

Introduction to Supersymmetry

Unreasonable effectiveness of the SM

$$\mathcal{L}_{\text{Yukawa}} = -\frac{y_t}{\sqrt{2}} H^0 \bar{t}_L t_R + h.c.$$

$$H^0 = \langle H^0 \rangle + h^0 = v + h^0$$

$$m_t = \frac{y_t v}{\sqrt{2}}$$

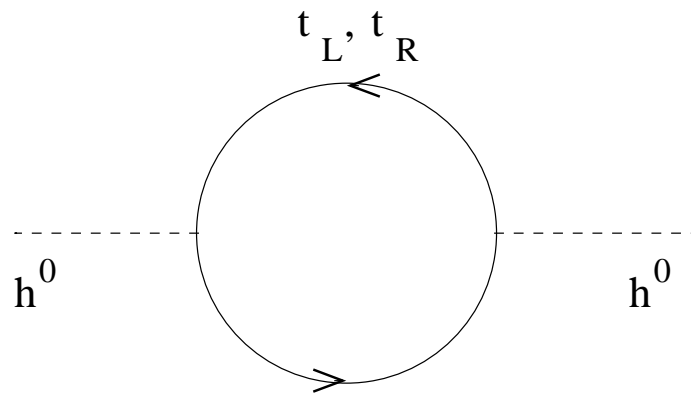


Figure 1: The top loop contribution to the Higgs mass term.

$$\begin{aligned}
-i\delta m_h^2|_{\text{top}} &= (-1)N_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[\frac{-iy_t}{\sqrt{2}} \frac{i}{\not{k}-m_t} \left(\frac{-iy_t^*}{\sqrt{2}} \right) \frac{i}{\not{k}-m_t} \right] \\
&= -2N_c |y_t|^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k^2 + m_t^2}{(k^2 - m_t^2)^2}
\end{aligned}$$

$$k_0 \rightarrow ik_4, \quad k^2 \rightarrow -k_E^2$$

$$-i\delta m_h^2|_{\text{top}} = \frac{iN_c |y_t|^2}{8\pi^2} \int_0^{\Lambda^2} dk_E^2 \frac{k_E^2 (k_E^2 - m_t^2)}{(k_E^2 + m_t^2)^2}$$

$$x = k_E^2 + m_t^2$$

$$\begin{aligned}
\delta m_h^2|_{\text{top}} &= -\frac{N_c |y_t|^2}{8\pi^2} \int_{m_t^2}^{\Lambda^2} dx \left(1 - \frac{3m_t^2}{x} + \frac{2m_t^4}{x^2} \right) \\
&= -\frac{N_c |y_t|^2}{8\pi^2} \left[\Lambda^2 - 3m_t^2 \ln \left(\frac{\Lambda^2 + m_t^2}{m_t^2} \right) + \dots \right]
\end{aligned}$$

$$\mathcal{L}_{\text{scalar}} = -\frac{\lambda}{2}(h^0)^2(|\phi_L|^2 + |\phi_R|^2) - h^0(\mu_L|\phi_L|^2 + \mu_R|\phi_R|^2) - m_L^2|\phi_L|^2 - m_R^2|\phi_R|^2$$

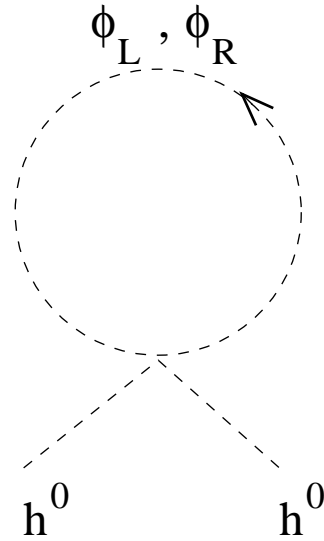


Figure 2: Scalar boson contribution to the Higgs mass term via the quartic coupling.

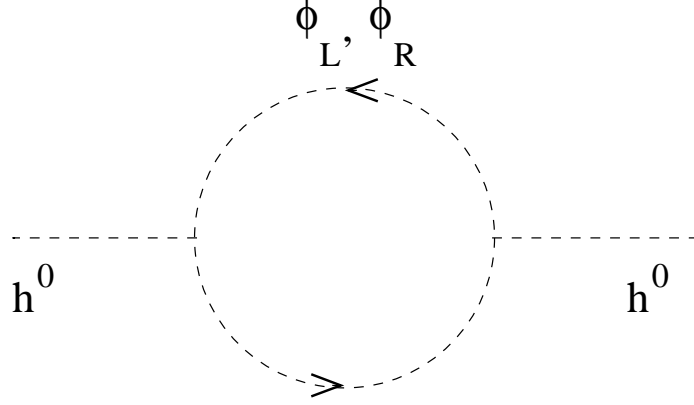


Figure 3: Scalar boson contribution to the Higgs mass term via the trilinear coupling.

$$\begin{aligned}
 -i\delta m_h^2|_2 &= -i\lambda N \int \frac{d^4 k}{(2\pi)^4} \left[\frac{i}{k^2 - m_L^2} + \frac{i}{k^2 - m_R^2} \right] \\
 \delta m_h^2|_2 &= \frac{\lambda N}{16\pi^2} \left[2\Lambda^2 - m_L^2 \ln \left(\frac{\Lambda^2 + m_L^2}{m_L^2} \right) - m_R^2 \ln \left(\frac{\Lambda^2 + m_R^2}{m_R^2} \right) + \dots \right]. \\
 -i\delta m_h^2|_3 &= N \int \frac{d^4 k}{(2\pi)^4} \left[\left(-i\mu_L \frac{i}{k^2 - m_L^2} \right)^2 + \left(-i\mu_R \frac{i}{k^2 - m_R^2} \right)^2 \right] \\
 \delta m_h^2|_3 &= -\frac{N}{16\pi^2} \left[\mu_L^2 \ln \left(\frac{\Lambda^2 + m_L^2}{m_L^2} \right) + \mu_R^2 \ln \left(\frac{\Lambda^2 + m_R^2}{m_R^2} \right) + \dots \right].
 \end{aligned}$$

If $N = N_c$ and $\lambda = |y_t|^2$ then Λ^2 cancels

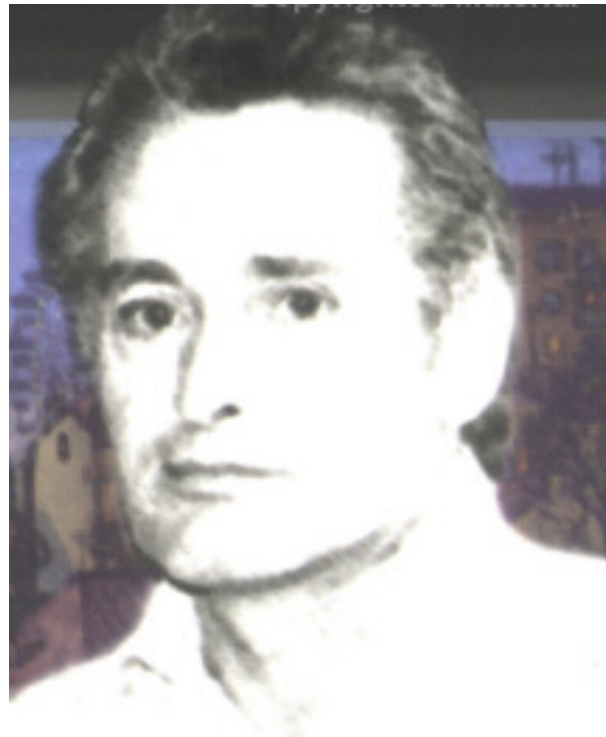
If $m_t = m_L = m_R$ and $\mu_L^2 = \mu_R^2 = 2\lambda m_t^2 \log \Lambda$ are canceled as well

SUSY will guarantee these relations

Coleman-Mandula



Golfand-Lichtman



Haag-Lopuszanski-Sohnius



SUSY algebra

$$\{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu,$$

$$\sigma_{\alpha\dot{\alpha}}^\mu = (1, \sigma^i) \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} = (1, -\sigma^i)$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[P_\mu, Q_\alpha] = [P_\mu, Q_{\dot{\alpha}}^\dagger] = 0$$

$$[Q_\alpha, R] = Q_\alpha \quad [Q_{\dot{\alpha}}^\dagger, R] = -Q_{\dot{\alpha}}^\dagger$$

$$H = P^0 = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2)$$

$$\begin{aligned}
(-1)^{\mathbf{F}} |\text{boson}\rangle &= +1 |\text{boson}\rangle \\
(-1)^{\mathbf{F}} |\text{fermion}\rangle &= -1 |\text{fermion}\rangle
\end{aligned}$$

$$\{(-1)^{\mathbf{F}}, Q_\alpha\} = 0$$

$$\sum_i |i\rangle\langle i| = 1$$

so

$$\begin{aligned}
\sum_i \langle i|(-1)^{\mathbf{F}} P^0|i\rangle &= \frac{1}{4} \left(\sum_i \langle i|(-1)^{\mathbf{F}} Q Q^\dagger|i\rangle + \sum_i \langle i|(-1)^{\mathbf{F}} Q^\dagger Q|i\rangle \right) \\
&= \frac{1}{4} \left(\sum_i \langle i|(-1)^{\mathbf{F}} Q Q^\dagger|i\rangle + \sum_{ij} \langle i|(-1)^{\mathbf{F}} Q^\dagger|j\rangle \langle j|Q|i\rangle \right) \\
&= \frac{1}{4} \left(\sum_i \langle i|(-1)^{\mathbf{F}} Q Q^\dagger|i\rangle + \sum_{ij} \langle j|Q|i\rangle \langle i|(-1)^{\mathbf{F}} Q^\dagger|j\rangle \right) \\
&= \frac{1}{4} \left(\sum_i \langle i|(-1)^{\mathbf{F}} Q Q^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{\mathbf{F}} Q^\dagger|j\rangle \right) \\
&= \frac{1}{4} \left(\sum_i \langle i|(-1)^{\mathbf{F}} Q Q^\dagger|i\rangle - \sum_j \langle j|(-1)^{\mathbf{F}} Q Q^\dagger|j\rangle \right) \\
&= 0.
\end{aligned}$$

SUSY:

$$Q_\alpha|0\rangle = 0$$

implies that the vacuum energy vanishes

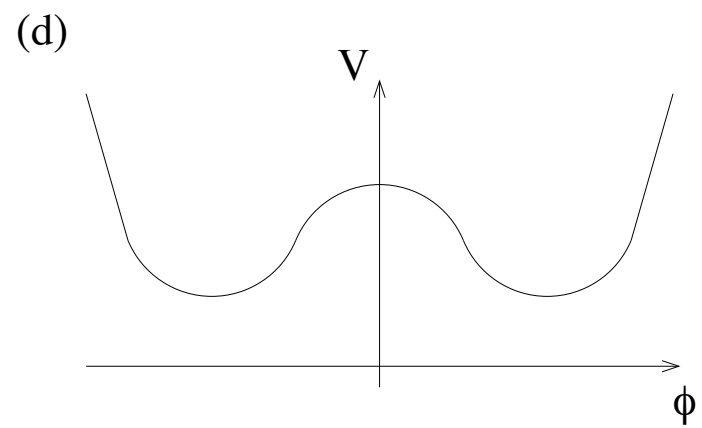
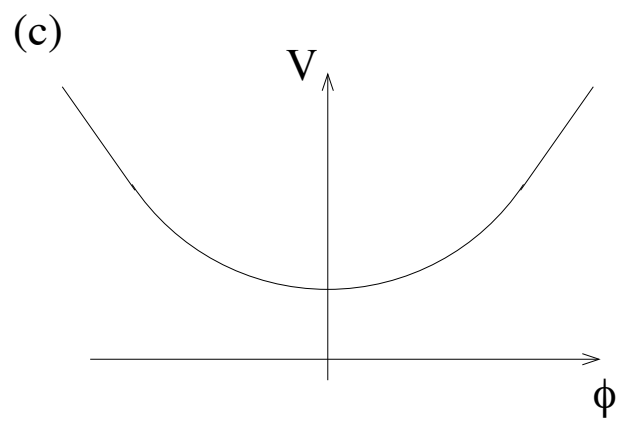
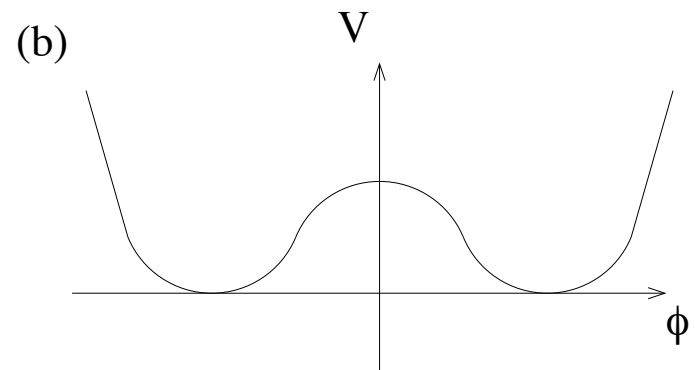
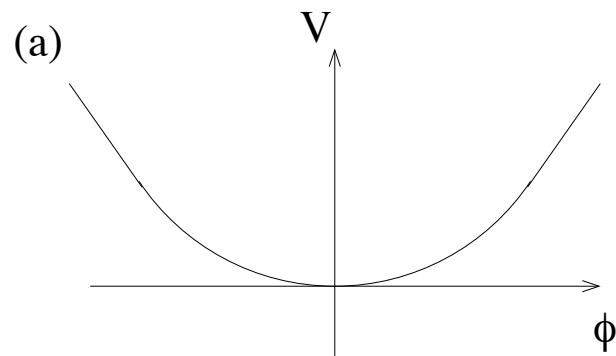
$$\langle 0|H|0\rangle = 0$$

SUSY breaking:

$$Q_\alpha|0\rangle \neq 0$$

and the vacuum energy is positive

$$\langle 0|H|0\rangle \neq 0$$



SUSY representations

massive particle rest frame: $p_\mu = (m, \vec{0})$.

$$\begin{aligned}\{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} &= 2m \delta_{\alpha\dot{\alpha}} \\ \{Q_\alpha, Q_\beta\} &= 0 \\ \{Q_{\dot{\alpha}}^\dagger, Q_{\dot{\beta}}^\dagger\} &= 0\end{aligned}$$

Clifford vacuum:

$$\begin{aligned}|\Omega_s\rangle &= Q_1 Q_2 |m, s', s'_3\rangle, \\ Q_1 |\Omega_s\rangle &= Q_2 |\Omega_s\rangle = 0\end{aligned}$$

massive multiplet:

$$\begin{aligned}&|\Omega_s\rangle \\ &Q_1^\dagger |\Omega_s\rangle, Q_2^\dagger |\Omega_s\rangle \\ &Q_1^\dagger Q_2^\dagger |\Omega_s\rangle\end{aligned}$$

massive “chiral” multiplet:

state	s_3
$ \Omega_0\rangle$	0
$Q_1^\dagger \Omega_0\rangle, Q_2^\dagger \Omega_0\rangle$	$\pm\frac{1}{2}$
$Q_1^\dagger Q_2^\dagger \Omega_0\rangle$	0

massive vector multiplet:

state	s_3
$ \Omega_{\frac{1}{2}}\rangle$	$\pm\frac{1}{2}$
$Q_1^\dagger \Omega_{\frac{1}{2}}\rangle, Q_2^\dagger \Omega_{\frac{1}{2}}\rangle$	0, 1, 0, -1
$Q_1^\dagger Q_2^\dagger \Omega_{\frac{1}{2}}\rangle$	$\pm\frac{1}{2}$

Massless particles

frame: $p_\mu = (E, 0, 0, -E)$

$$\begin{aligned}\{Q_1, Q_1^\dagger\} &= 4E \\ \{Q_2, Q_2^\dagger\} &= 0 \\ \{Q_\alpha, Q_\beta\} &= 0 \\ \{Q_{\dot{\alpha}}, Q_{\dot{\beta}}\} &= 0\end{aligned}$$

Clifford vacuum:

$$\begin{aligned}|\Omega_\lambda\rangle &= Q_1|E, \lambda'\rangle, \\ Q_1|\Omega_\lambda\rangle &= 0\end{aligned}$$

$$\langle\Omega_\lambda|Q_2Q_2^\dagger|\Omega_\lambda\rangle + \langle\Omega_\lambda|Q_2^\dagger Q_2|\Omega_\lambda\rangle = 0$$

$$\langle\Omega_\lambda|Q_2Q_2^\dagger|\Omega_\lambda\rangle = 0$$

massless supermultiplet

state	helicity
$ \Omega_\lambda\rangle$	λ
$Q_1^\dagger \Omega_\lambda\rangle$	$\lambda + \frac{1}{2}$

CPT invariance requires:

state	helicity
$ \Omega_{-\lambda-\frac{1}{2}}\rangle$	$-\lambda - \frac{1}{2}$
$Q_1^\dagger \Omega_{-\lambda-\frac{1}{2}}\rangle$	$-\lambda$

massless chiral multiplet

state	helicity
$ \Omega_0\rangle$	0
$Q_1^\dagger \Omega_0\rangle$	$\frac{1}{2}$

include CPT conjugate states:

state	helicity
$ \Omega_{-\frac{1}{2}}\rangle$	$-\frac{1}{2}$
$Q_1^\dagger \Omega_{-\frac{1}{2}}\rangle$	0

massless vector multiplet

state	helicity
$ \Omega_{\frac{1}{2}}\rangle$	$\frac{1}{2}$
$Q_1^\dagger \Omega_{\frac{1}{2}}\rangle$	1

and its CPT conjugate:

state	helicity
$ \Omega_{-1}\rangle$	-1
$Q_1^\dagger \Omega_{-1}\rangle$	$-\frac{1}{2}$

Superpartners

fermion	\leftrightarrow	sfermion
quark	\leftrightarrow	squark
gauge boson	\leftrightarrow	gaugino
gluon	\leftrightarrow	gluino

Extended SUSY

$$\begin{aligned}\{Q_\alpha^a, Q_{\dot{\alpha}b}^\dagger\} &= 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_b^a \\ \{Q_\alpha^a, Q_\beta^b\} &= 0 \\ \{Q_{\dot{\alpha}a}^\dagger, Q_{\dot{\beta}b}^\dagger\} &= 0\end{aligned}$$

where

$$a, b = 1, \dots, \mathcal{N}$$

$U(\mathcal{N})_R$ R-symmetry

massless multiplets: $p_\mu = (E, 0, 0, -E)$

$$\begin{aligned}\{Q_1^a, Q_{1b}^\dagger\} &= 4E\delta_b^a, \\ \{Q_2^a, Q_{2b}^\dagger\} &= 0.\end{aligned}$$

general massless multiplet

state	helicity	degeneracy
$ \Omega_\lambda\rangle$	λ	1
$Q_{1a}^\dagger \Omega_\lambda\rangle$	$\lambda + \frac{1}{2}$	\mathcal{N}
$Q_{1a}^\dagger Q_{1b}^\dagger \Omega_\lambda\rangle$	$\lambda + 1$	$\mathcal{N}(\mathcal{N} - 1)/2$
\vdots	\vdots	\vdots
$Q_{11}^\dagger Q_{12}^\dagger \dots Q_{1\mathcal{N}}^\dagger \Omega_\lambda\rangle$	$\lambda + \mathcal{N}/2$	1

$\mathcal{N} = 2$ massless vector multiplet

state	helicity	degeneracy
$ \Omega_{-1}\rangle$	-1	1
$Q^\dagger \Omega_{-1}\rangle$	$-\frac{1}{2}$	2
$Q^\dagger Q^\dagger \Omega_{-1}\rangle$	0	1

with the addition of the CPT conjugate:

state	helicity	degeneracy
$ \Omega_0\rangle$	0	1
$Q^\dagger \Omega_0\rangle$	$\frac{1}{2}$	2
$Q^\dagger Q^\dagger \Omega_0\rangle$	1	1

built from one $\mathcal{N} = 1$ vector multiplet and one $\mathcal{N} = 1$ chiral multiplet.

$\mathcal{N} = 2$ Hypermultiplet

state	helicity	degeneracy	
$ \Omega_{-\frac{1}{2}}\rangle$	$-\frac{1}{2}$	1	χ_α
$Q^\dagger \Omega_{-\frac{1}{2}}\rangle$	0	2	ϕ
$Q^\dagger Q^\dagger \Omega_{-\frac{1}{2}}\rangle$	$\frac{1}{2}$	1	$\psi^{\dagger\dot{\alpha}}$

gauge-invariant mass term: $\psi^\alpha \chi_\alpha$

$\mathcal{N} = 2$ is vector-like

$\mathcal{N} = 3$ massless supermultiplet

state	helicity	degeneracy
$ \Omega_{-1}\rangle$	-1	1
$Q^\dagger \Omega_{-1}\rangle$	$-\frac{1}{2}$	3
$Q^\dagger Q^\dagger \Omega_{-1}\rangle$	0	3
$Q^\dagger Q^\dagger Q^\dagger \Omega_{-1}\rangle$	$\frac{1}{2}$	1

plus CPT conjugate

state	helicity	degeneracy
$ \Omega_{-\frac{1}{2}}\rangle$	$-\frac{1}{2}$	1
$Q^\dagger \Omega_{-\frac{1}{2}}\rangle$	0	3
$Q^\dagger Q^\dagger \Omega_{-\frac{1}{2}}\rangle$	$\frac{1}{2}$	3
$Q^\dagger Q^\dagger Q^\dagger \Omega_{-\frac{1}{2}}\rangle$	1	1

$\mathcal{N} = 3$ is vector-like

$\mathcal{N} = 4$ massless vector supermultiplet

state	helicity	\mathbf{R}
$ \Omega_{-1}\rangle$	-1	$\mathbf{1}$
$Q^\dagger \Omega_{-1}\rangle$	$-\frac{1}{2}$	$\mathbf{4}$
$Q^\dagger Q^\dagger \Omega_{-1}\rangle$	0	$\mathbf{6}$
$Q^\dagger Q^\dagger Q^\dagger \Omega_{-1}\rangle$	$\frac{1}{2}$	$\overline{\mathbf{4}}$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_{-1}\rangle$	1	$\mathbf{1}$

vector-like theory

Massive Supermultiplets

$$\{Q_\alpha^a, Q_{\dot{\alpha}b}^\dagger\} = 2m \delta_{\alpha\dot{\alpha}} \delta_b^a$$

state	spin
$ \Omega_s\rangle$	s
$Q_{\dot{\alpha}a}^\dagger \Omega_s\rangle$	$s + \frac{1}{2}$
$Q_{\dot{\alpha}a}^\dagger Q_{\dot{\beta}b}^\dagger \Omega_s\rangle$	$s + 1$
\vdots	
$Q_{11}^\dagger Q_{21}^\dagger Q_{12}^\dagger Q_{22}^\dagger \cdots Q_{1\mathcal{N}}^\dagger Q_{2\mathcal{N}}^\dagger \Omega_\lambda\rangle$	s

$\mathcal{N} = 2$ massive supermultiplet

state	$(d_R, 2j + 1)$
$ \Omega_0\rangle$	$(1, 1)$
$Q^\dagger \Omega_0\rangle$	$(2, 2)$
$Q^\dagger Q^\dagger \Omega_0\rangle$	$(3, 1) + (1, 3)$
$Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(2, 2)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(1, 1)$

16 states: five of spin 0, four of spin $\frac{1}{2}$, and one of spin 1.

$\mathcal{N} = 4$ massive supermultiplet

state	$(\mathbf{R}, 2j + 1)$
$ \Omega_0\rangle$	$(\mathbf{1}, 1)$
$Q^\dagger \Omega_0\rangle$	$(\mathbf{4}, 2)$
$Q^\dagger Q^\dagger \Omega_0\rangle$	$(\mathbf{10}, 1) + (\mathbf{6}, 3)$
$Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\overline{\mathbf{20}}, 2) + (\overline{\mathbf{4}}, 4)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\mathbf{20}', 1) + (\mathbf{15}, 3) + (\mathbf{1}, 5)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\mathbf{20}, 2) + (\mathbf{4}, 4)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\overline{\mathbf{10}}, 1) + (\mathbf{6}, 3)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\overline{\mathbf{4}}, 2)$
$Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger Q^\dagger \Omega_0\rangle$	$(\mathbf{1}, 1)$

which contains 256 states, including eight spin $\frac{3}{2}$ states and one spin 2 state

Central Charges

$$\begin{aligned}\{Q_\alpha^a, Q_{\dot{\alpha}b}^\dagger\} &= 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_b^a \\ \{Q_\alpha^a, Q_\beta^b\} &= 2\sqrt{2}\epsilon_{\alpha\beta} Z^{ab} \\ \{Q_{\dot{\alpha}a}^\dagger, Q_{\dot{\beta}b}^\dagger\} &= 2\sqrt{2}\epsilon_{\dot{\alpha}\dot{\beta}} Z_{ab}^*\end{aligned}$$

where

$$\epsilon = i\sigma^2$$

for $\mathcal{N} = 2$

$$\begin{aligned}\{Q_\alpha^a, Q_{\dot{\alpha}b}^\dagger\} &= 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_b^a \\ \{Q_\alpha^a, Q_\beta^b\} &= 2\sqrt{2}\epsilon_{\alpha\beta}\epsilon^{ab} Z \\ \{Q_{\dot{\alpha}a}^\dagger, Q_{\dot{\beta}b}^\dagger\} &= 2\sqrt{2}\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{ab} Z\end{aligned}$$

Defining

$$A_\alpha = \frac{1}{2} \left[Q_\alpha^1 + \epsilon_{\alpha\beta} \left(Q_\beta^2 \right)^\dagger \right]$$

$$B_\alpha = \frac{1}{2} \left[Q_\alpha^1 - \epsilon_{\alpha\beta} \left(Q_\beta^2 \right)^\dagger \right]$$

reduces the algebra to

$$\{A_\alpha, A_\beta^\dagger\} = \delta_{\alpha\beta}(M + \sqrt{2}Z)$$

$$\{B_\alpha, B_\beta^\dagger\} = \delta_{\alpha\beta}(M - \sqrt{2}Z)$$

$$\langle M, Z | B_\alpha B_\alpha^\dagger | M, Z \rangle + \langle M, Z | B_\alpha^\dagger B_\alpha | M, Z \rangle = (M - \sqrt{2}Z) ,$$

$$M \geq \sqrt{2}Z$$

for $M = \sqrt{2}Z$ (short multiplets): B_α produces states of zero norm

$M > \sqrt{2}Z$ (long multiplets)

short (BPS) multiplet:

state	$2j + 1$
$ \Omega_0\rangle$	1
$A^\dagger \Omega_0\rangle$	2
$(A^\dagger)^2 \Omega_0\rangle$	1

state	$2j + 1$
$ \Omega_{\frac{1}{2}}\rangle$	2
$A^\dagger \Omega_{\frac{1}{2}}\rangle$	1 + 3
$(A^\dagger)^2 \Omega_{\frac{1}{2}}\rangle$	2

short multiplet has 8 states as opposed to 32 states for the corresponding long multiplet

BPS state:

$$M = \sqrt{2}Z$$

is exact