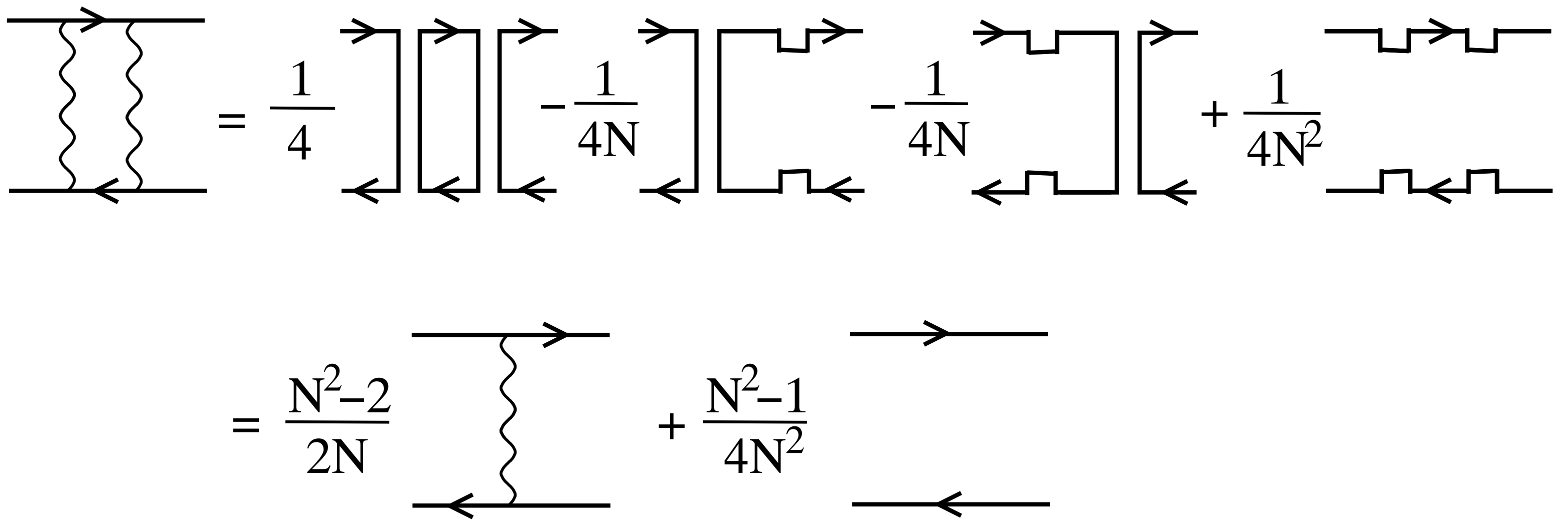


Figure 8: The bird-track diagram for the sum over four generators quickly reduces to the sum over two generators and a product of identity matrices.

SUSY QCD Quartic RG

$$(\phi^{*in}(T^a)^m_n\phi_{mi} - \overline{\phi}^{in}(T^a)^m_n\overline{\phi}^*_{mi})(\phi^{*jq}(T^a)^p_q\phi_{pj} - \overline{\phi}^{jq}(T^a)^p_q\overline{\phi}^*_{pj}) \ ,$$

(with flavor indices $i \neq j$, the case $i = j$ is left as an exercise) we have

$$\begin{aligned}
\Lambda^{(2)} &= \left(2F + N - \tfrac{6}{N}\right)(T^a)^m_n(T^a)^p_q + \left(1 - \tfrac{1}{N^2}\right)\delta^m_n\delta^p_q \ , \\
-4H &= -8\left(N - \tfrac{2}{N}\right)(T^a)^m_n(T^a)^p_q - 4\left(1 - \tfrac{1}{N^2}\right)\delta^m_n\delta^p_q \ , \\
3A &= 3\left(N - \tfrac{4}{N}\right)(T^a)^m_n(T^a)^p_q + 3\left(1 - \tfrac{1}{N^2}\right)\delta^m_n\delta^p_q \ , \\
\Lambda^Y &= 4\left(N - \tfrac{1}{N}\right)(T^a)^m_n(T^a)^p_q \ , \\
-3\Lambda^S &= -6\left(N - \tfrac{1}{N}\right)(T^a)^m_n(T^a)^p_q \ .
\end{aligned}$$

individual diagrams that renormalize the gauge invariant, SUSY breaking, operator $(\phi^{*mi}\phi_{mi})(\phi^{*pj}\phi_{pj})$ but the full β function for this operator vanishes and the D-term β function satisfies

$$\begin{aligned}
\beta_\lambda &= \beta_{g^2}T^aT^a \ , \\
\beta_{g^2} &= 2g\beta_g \ .
\end{aligned}$$

So SUSY is not anomalous at one-loop, and the β functions preserve the relations between couplings at all scales.

SUSY RG

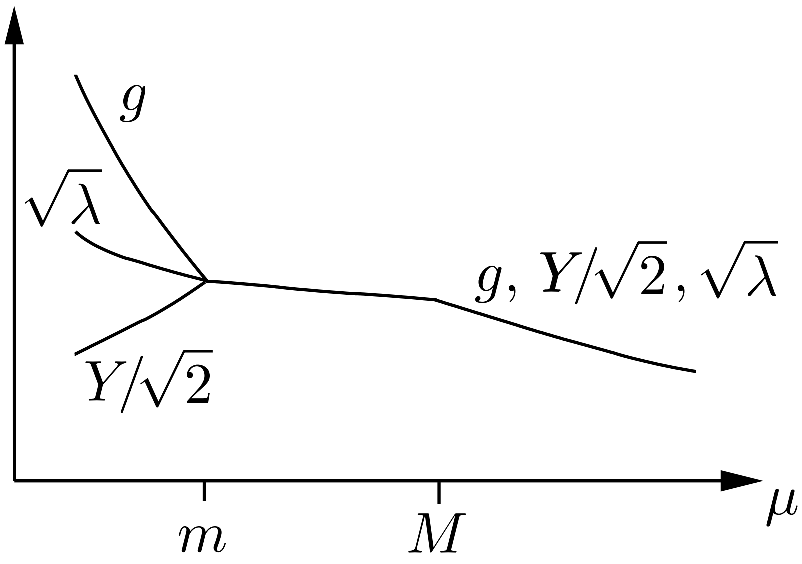


Figure 9: The couplings remain equal as we run below the SUSY threshold M, but split apart below the non-SUSY threshold m.

If we had added dimension 4 SUSY breaking terms to the theory then the couplings would have run differently at all scales

one-loop squark mass

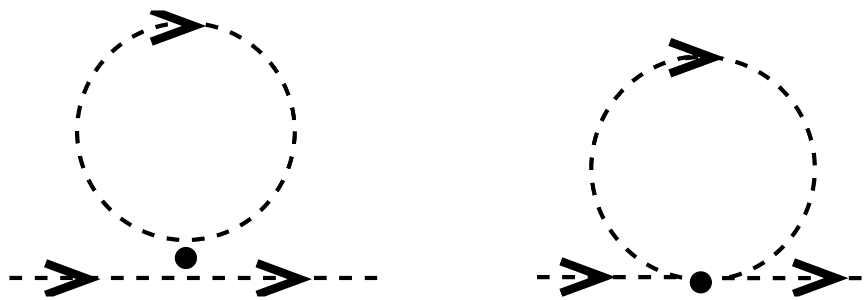


Figure 10: The squark loop correction to the squark mass.

$$\begin{aligned}\Sigma_{\rm squark}(0) &= -ig^2 (T^a)^l_n (T^a)^m_l \int \tfrac{d^4k}{(2\pi)^4} \tfrac{i}{k^2} \\ &= \tfrac{-ig^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2\ .\end{aligned}$$

one-loop squark mass

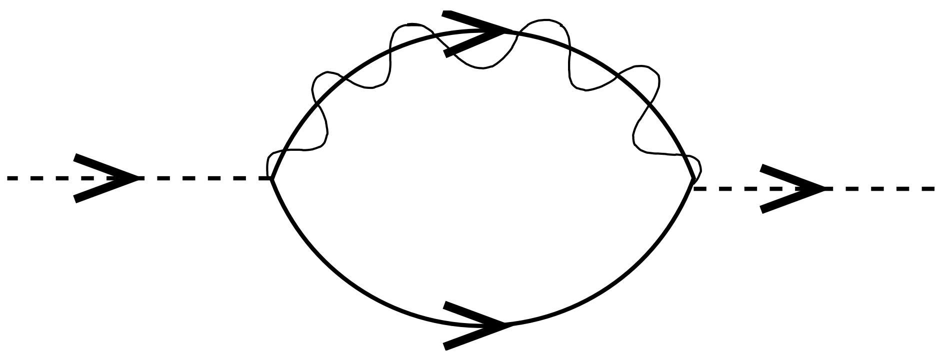


Figure 11: The quark–gluino loop correction to the squark mass.

$$
\begin{aligned}
\Sigma_{\mathrm{quark-gluino}}(0) &= (-i\sqrt{2}g)^2 (T^a)^l_n (T^a)^m_l (-1) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\, \frac{ik\cdot\sigma}{k^2}\, \frac{ik\cdot\overline{\sigma}}{k^2} \\
&= -2g^2 C_2(\Box)\delta^m_n \int \frac{d^4k}{(2\pi)^4}\, \frac{2k^2}{k^4} \\
&= \frac{4ig^2}{16\pi^2} C_2(\Box)\delta^m_n \int_0^{\Lambda^2} dk^2 \ .
\end{aligned}
$$

one-loop squark mass

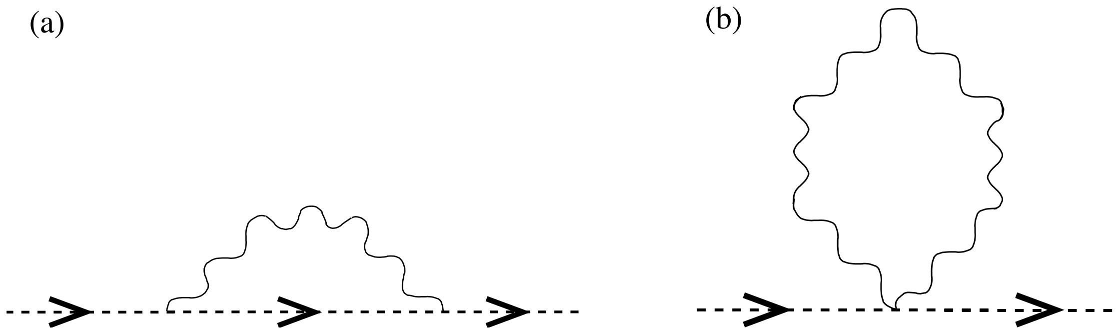


Figure 12: (a) The squark–gluon loop and (b) the gluon loop.

$$\Sigma_{\text{quark-gluino}}(0) = (ig)^2 (T^a)^l_n (T^a)^m_l \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} k^\mu (-i) \frac{(g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2})}{k^2} k^\nu$$
$$= \frac{\xi i g^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2 \ ,$$

$$\Sigma_{\text{gluon}}(0) = \frac{1}{2} i g^2 \left\{ (T^a)^l_n, (T^b)^m_l \right\} \delta^{ab} g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (-i) \frac{(g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2})}{k^2}$$
$$= \frac{-(3+\xi) i g^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2 \ .$$

one-loop squark mass

Adding all the terms together we have

$$\Sigma(0) \quad = \quad (-1+4+\xi-(3+\xi))\,\frac{ig^2}{16\pi^2}C_2(\square)\delta_n^m\int_0^{\Lambda^2}dk^2 = 0\ .$$

The quadratic divergence in the squark mass cancels! In fact for a massless squark all the mass corrections cancel. This means that in a SUSY theory with a Higgs the Higgs mass is protected from quadratic divergences from gauge interactions as well as from Yukawa interactions

Flat directions $F < N$

$$D^a = g(\phi^{*in}(T^a)^m_n \phi_{mi} - \overline{\phi}^{in}(T^a)^m_n \overline{\phi}^*_{mi})$$

and the scalar potential is:

$$V = \frac{1}{2} D^a D^a$$

$$\text{define } \; d^n_m \equiv \langle \phi^{*in} \phi_{mi} \rangle$$
$$\overline{d}^n_m = \langle \overline{\phi}^{in} \overline{\phi}^*_{mi} \rangle$$

maximal rank F. In a SUSY vacua:

$$D^a = T^{am}_n (d^n_m - \overline{d}^n_m) = 0$$

Since T^a is a complete basis for traceless matrices, we must therefore have that the difference of the two matrices is proportional to the identity matrix:

$$d^n_m - \overline{d}^n_m = \alpha I$$

Flat directions $F < N$

d^n_m can be diagonalized by an $SU(N)$ gauge transformation

$$U^\dagger d U$$

In this diagonal basis there will be at least $N - F$ zero eigenvalues

$$d = \begin{pmatrix} v_1^2 & & & & & & \\ & v_2^2 & & & & & \\ & & \ddots & & & & \\ & & & v_F^2 & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix}$$

where $v_i^2 \geq 0$. In this basis $\overline{d}^n_m$ must also be diagonal, and it must also have $N - F$ zero eigenvalues. This tells us that $\alpha = 0$, and hence that

$$\overline{d}^n_m = d^n_m$$

Flat directions $F < N$

d_m^n and $\overline{d}_m^n$ are invariant under $SU(F) \times SU(F)$ transformations since

$$\begin{array}{rcl} \phi_{mi} & \rightarrow & \phi_{mi} V^i_j \ , \\ d_m^n & \rightarrow & V_i^{*j} \langle \phi^{*in} \rangle \langle \phi_{mi} \rangle V^i_j \ , \\ & \rightarrow & \langle \phi^{*jn} \phi_{mj} \rangle = d_m^n \ . \end{array}$$

Thus, up to a flavor transformation, we can write

$$\langle \overline{\phi}^* \rangle = \langle \phi \rangle = \left(\begin{array}{ccc} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & 0 \end{array} \right) \ .$$

D-term potential has flat directions, as we change the VEVs, we move between different vacua with different particle spectra,
generically $SU(N - F)$ gauge symmetry

Flat directions $F \geq N$

d^n_m and $\overline{d}^n_m$ are $N \times N$ positive semi-definite Hermitian matrices of maximal rank N in a SUSY vacuum :

$$d^n_m - \overline{d}^n_m = \rho I \ .$$

d^n_m can be diagonalized by an $SU(N)$ gauge transformation:

$$d = \begin{pmatrix} |v_1|^2 & & & \\ & |v_2|^2 & & \\ & & \ddots & \\ & & & |v_N|^2 \end{pmatrix}$$

In this basis, $\overline{d}^n_m$ must also be diagonal, with eigenvalues $|\overline{v}_i|^2$, so

$$|v_i|^2 = |\overline{v}_i|^2 + \rho \ .$$

Flat directions $F \geq N$

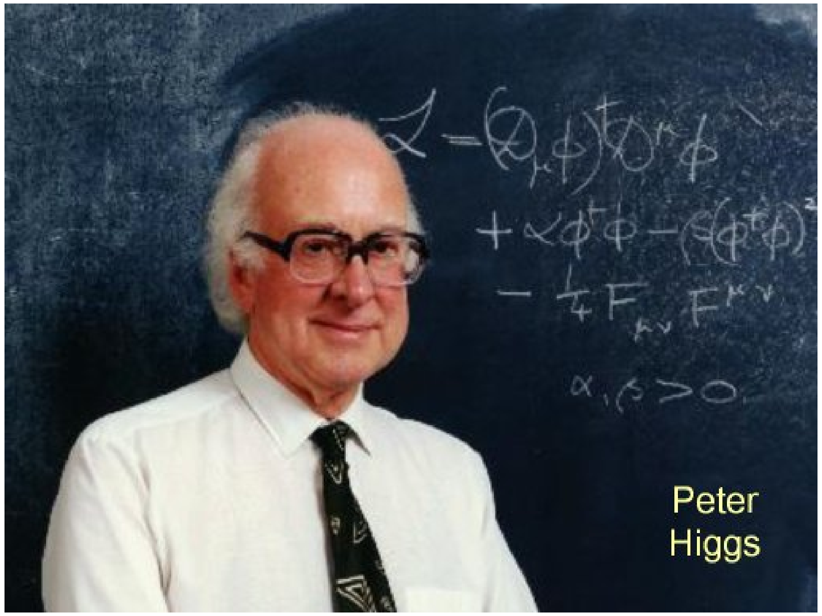
Since d^n_m and $\overline{d}^n_m$ are invariant under flavor transformations, we can use $SU(F) \times SU(F)$ transformations to put $\langle\phi\rangle$ and $\langle\overline{\phi}\rangle$ in the form

$$\langle\Phi\rangle = \begin{pmatrix} v_1 & & & 0 & \dots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \dots & 0 \end{pmatrix}, \quad \langle\overline{\Phi}\rangle = \begin{pmatrix} \overline{v}_1 & & \\ & \ddots & \\ & & \overline{v}_N \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}.$$

Again we have a space of degenerate vacua. At a generic point in the moduli space the $SU(N)$ gauge symmetry is completely broken.

The super Higgs mechanism

a massless vector supermultiplet eats a chiral supermultiplet to form a massive vector supermultiplet



Fayet

The super Higgs mechanism

Consider the case when $v_1 = \overline{v}_1 = v$ and $v_i = \overline{v}_i = 0$, for $i > 1$
$SU(N) \to SU(N-1)$ and $SU(F) \times SU(F) \to SU(F-1) \times SU(F-1)$.
The number of broken gauge generators is

$$N^2 - 1 - ((N-1)^2 - 1) = 2(N-1) + 1 \ ,$$

decompose the adjoint of $SU(N)$ under $SU(N-1)$, we have

$$\mathbf{Ad_N} = \mathbf{1} + \Box + \overline{\Box} + \mathbf{Ad_{N-1}}$$

convenient basis of gauge generators is $G^A = X^0, X_1^\alpha, X_2^\alpha, T^a$ where $A = 1, \ldots, N^2 - 1$, $\alpha = 1, \ldots, N-1$, and $a = 1, \ldots, (N-1)^2 - 1$.
Xs are the broken generators (span the coset of $SU(N)/SU(N-1)$),
Ts are the unbroken $SU(N-1)$ generators

The super Higgs mechanism

The Xs are analogs of the Pauli matrices:

$$X^0 = \frac{1}{\sqrt{2(N^2-N)}} \begin{pmatrix} N-1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & \ddots & \\ & & & & -1 \end{pmatrix},$$

$$X_1^\alpha = \frac{1}{2} \begin{pmatrix} 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ 0 & & & & & & \\ \vdots & & & & & & \\ 0 & & & & & & \\ 1 & & & \mathbf{0} & & & \\ 0 & & & & & & \\ \vdots & & & & & & \\ 0 & & & & & & \end{pmatrix}, \quad X_2^\alpha = \frac{1}{2} \begin{pmatrix} 0 & \dots & 0 & i & 0 & \dots \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & & & & \\ -i & & & \mathbf{0} & & \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & & & & \end{pmatrix}$$

The super Higgs mechanism

We can also define raising and lowering operators:

$$X^{\pm\alpha} = \frac{1}{\sqrt{2}}(X_1^\alpha \mp i X_2^\alpha)$$

so that

$$X^{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \ldots & 0 & 1 & 0 & \ldots & 0 \\ & & & & & & \\ & & & \mathbf{0} & & & \\ & & & & & & \end{pmatrix}, \quad X^{-\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & & \\ 0 & & \\ \vdots & & \\ 0 & & \\ 1 & & \mathbf{0} \\ 0 & & \\ \vdots & & \\ 0 & & \end{pmatrix}$$

The super Higgs mechanism

We can then write the sum of the product of two generators as:

$$G^A G^A = X^0 X^0 + X^{+\alpha} X^{-\alpha} + X^{-\alpha} X^{+\alpha} + T^a T^a$$

Expanding the squark field around its VEV $\langle\phi\rangle$

$$\phi \rightarrow \langle\phi\rangle + \phi \ ,$$

we have

$$\begin{aligned} \textstyle\sum_A G^A \langle\phi\rangle &= X^0 \langle\phi\rangle + \textstyle\sum_\alpha X^{-\alpha} \langle\phi\rangle \ , \\ \langle\phi\rangle \textstyle\sum_A G^A &= \langle\phi\rangle X^0 + \langle\phi\rangle \textstyle\sum_\alpha X^{+\alpha} \ , \end{aligned}$$

since T^a annihilates $\langle\phi\rangle$. label the components of the gluino field as

$$G^A \lambda^A = X^0 \Lambda^0 + X^{+\alpha} \Lambda^{+\alpha} + X^{-\alpha} \Lambda^{-\alpha} + T^a \lambda^a \ ,$$

The super Higgs mechanism

write the quark field as

$$Q = \begin{pmatrix} \omega^0 & \psi_i \\ \omega_\alpha & Q'_{mi} \end{pmatrix}, \quad \overline{Q} = \begin{pmatrix} \overline{\omega}^0 & \overline{\omega}^\alpha \\ \overline{\psi}^i & \overline{Q}'^{im} \end{pmatrix},$$

where i is a flavor index, α and m are color indices, Q' is a matrix with $N-1$ rows and $F-1$ columns, and $\overline{Q}$ is a matrix with $F-1$ rows and $N-1$ columns.

fermion mass terms generated by the Yukawa interactions:

$$\begin{aligned} \mathcal{L}_{\text{F mass}} &= -\sqrt{2}g\left[\left(\langle\phi^*\rangle X^0\Lambda^0 + \langle\phi^*\rangle X^{+\alpha}\Lambda^{+\alpha}\right) Q \right. \\ &\qquad \left. -\overline{Q}\left(X^0\Lambda^0\langle\overline{\phi}^*\rangle + X^{-\alpha}\Lambda^{-\alpha}\langle\overline{\phi}^*\rangle\right) + h.c.\right] \\ &= -gv\left[\sqrt{\tfrac{N-1}{N}}\left(\omega^0\Lambda^0 - \overline{\omega}^0\Lambda^0\right) + \omega^\alpha\Lambda^{+\alpha} - \overline{\omega}^\alpha\Lambda^{-\alpha} + h.c.\right]. \end{aligned}$$

So we have a Dirac fermion $(\Lambda^0, (1/\sqrt{2})(\omega^0 - \overline{\omega}^0))$ with mass $gv\sqrt{2(N-1)/N}$, two sets of $N-1$ Dirac fermions $(\Lambda^{+\alpha}, \omega^\alpha)$, $(\Lambda^{-\alpha}, -\overline{\omega}^\alpha))$ with mass gv, and massless Weyl fermions Q', $\overline{Q'}$, ψ, $\overline{\psi}$, and $(1/\sqrt{2})(\omega^0 + \overline{\omega}^0))$.

The super Higgs mechanism

decompose the squark field as

$$\phi = \begin{pmatrix} h & \sigma_i \\ H_\alpha & \phi'_{mi} \end{pmatrix}, \quad \overline{\phi} = \begin{pmatrix} \overline{h} & \overline{H}^\alpha \\ \overline{\sigma}^i & \overline{\phi}'^{im} \end{pmatrix},$$

where ϕ' is a matrix with $N-1$ rows and $F-1$ columns. Shifting the scalar field by its VEV so that $\phi \to \langle\phi\rangle + \phi$ we have that the auxiliary D^A field is given by

$$\begin{aligned} \frac{D^A}{g} &= \langle\phi^*\rangle G^A\langle\phi\rangle - \langle\overline{\phi}\rangle G^A\langle\overline{\phi^*}\rangle + \langle\phi^*\rangle G^A\phi - \langle\overline{\phi}\rangle G^A\overline{\phi^*} \\ &\quad +\phi^* G^A\langle\phi\rangle - \overline{\phi} G^A\langle\overline{\phi^*}\rangle + \phi^* G^A\phi - \overline{\phi} G^A\overline{\phi^*}\ . \end{aligned}$$

The super Higgs mechanism

picking out the mass terms in the scalar potential $V = \frac{1}{2} D^A D^A$:

$$\begin{aligned} V_{\text{mass}} &= \frac{g^2}{2} \Big[\Big(\langle \phi^* \rangle X^0 \phi + \phi^* X^0 \langle \phi \rangle - \langle \overline{\phi} \rangle X^0 \overline{\phi}^* - \overline{\phi} X^0 \langle \overline{\phi}^* \rangle \Big)^2 \\ &\qquad + 2(\langle \phi^* \rangle X^{+\alpha} \phi - \langle \overline{\phi} \rangle X^{+\alpha} \overline{\phi}^*)(\phi^* X^{-\alpha} \langle \phi \rangle - \overline{\phi} X^{-\alpha} \langle \overline{\phi}^* \rangle) \Big] \\ &= \frac{g^2 v^2}{2} \Big[\frac{(N-1)^2}{2(N^2-N)} \Big(h + h^* - (\overline{h}^* + \overline{h}) \Big)^2 \\ &\qquad + (H^\alpha - \overline{H}^{*\alpha})(H^{*\alpha} - \overline{H}^\alpha) \Big] . \end{aligned}$$

diagonalize the mass matrix:

$$\begin{aligned} H^{+\alpha} &= \tfrac{1}{\sqrt{2}}(H^\alpha - \overline{H}^{*\alpha}), & \pi^{+\alpha} &= \tfrac{1}{\sqrt{2}}(H^\alpha + \overline{H}^{*\alpha}), \\ H^{-\alpha} &= \tfrac{1}{\sqrt{2}}(H^{*\alpha} - \overline{H}^{\alpha}), & \pi^{-\alpha} &= \tfrac{1}{\sqrt{2}}(H^{*\alpha} + \overline{H}^{\alpha}), \\ h^0 &= \mathrm{Re}(h - \overline{h}) , & \pi^0 &= \mathrm{Im}(h - \overline{h}) , \\ \Omega &= \tfrac{1}{\sqrt{2}}(h + \overline{h}). \end{aligned}$$

The super Higgs mechanism

mass terms reduce to

$$V_{\text{mass}} = g^2 v^2 \left[\tfrac{N-1}{N} (h^0)^2 + H^{+\alpha} H^{-\alpha} \right].$$

real scalar h^0 with mass $gv\sqrt{2(N-1)/N}$,
a complex scalar $H^{+\alpha}$ (and its conjugate $H^{-\alpha}$) with mass gv,
massless complex scalars σ_i, $\overline{\sigma}^i$, and Ω.

πs become the longitudinal components of the massive gauge bosons, can be removed by going to Unitary gauge

The super Higgs mechanism

We can write the gauge fields as:

$$G^B A^B_\mu = X^0 W^0_\mu + X^{+\alpha} W^{+\alpha}_\mu + X^{-\alpha} W^{-\alpha}_\mu + T^a A^a_\mu \ .$$

Then the $A^2\phi^2$ terms which lead to gauge boson masses are

$$\begin{aligned} \mathcal{L}_{A^2\phi^2} &= g^2 A^A_\mu A^B_\nu g^{\mu\nu} \langle \phi^* \rangle G^A G^B \langle \phi \rangle \\ &= g^2 g^{\mu\nu} \langle \phi^* \rangle (X^0 W^0_\mu X^0 W^0_\nu + X^{+\alpha} W^{+\alpha}_\mu X^{-\alpha} W^{-\alpha}_\nu + X^{-\alpha} W^{-\alpha}_\mu X^{+\alpha} \\ &= g^2 v^2 g^{\mu\nu} \left(\tfrac{N-1}{2N} W^0_\mu W^0_\nu + \tfrac{1}{2} W^{+\alpha}_\mu W^{-\alpha}_\nu \right) . \end{aligned}$$

identical term arising from $\mathcal{L}_{A^2\overline{\phi}^2}$

gauge boson W^0_μ with mass $gv\sqrt{2(N-1)/N}$,

gauge bosons $W^{+\alpha}_\mu$ and $W^{-\alpha}_\mu$ with mass gv,

the massless gauge bosons A^a_μ of the unbroken $SU(N-1)$ gauge group.

all the particles fall into supermultiplets

The super Higgs mechanism

v=0

	$SU(N)$	$SU(F)$	$SU(F)$	b.d.o.f.
$\mathcal{Q}$	$\square$	$\square$	$\mathbf{1}$	$2NF$
$\overline{\mathcal{Q}}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$2NF$

for $v \neq 0$ we have massive states (in Unitary gauge):

	$SU(N-1)$	$SU(F-1)$	$SU(F-1)$	b.d.o.f.
W^0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	4
$W+$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$4(N-1)$
W^-	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$4(N-1)$

massive vector supermultiplet $(W^0_\mu,\, h^0,\, \Lambda^0,\, (1/\sqrt{2})(\omega^0 - \overline{\omega}^0))$

$$m_{W^0} = gv\sqrt{\frac{2(N-1)}{N}}\ ,$$

massive vector supermultiplets $(W^{+\alpha}_\mu, H^{+\alpha}, \Lambda^{+\alpha}, \omega^\alpha)$ and $(W^{-\alpha}_\mu, H^{-\alpha}, \Lambda^{-\alpha}, \overline{\omega}^{\alpha}$

$$m_{W^\pm} = gv.$$

The super Higgs mechanism

for $v \neq 0$ also have the massless states:

	$SU(N-1)$	$SU(F-1)$	$SU(F-1)$	b.d.o.f.
$\mathcal{Q}'$	$\square$	$\square$	$\mathbf{1}$	$2(N-1)(F-1)$
$\overline{\mathcal{Q}}'$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$2(N-1)(F-1)$
ψ	$\mathbf{1}$	$\square$	$\mathbf{1}$	$2(F-1)$
$\overline{\psi}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$2(F-1)$
S	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	2

quark chiral supermultiplet $\mathcal{Q}' = (\phi', Q')$
gauge singlets $\psi = (\sigma, \psi)$ and $S = (1/\sqrt{2})(h + \overline{h}), (1/\sqrt{2})(\omega^0 + \overline{\omega}^0)$

In both cases ($v = 0$ and $v \neq 0$) a total of $2(N^2 - 1) + 4FN$ b.d.o.f. (and, of course, the same number of fermionic degrees of freedom).