

# SUSY gauge theories

# SUSY QCD

Consider a SUSY  $SU(N)$  with  $F$  “flavors” of “quarks” and squarks

$$Q_i = (\phi_i, Q_i, \mathcal{F}_i), i = 1, \dots, F ,$$

where  $\phi$  is the squark and  $Q$  is the quark.

$$\overline{Q}_i = (\overline{\phi}_i, \overline{Q}_i, \overline{\mathcal{F}}_i) ,$$

in the antifundamental representation. Note the the bar ( $\overline{\quad}$ ) is part of the name not a conjugation, the conjugate fields are

$$Q_i^\dagger = (\phi_i^*, Q_i^\dagger, \mathcal{F}_i^*), \quad \overline{Q}_i^\dagger = (\overline{\phi}_i^*, \overline{Q}_i^\dagger, \overline{\mathcal{F}}_i^*).$$

# SUSY QCD

matter content is:

|           | $SU(N)$         | $SU(F)$      | $SU(F)$         | $U(1)_B$ | $U(1)_R$        |
|-----------|-----------------|--------------|-----------------|----------|-----------------|
| $Q$       | $\square$       | $\square$    | $\mathbf{1}$    | 1        | $\frac{F-N}{F}$ |
| $\bar{Q}$ | $\bar{\square}$ | $\mathbf{1}$ | $\bar{\square}$ | -1       | $\frac{F-N}{F}$ |

$$W = 0$$

# *R*-charge

$$[R, Q_\alpha] = -Q_\alpha.$$

chiral supermultiplet:

$$R_\psi = R_\phi - 1,$$

normalize the *R*-charge by

$$R\lambda^a = \lambda^a,$$

*R*-charge of the gluino is 1, and the *R*-charge of the gluon is 0.

# Group Theory: Bird Tracks

Identify the group generator with a vertex as in Fig. 1.

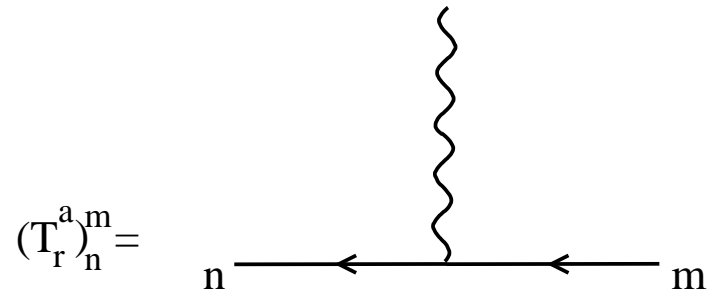


Figure 1: Bird-track notation for the group generator  $T^a$ .

# Bird Tracks

quadratic Casimir  $C_2(\mathbf{r})$  and the index  $T(\mathbf{r})$  of the representation  $\mathbf{r}$ ,

$$\begin{aligned} (T_{\mathbf{r}}^a)^m_l (T_{\mathbf{r}}^a)^l_n &= C_2(\mathbf{r}) \delta_n^m, \\ (T_{\mathbf{r}}^a)^m_n (T_{\mathbf{r}}^b)^n_m &= T(\mathbf{r}) \delta^{ab}, \end{aligned}$$

are given diagrammatically as

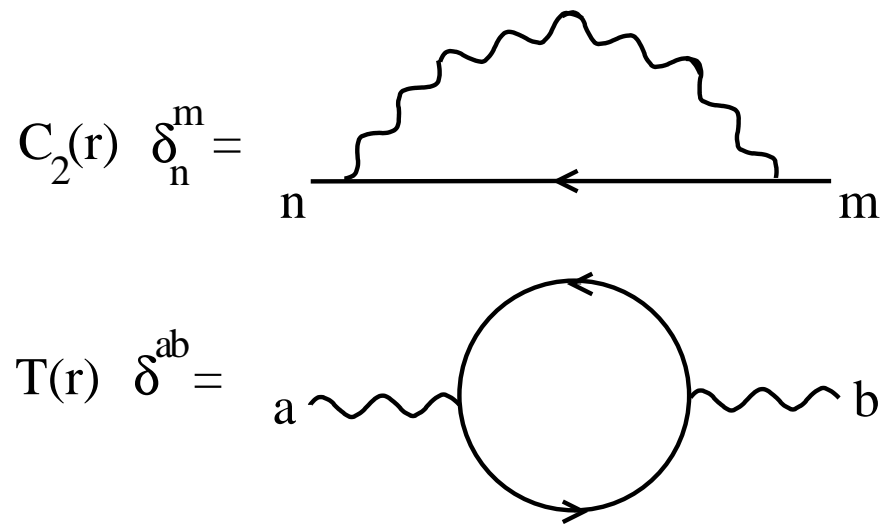


Figure 2:

# Bird Tracks

Contracting the external legs: In the first diagram setting  $m$  equal

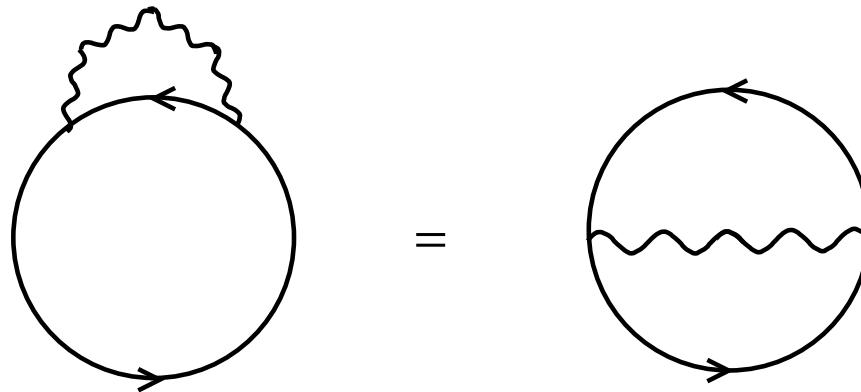


Figure 3:

to  $n$  and summing over  $n$  yields a factor of  $d(\mathbf{r})$ . In the second diagram setting  $a$  equal to  $b$  and summing yields a factor  $d(\mathbf{Ad})$ .

$$d(\mathbf{r})C_2(\mathbf{r}) = d(\mathbf{Ad})T(\mathbf{r}) .$$

# Casimirs

$$\begin{aligned}d(\square) &= N, & d(\mathbf{Ad}) &= N^2 - 1 \\T(\square) &= \frac{1}{2}, & T(\mathbf{Ad}) &= N\end{aligned}$$

so

$$C_2(\square) = \frac{N^2 - 1}{2N}, \quad C_2(\mathbf{Ad}) = N .$$

# Sum over Generators

For the fundamental representation  $\square$ :

$$(T^a)_p^l (T^a)_n^m = \frac{1}{2} (\delta_n^l \delta_p^m - \frac{1}{N} \delta_p^l \delta_n^m) .$$

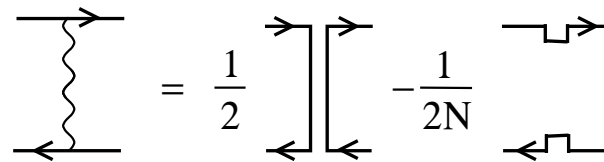


Figure 4:

We can reduce the sums over multiple generators to an essentially topological exercise

# Anomalies

Since we can define an  $R$ -charge by taking arbitrary linear combinations of the  $U(1)_R$  and  $U(1)_B$  charges we can choose  $Q_i$  and  $\bar{Q}_i$  to have the same  $R$ -charge. For a  $U(1)$  not to be broken by instanton effects the  $SU(N)^2U(1)_R$  anomaly diagram vanishes

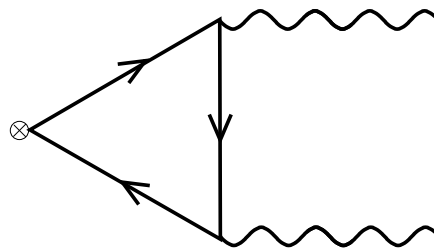


Figure 5:

fermion contributes its  $R$ -charge times  $T(r)$ . Sum over gluino, quarks:

$$1 \cdot T(\mathbf{Ad}) + (R - 1)T(\square) 2F = 0 ,$$

$$\text{so } R = \frac{F - N}{F}$$

# Renormalization group

tree-level SUSY:  $Y = \sqrt{2}g$ ,  $\lambda = g^2$ . For SUSY to be a consistent quantum symmetry these relations must be preserved under RG running. the  $\beta$  function for the gauge coupling at one-loop is

$$\beta_g = \mu \frac{dg}{d\mu} = -\frac{g^3}{16\pi^2} \left( \frac{11}{3}T(\text{Ad}) - \frac{2}{3}T(F) - \frac{1}{3}T(S) \right) \equiv -\frac{g^3 b}{16\pi^2} ,$$

For SUSY QCD:

$$b = (3N - F)$$

# Renormalization group

the  $\beta$  function for the Yukawa coupling is :

$$(4\pi)^2 \beta_Y^j = \frac{1}{2} \left[ Y_2^\dagger(F) Y^j + Y^j Y_2(F) \right] + 2Y^k Y^{j\dagger} Y^k + Y^k \text{Tr} Y^{k\dagger} Y^j - 3g_m^2 \{C_2^m(F), Y^j\} ,$$

where

$$Y_2(F) \equiv Y^{j\dagger} Y^j$$

$Y_2^\dagger(F) Y^j$  represents the scalar loop corrections to the fermion legs

$2Y^k Y^{j\dagger} Y^k$  contains the 1PI vertex corrections

$Y^k \text{Tr} Y^{k\dagger} Y^j$  represents fermion loop corrections to the scalar leg

$C_2^m(F)$  is the quadratic Casimir of the fermion fields in the  $m$ th gauge group, and represents gauge loop corrections to the fermion legs

# SUSY QCD RG

For SUSY QCD the Yukawa coupling of quark  $i$  with color index  $m$ , gluino  $a$ , and antiquark  $j$  with color index  $n$  is given by

$$Y_{im,a}^{jn} = \sqrt{2}g(T^a)_m^n \delta_i^j .$$

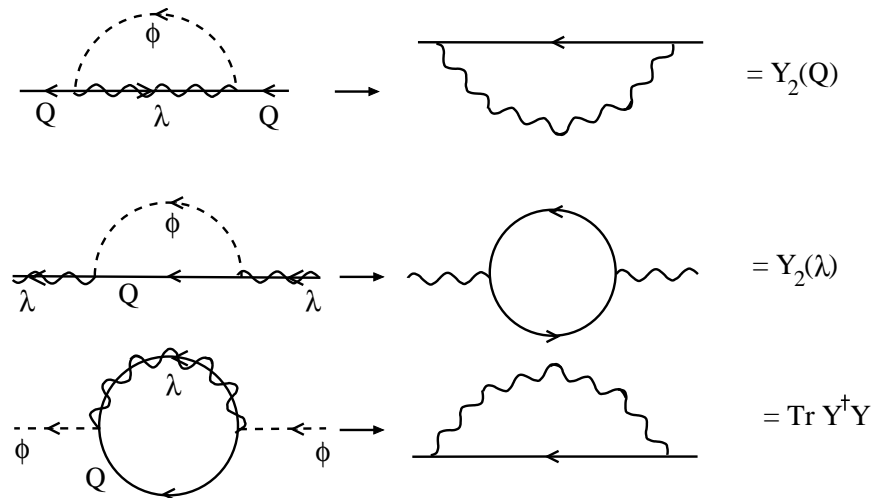


Figure 6: Feynman diagrams and associated bird-track diagrams.

$$Y_2(Q) = 2g^2 C_2(\square), \quad Y_2(\lambda) = 2g^2 2F T(\square)$$

# SUSY QCD RG

no scalar corrections corresponding to  $Y^k Y^{\dagger j} Y^k$ . As for the fermion loop correction it always has a quark (antiquark) and gluino for the internal lines so we have

$$Y^k \text{Tr} Y^{k\dagger} Y^j = Y_{im,a}^{kq} (Y_{fp,b}^{kq})^\dagger Y_{fp,b}^{jn} = 2g^2 C_2(\square) (T^a)_m^n \delta_i^j ,$$

gauge loop corrections are

$$\{C_2^m(F), Y^j\} = (C_2(\square) + C_2(\mathbf{Ad})) Y^j .$$

all the terms in  $\beta_Y^j$  proportional to  $C_2(\square)$  cancel:

$$\begin{aligned} (4\pi)^2 \beta_Y^j &= \sqrt{2} g^3 (C_2(\square) + F + 2C_2(\square) - 3C_2(\square) - 3N) \\ &= -\sqrt{2} g^3 (3N - F) \\ &= \sqrt{2} (4\pi)^2 \beta_g \end{aligned}$$

so the relation between the Yukawa and gauge couplings is preserved under RG running

# SUSY QCD Quartic RG

SUSY also requires the  $D$ -term quartic coupling  $\lambda = g^2$ . The auxiliary  $D^a$  field is given by

$$D^a = g(\phi^{*in}(T^a)_n^m \phi_{mi} - \bar{\phi}^{in}(T^a)_n^m \bar{\phi}_{mi}^*)$$

and the  $D$ -term potential is

$$V = \frac{1}{2} D^a D^a$$

The  $\beta$  function for a quartic scalar coupling at one-loop is

$$(4\pi)^2 \beta_\lambda = \Lambda^{(2)} - 4H + 3A + \Lambda^Y - 3\Lambda^S,$$

$\Lambda^{(2)}$  corresponds to the 1PI contribution from the quartic interactions

$H$  corresponds to the fermion box graphs

$A$  to the two gauge boson exchange graphs

$\Lambda^Y$  to the Yukawa leg corrections

$\Lambda^S$  corresponds to the gauge leg corrections

# SUSY QCD Quartic RG

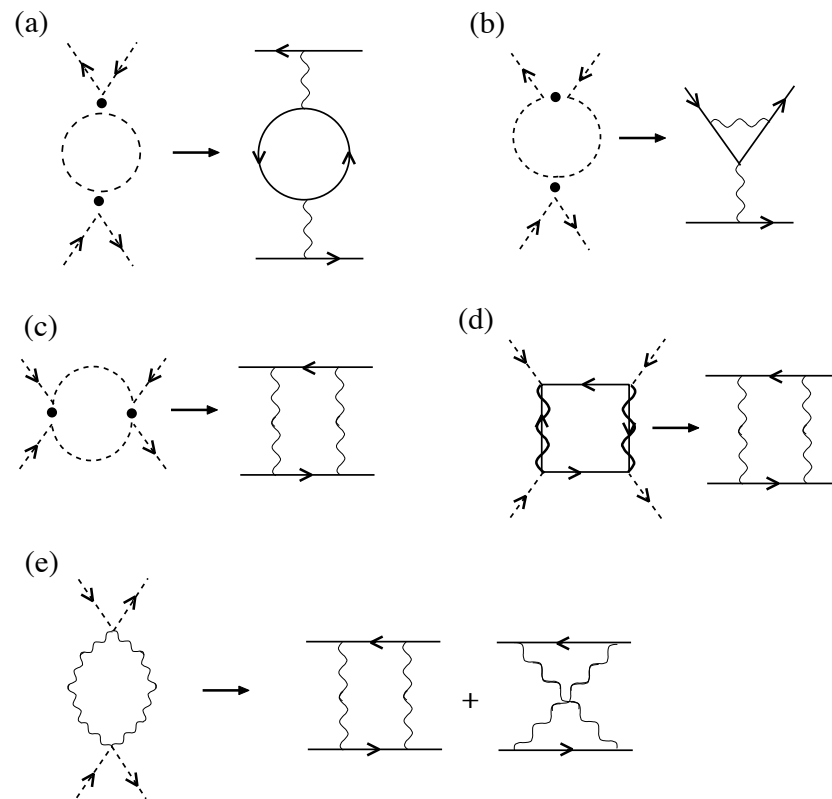


Figure 7:

# SUSY QCD Quartic RG

$$\begin{aligned}
 & \text{Diagram 1} = \frac{1}{4} \text{Diagram 2} - \frac{1}{4N} \text{Diagram 3} - \frac{1}{4N} \text{Diagram 4} + \frac{1}{4N^2} \text{Diagram 5} \\
 & = \frac{N^2-2}{2N} \text{Diagram 6} + \frac{N^2-1}{4N^2} \text{Diagram 7}
 \end{aligned}$$

Figure 8: The bird-track diagram for the sum over four generators quickly reduces to the sum over two generators and a product of identity matrices.

# SUSY QCD Quartic RG

$$(\phi^{*in}(T^a)_n^m \phi_{mi} - \bar{\phi}^{-in}(T^a)_n^m \bar{\phi}_{mi}^*)(\phi^{*jq}(T^a)_q^p \phi_{pj} - \bar{\phi}^{-jq}(T^a)_q^p \bar{\phi}_{pj}^*) ,$$

(with flavor indices  $i \neq j$ , the case  $i = j$  is left as an exercise) we have

$$\begin{aligned} \Lambda^{(2)} &= (2F + N - \frac{6}{N}) (T^a)_n^m (T^a)_q^p + (1 - \frac{1}{N^2}) \delta_n^m \delta_q^p , \\ -4H &= -8 (N - \frac{2}{N}) (T^a)_n^m (T^a)_q^p - 4 (1 - \frac{1}{N^2}) \delta_n^m \delta_q^p , \\ 3A &= 3 (N - \frac{4}{N}) (T^a)_n^m (T^a)_q^p + 3 (1 - \frac{1}{N^2}) \delta_n^m \delta_q^p , \\ \Lambda^Y &= 4 (N - \frac{1}{N}) (T^a)_n^m (T^a)_q^p , \\ -3\Lambda^S &= -6 (N - \frac{1}{N}) (T^a)_n^m (T^a)_q^p . \end{aligned}$$

individual diagrams that renormalize the gauge invariant, SUSY breaking, operator  $(\phi^{*mi} \phi_{mi})(\phi^{*pj} \phi_{pj})$  but the full  $\beta$  function for this operator vanishes and the  $D$ -term  $\beta$  function satisfies

$$\begin{aligned} \beta_\lambda &= \beta_{g^2} T^a T^a , \\ \beta_{g^2} &= 2g\beta_g . \end{aligned}$$

So SUSY is not anomalous at one-loop, and the  $\beta$  functions preserve the relations between couplings at all scales.

# SUSY RG

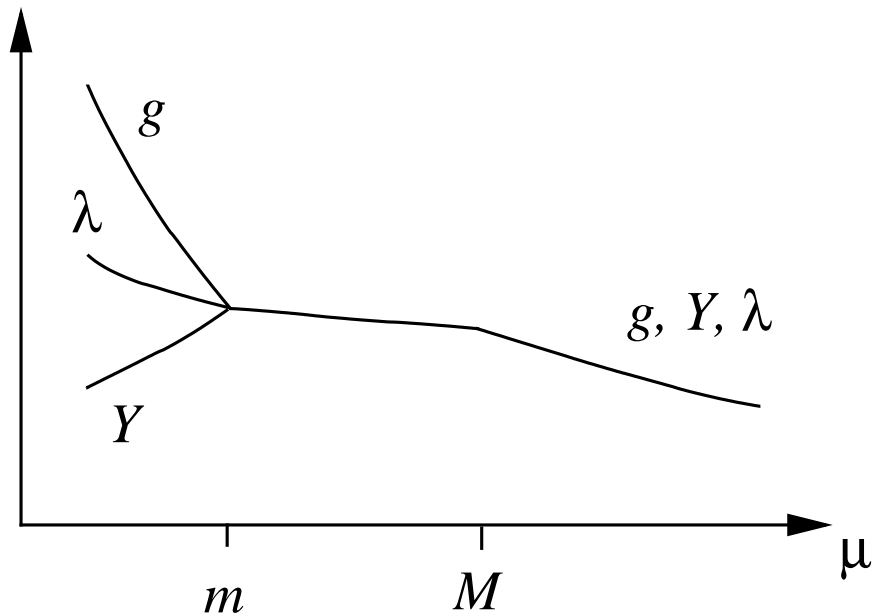


Figure 9: The couplings remain equal as we run below the SUSY threshold  $M$ , but split apart below the non-SUSY threshold  $m$ .

If we had added dimension 4 SUSY breaking terms to the theory then the couplings would have run differently at all scales

# one-loop squark mass

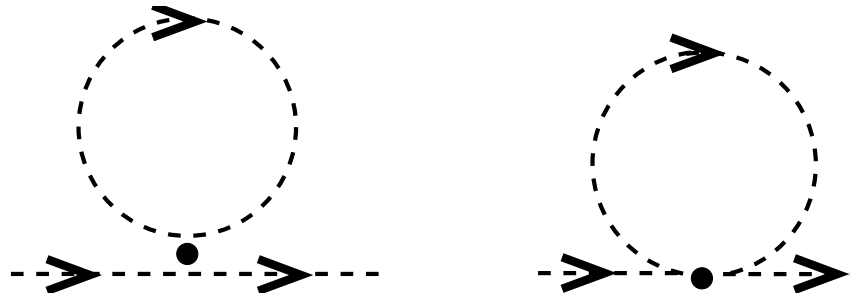


Figure 10: The squark loop correction to the squark mass.

$$\begin{aligned}\Sigma_{\text{squark}}(0) &= -ig^2 (T^a)_n^l (T^a)_l^m \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2} \\ &= \frac{-ig^2}{16\pi^2} C_2(\square) \delta_n^m \int_0^{\Lambda^2} dk^2 .\end{aligned}$$

# one-loop squark mass

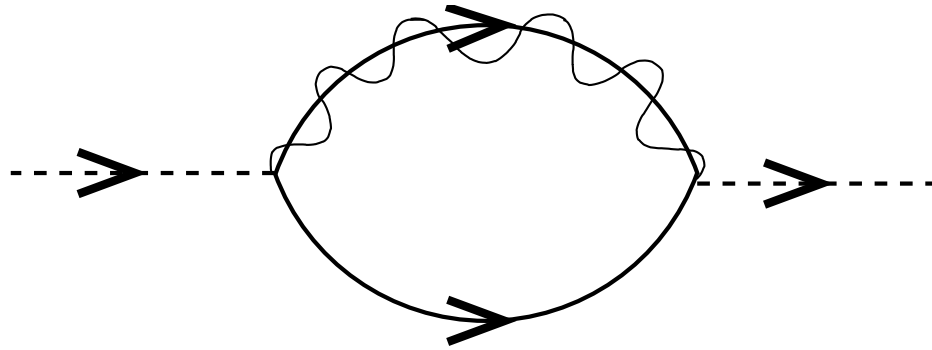


Figure 11: The quark–gluino loop correction to the squark mass.

$$\begin{aligned}
 \Sigma_{\text{quark–gluino}}(0) &= (-i\sqrt{2}g)^2 (T^a)_n^l (T^a)_l^m (-1) \int \frac{d^4k}{(2\pi)^4} \text{Tr} \frac{ik \cdot \sigma}{k^2} \frac{ik \cdot \bar{\sigma}}{k^2} \\
 &= -2g^2 C_2(\square) \delta_n^m \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{k^4} \\
 &= \frac{4ig^2}{16\pi^2} C_2(\square) \delta_n^m \int_0^{\Lambda^2} dk^2 .
 \end{aligned}$$

# one-loop squark mass

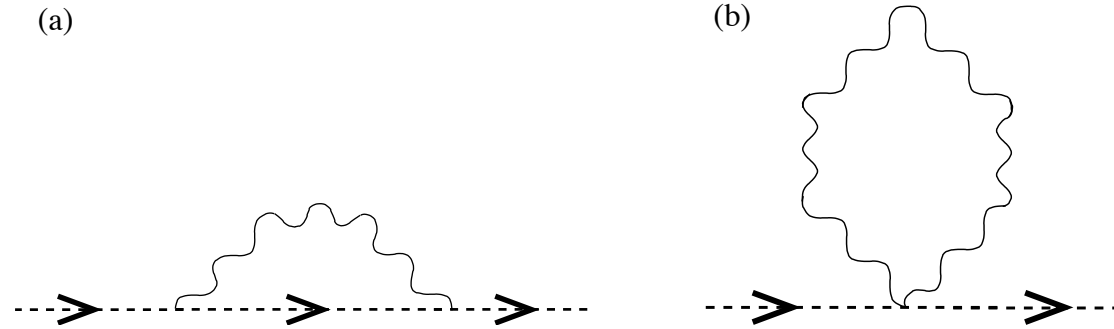


Figure 12: (a) The squark–gluon loop and (b) the gluon loop.

$$\begin{aligned}
 \Sigma_{\text{quark-gluino}}(0) &= (ig)^2 (T^a)_n^l (T^a)_l^m \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2} k^\mu (-i) \frac{(g_{\mu\nu} + (\xi-1) \frac{k_\mu k_\nu}{k^2})}{k^2} k^\nu \\
 &= \frac{\xi ig^2}{16\pi^2} C_2(\square) \delta_n^m \int_0^{\Lambda^2} dk^2,
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{\text{gluon}}(0) &= \frac{1}{2} ig^2 \{ (T^a)_n^l, (T^b)_l^m \} \delta^{ab} g^{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2} (-i) \frac{(g_{\mu\nu} + (\xi-1) \frac{k_\mu k_\nu}{k^2})}{k^2} \\
 &= \frac{-(3+\xi)ig^2}{16\pi^2} C_2(\square) \delta_n^m \int_0^{\Lambda^2} dk^2.
 \end{aligned}$$

# one-loop squark mass

Adding all the terms together we have

$$\Sigma(0) = (-1 + 4 + \xi - (3 + \xi)) \frac{ig^2}{16\pi^2} C_2(\square) \delta_n^m \int_0^{\Lambda^2} dk^2 = 0 .$$

The quadratic divergence in the squark mass cancels! In fact for a massless squark all the mass corrections cancel. This means that in a SUSY theory with a Higgs the Higgs mass is protected from quadratic divergences from gauge interactions as well as from Yukawa interactions

# Flat directions $F < N$

$$D^a = g(\phi^{*in} (T^a)_n^m \phi_{mi} - \bar{\phi}^{in} (T^a)_n^m \bar{\phi}_{mi}^*)$$

and the scalar potential is:

$$V = \frac{1}{2} D^a D^a$$

$$\begin{aligned} \text{define } d_m^n &\equiv \langle \phi^{*in} \phi_{mi} \rangle \\ \bar{d}_m^n &= \langle \bar{\phi}^{in} \bar{\phi}_{mi}^* \rangle \end{aligned}$$

maximal rank  $F$ . In a SUSY vacua:

$$D^a = T_n^{am} (d_m^n - \bar{d}_m^n) = 0$$

Since  $T^a$  is a complete basis for traceless matrices, we must therefore have that the difference of the two matrices is proportional to the identity matrix:

$$d_m^n - \bar{d}_m^n = \alpha I$$



## Flat directions $F < N$

$d_m^n$  and  $\bar{d}_m^n$  are invariant under  $SU(F) \times SU(F)$  transformations since

$$\begin{aligned} \phi_{mi} &\rightarrow \phi_{mi} V_j^i, \\ d_m^n &\rightarrow V_i^{*j} \langle \phi^{*in} \rangle \langle \phi_{mi} \rangle V_j^i, \\ &\rightarrow \langle \phi^{*jn} \phi_{mj} \rangle = d_m^n. \end{aligned}$$

Thus, up to a flavor transformation, we can write

$$\langle \bar{\phi}^* \rangle = \langle \phi \rangle = \begin{pmatrix} v_1 & & & \\ & \ddots & & \\ & & v_F & \\ 0 & \dots & 0 & \\ \vdots & & \vdots & \\ 0 & \dots & 0 & \end{pmatrix}.$$

$D$ -term potential has flat directions, as we change the VEVs, we move between different vacua with different particle spectra, generically  $SU(N - F)$  gauge symmetry

## Flat directions $F \geq N$

$d_m^n$  and  $\bar{d}_m^n$  are  $N \times N$  positive semi-definite Hermitian matrices of maximal rank  $N$  in a SUSY vacuum :

$$d_m^n - \bar{d}_m^n = \rho I .$$

$d_m^n$  can be diagonalized by an  $SU(N)$  gauge transformation:

$$d = \begin{pmatrix} |v_1|^2 & & & \\ & |v_2|^2 & & \\ & & \ddots & \\ & & & |v_N|^2 \end{pmatrix}$$

In this basis,  $\bar{d}_m^n$  must also be diagonal, with eigenvalues  $|\bar{v}_i|^2$ , so

$$|v_i|^2 = |\bar{v}_i|^2 + \rho .$$

## Flat directions $F \geq N$

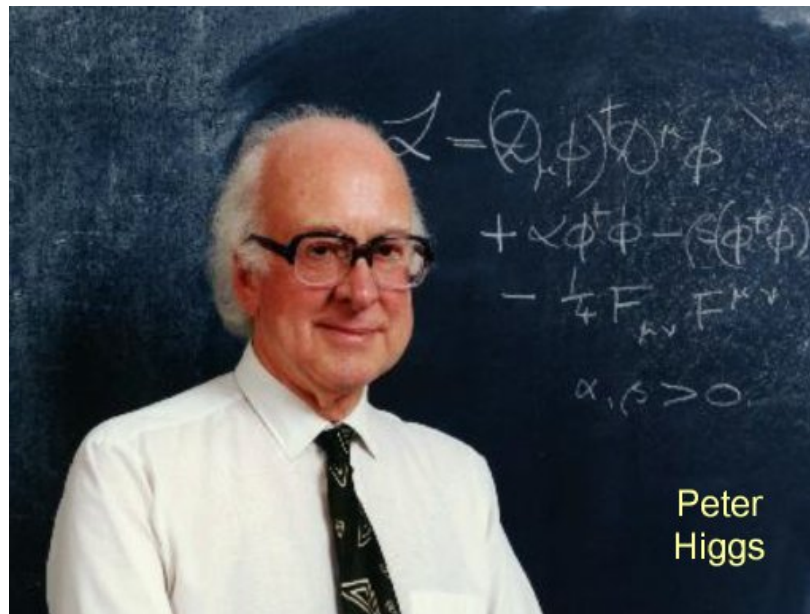
Since  $d_m^n$  and  $\bar{d}_m^n$  are invariant under flavor transformations, we can use  $SU(F) \times SU(F)$  transformations to put  $\langle \phi \rangle$  and  $\langle \bar{\phi} \rangle$  in the form

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & 0 & \dots & 0 \\ & \ddots & \vdots & & \vdots \\ & & v_N & 0 & \dots & 0 \end{pmatrix}, \quad \langle \bar{\Phi} \rangle = \begin{pmatrix} \bar{v}_1 & & & & \\ & \ddots & & & \\ & & & & \bar{v}_N \\ 0 & \dots & 0 & & \\ \vdots & & \vdots & & \\ 0 & \dots & 0 & & \end{pmatrix}.$$

Again we have a space of degenerate vacua. At a generic point in the moduli space the  $SU(N)$  gauge symmetry is completely broken.

# The super Higgs mechanism

a massless vector supermultiplet eats a chiral supermultiplet to form a massive vector supermultiplet



# The super Higgs mechanism

Consider the case when  $v_1 = \bar{v}_1 = v$  and  $v_i = \bar{v}_i = 0$ , for  $i > 1$   
 $SU(N) \rightarrow SU(N-1)$  and  $SU(F) \times SU(F) \rightarrow SU(F-1) \times SU(F-1)$ .  
The number of broken gauge generators is

$$N^2 - 1 - ((N-1)^2 - 1) = 2(N-1) + 1 ,$$

decompose the adjoint of  $SU(N)$  under  $SU(N-1)$ , we have

$$\mathbf{Ad}_N = \mathbf{1} + \square + \bar{\square} + \mathbf{Ad}_{N-1}$$

convenient basis of gauge generators is  $G^A = X^0, X_1^\alpha, X_2^\alpha, T^a$  where  
 $A = 1, \dots, N^2 - 1$ ,  $\alpha = 1, \dots, N - 1$ , and  $a = 1, \dots, (N-1)^2 - 1$ .  
 $X$ s are the broken generators (span the coset of  $SU(N)/SU(N-1)$ ),  
 $T$ s are the unbroken  $SU(N-1)$  generators

# The super Higgs mechanism

The  $X$ s are analogs of the Pauli matrices:

$$X^0 = \frac{1}{\sqrt{2(N^2-N)}} \begin{pmatrix} N-1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & \ddots & & & \\ & & & & -1 & & \\ & & & & & & \\ & & & & & & -1 \end{pmatrix},$$

$$X_1^\alpha = \frac{1}{2} \begin{pmatrix} 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ 0 & & & & & & \\ \vdots & & & & & & \\ 0 & & & & & & \\ 1 & & & \mathbf{0} & & & \\ 0 & & & & & & \\ \vdots & & & & & & \\ 0 & & & & & & \end{pmatrix}, \quad X_2^\alpha = \frac{1}{2} \begin{pmatrix} 0 & \dots & 0 & i & 0 & \dots \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & & & & \\ -i & & & \mathbf{0} & & \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & & & & \end{pmatrix}$$



# The super Higgs mechanism

We can then write the sum of the product of two generators as:

$$G^A G^A = X^0 X^0 + X^{+\alpha} X^{-\alpha} + X^{-\alpha} X^{+\alpha} + T^a T^a$$

Expanding the squark field around its VEV  $\langle\phi\rangle$

$$\phi \rightarrow \langle\phi\rangle + \phi ,$$

we have

$$\begin{aligned} \sum_A G^A \langle\phi\rangle &= X^0 \langle\phi\rangle + \sum_\alpha X^{-\alpha} \langle\phi\rangle , \\ \langle\phi\rangle \sum_A G^A &= \langle\phi\rangle X^0 + \langle\phi\rangle \sum_\alpha X^{+\alpha} , \end{aligned}$$

since  $T^a$  annihilates  $\langle\phi\rangle$ . label the components of the gluino field as

$$G^A \lambda^A = X^0 \Lambda^0 + X^{+\alpha} \Lambda^{+\alpha} + X^{-\alpha} \Lambda^{-\alpha} + T^a \lambda^a ,$$

# The super Higgs mechanism

write the quark field as

$$Q = \begin{pmatrix} \omega^0 & \psi_i \\ \omega_\alpha & Q'_{mi} \end{pmatrix}, \quad \bar{Q} = \begin{pmatrix} \bar{\omega}^0 & \bar{\omega}^\alpha \\ \bar{\psi}^i & \bar{Q}'^{im} \end{pmatrix},$$

where  $i$  is a flavor index,  $\alpha$  and  $m$  are color indices,  $Q'$  is a matrix with  $N - 1$  rows and  $F - 1$  columns, and  $\bar{Q}$  is a matrix with  $F - 1$  rows and  $N - 1$  columns.

fermion mass terms generated by the Yukawa interactions:

$$\begin{aligned} \mathcal{L}_{\text{F mass}} &= -\sqrt{2}g \left[ (\langle \phi^* \rangle X^0 \Lambda^0 + \langle \phi^* \rangle X^{+\alpha} \Lambda^{+\alpha}) Q \right. \\ &\quad \left. - \bar{Q} \left( X^0 \Lambda^0 \langle \bar{\phi}^* \rangle + X^{-\alpha} \Lambda^{-\alpha} \langle \bar{\phi}^* \rangle \right) + h.c. \right] \\ &= -gv \left[ \sqrt{\frac{N-1}{N}} (\omega^0 \Lambda^0 - \bar{\omega}^0 \Lambda^0) + \omega^\alpha \Lambda^{+\alpha} - \bar{\omega}^\alpha \Lambda^{-\alpha} + h.c. \right]. \end{aligned}$$

So we have a Dirac fermion  $(\Lambda^0, (1/\sqrt{2})(\omega^0 - \bar{\omega}^0))$  with mass  $gv\sqrt{2(N-1)/N}$ , two sets of  $N - 1$  Dirac fermions  $(\Lambda^{+\alpha}, \omega^\alpha)$ ,  $(\Lambda^{-\alpha}, -\bar{\omega}^\alpha)$  with mass  $gv$ , and massless Weyl fermions  $Q'$ ,  $\bar{Q}'$ ,  $\psi$ ,  $\bar{\psi}$ , and  $(1/\sqrt{2})(\omega^0 + \bar{\omega}^0)$ .

# The super Higgs mechanism

decompose the squark field as

$$\phi = \begin{pmatrix} h & \sigma_i \\ H_\alpha & \phi'_{mi} \end{pmatrix}, \quad \bar{\phi} = \begin{pmatrix} \bar{h} & \bar{H}^\alpha \\ \bar{\sigma}^i & \bar{\phi}'^{im} \end{pmatrix},$$

where  $\phi'$  is a matrix with  $N - 1$  rows and  $F - 1$  columns. Shifting the scalar field by its VEV so that  $\phi \rightarrow \langle \phi \rangle + \phi$  we have that the auxiliary  $D^A$  field is given by

$$\begin{aligned} \frac{D^A}{g} &= \langle \phi^* \rangle G^A \langle \phi \rangle - \langle \bar{\phi} \rangle G^A \langle \bar{\phi}^* \rangle + \langle \phi^* \rangle G^A \phi - \langle \bar{\phi} \rangle G^A \bar{\phi}^* \\ &+ \phi^* G^A \langle \phi \rangle - \bar{\phi} G^A \langle \bar{\phi}^* \rangle + \phi^* G^A \phi - \bar{\phi} G^A \bar{\phi}^* . \end{aligned}$$

# The super Higgs mechanism

picking out the mass terms in the scalar potential  $V = \frac{1}{2}D^A D^A$  :

$$\begin{aligned}
 V_{\text{mass}} &= \frac{g^2}{2} \left[ \left( \langle \phi^* \rangle X^0 \phi + \phi^* X^0 \langle \phi \rangle - \langle \bar{\phi} \rangle X^0 \bar{\phi}^* - \bar{\phi} X^0 \langle \bar{\phi}^* \rangle \right)^2 \right. \\
 &\quad \left. + 2(\langle \phi^* \rangle X^{+\alpha} \phi - \langle \bar{\phi} \rangle X^{+\alpha} \bar{\phi}^*) (\phi^* X^{-\alpha} \langle \phi \rangle - \bar{\phi} X^{-\alpha} \langle \bar{\phi}^* \rangle) \right] \\
 &= \frac{g^2 v^2}{2} \left[ \frac{(N-1)^2}{2(N^2-N)} \left( h + h^* - (\bar{h}^* + \bar{h}) \right)^2 \right. \\
 &\quad \left. + (H^\alpha - \bar{H}^{*\alpha})(H^{*\alpha} - \bar{H}^\alpha) \right].
 \end{aligned}$$

diagonalize the mass matrix:

$$\begin{aligned}
 H^{+\alpha} &= \frac{1}{\sqrt{2}} (H^\alpha - \bar{H}^{*\alpha}), & \pi^{+\alpha} &= \frac{1}{\sqrt{2}} (H^\alpha + \bar{H}^{*\alpha}), \\
 H^{-\alpha} &= \frac{1}{\sqrt{2}} (H^{*\alpha} - \bar{H}^\alpha), & \pi^{-\alpha} &= \frac{1}{\sqrt{2}} (H^{*\alpha} + \bar{H}^\alpha), \\
 h^0 &= \text{Re}(h - \bar{h}), & \pi^0 &= \text{Im}(h - \bar{h}), \\
 \Omega &= \frac{1}{\sqrt{2}} (h + \bar{h}).
 \end{aligned}$$

# The super Higgs mechanism

mass terms reduce to

$$V_{\text{mass}} = g^2 v^2 \left[ \frac{N-1}{N} (h^0)^2 + H^{+\alpha} H^{-\alpha} \right].$$

real scalar  $h^0$  with mass  $gv \sqrt{2(N-1)/N}$ ,  
a complex scalar  $H^{+\alpha}$  (and its conjugate  $H^{-\alpha}$ ) with mass  $gv$ ,  
massless complex scalars  $\sigma_i, \bar{\sigma}^i$ , and  $\Omega$ .

$\pi$ s become the longitudinal components of the massive gauge bosons,  
can be removed by going to Unitary gauge

# The super Higgs mechanism

We can write the gauge fields as:

$$G^B A_\mu^B = X^0 W_\mu^0 + X^{+\alpha} W_\mu^{+\alpha} + X^{-\alpha} W_\mu^{-\alpha} + T^a A_\mu^a .$$

Then the  $A^2 \phi^2$  terms which lead to gauge boson masses are

$$\begin{aligned} \mathcal{L}_{A^2 \phi^2} &= g^2 A_\mu^A A_\nu^B g^{\mu\nu} \langle \phi^* \rangle G^A G^B \langle \phi \rangle \\ &= g^2 g^{\mu\nu} \langle \phi^* \rangle (X^0 W_\mu^0 X^0 W_\nu^0 + X^{+\alpha} W_\mu^{+\alpha} X^{-\alpha} W_\nu^{-\alpha} + X^{-\alpha} W_\mu^{-\alpha} X^{+\alpha} W_\nu^{+\alpha}) \\ &= g^2 v^2 g^{\mu\nu} \left( \frac{N-1}{2N} W_\mu^0 W_\nu^0 + \frac{1}{2} W_\mu^{+\alpha} W_\nu^{-\alpha} \right) . \end{aligned}$$

identical term arising from  $\mathcal{L}_{A^2 \bar{\phi}^2}$

gauge boson  $W_\mu^0$  with mass  $gv \sqrt{2(N-1)/N}$ ,

gauge bosons  $W_\mu^{+\alpha}$  and  $W_\mu^{-\alpha}$  with mass  $gv$ ,

the massless gauge bosons  $A_\mu^a$  of the unbroken  $SU(N-1)$  gauge group.

all the particles fall into supermultiplets

# The super Higgs mechanism

|       |                | $SU(N)$              | $SU(F)$      | $SU(F)$              | b.d.o.f. |
|-------|----------------|----------------------|--------------|----------------------|----------|
| $v=0$ | $Q$            | $\square$            | $\square$    | $\mathbf{1}$         | $2NF$    |
|       | $\overline{Q}$ | $\overline{\square}$ | $\mathbf{1}$ | $\overline{\square}$ | $2NF$    |

for  $v \neq 0$  we have massive states (in Unitary gauge):

|       | $SU(N-1)$            | $SU(F-1)$    | $SU(F-1)$    | b.d.o.f. |
|-------|----------------------|--------------|--------------|----------|
| $W^0$ | $\mathbf{1}$         | $\mathbf{1}$ | $\mathbf{1}$ | 4        |
| $W_+$ | $\square$            | $\mathbf{1}$ | $\mathbf{1}$ | $4(N-1)$ |
| $W_-$ | $\overline{\square}$ | $\mathbf{1}$ | $\mathbf{1}$ | $4(N-1)$ |

massive vector supermultiplet  $(W_\mu^0, h^0, \Lambda^0, (1/\sqrt{2})(\omega^0 - \overline{\omega}^0))$

$$m_{W^0} = gv \sqrt{\frac{2(N-1)}{N}},$$

massive vector supermultiplets  $(W_\mu^{+\alpha}, H^{+\alpha}, \Lambda^{+\alpha}, \omega^\alpha)$  and  $(W_\mu^{-\alpha}, H^{-\alpha}, \Lambda^{-\alpha}, \overline{\omega}^\alpha)$

$$m_{W^\pm} = gv.$$

# The super Higgs mechanism

for  $v \neq 0$  also have the massless states:

|                   | $SU(N - 1)$          | $SU(F - 1)$  | $SU(F - 1)$          | b.d.o.f.          |
|-------------------|----------------------|--------------|----------------------|-------------------|
| $Q'$              | $\square$            | $\square$    | $\mathbf{1}$         | $2(N - 1)(F - 1)$ |
| $\overline{Q}'$   | $\overline{\square}$ | $\mathbf{1}$ | $\overline{\square}$ | $2(N - 1)(F - 1)$ |
| $\psi$            | $\mathbf{1}$         | $\square$    | $\mathbf{1}$         | $2(F - 1)$        |
| $\overline{\psi}$ | $\mathbf{1}$         | $\mathbf{1}$ | $\overline{\square}$ | $2(F - 1)$        |
| $S$               | $\mathbf{1}$         | $\mathbf{1}$ | $\mathbf{1}$         | 2                 |

quark chiral supermultiplet  $Q' = (\phi', Q')$

gauge singlets  $\psi = (\sigma, \psi)$  and  $S = (1/\sqrt{2})(h + \bar{h}), (1/\sqrt{2})(\omega^0 + \bar{\omega}^0)$

In both cases ( $v = 0$  and  $v \neq 0$ ) a total of  $2(N^2 - 1) + 4FN$  b.d.o.f. (and, of course, the same number of fermionic degrees of freedom).