Anomaly and gaugino mediation
“Supergravity” mediation

$X$ is in the hidden sector, $M_{Pl}$ suppressed couplings

$$W = W_{\text{hid}}(X) + W_{\text{vis}}(\psi)$$

$$f = \left( \delta^i_j - \frac{c^i_j}{M_{Pl}^2} X^\dagger X \right) \psi^j V \psi_i + \ldots$$

$$\tau = \frac{\theta_{\text{YM}}}{2\pi} + i \frac{4\pi}{g^2} + i \frac{k}{M_{Pl}} X + \ldots$$

SUSY breaking VEV

$$\langle X \rangle = M + \mathcal{F}_X \theta^2 ,$$

induced squark and gluino masses:

$$(M^2_q)^i_j = c^i_j \frac{\mathcal{F}_X^2}{M_{Pl}^2}, \quad M_{\lambda} = k \frac{\mathcal{F}_X}{M_{Pl}}$$

no reason for the $c^i_j$ to respect flavor symmetries $\Rightarrow$ FCNCs
Naive Expectation

Kähler function might have flavor-blind form:

$$K = X^\dagger X + \psi_i^i e^V \psi_i$$

$\Rightarrow$

$$f = -3 + \frac{1}{M_{P1}^2} \left[ X^\dagger X + \left(1 + \frac{X^\dagger X}{M_{P1}^2}\right) \psi_i^i e^V \psi_i + \ldots \right]$$

interactions are flavor-blind but there are direct interactions induced by Planck scale (string) states which have been integrated out these interactions should not be flavor-blind they must generate Yukawa couplings
Extra Dimensions

SUSY breaking sector separated by a distance $r$ from the MSSM two sectors on different 3-branes embedded in the higher dimensional theory Interactions suppressed by

$$e^{-Mr}$$

where $M$ is the higher dimensional Planck/string scale

If only supergravity states propagate in bulk

then setting $e^a_\mu = 0$ and $\Sigma = M$ must decouple the two sectors

Lagrangian must have the form

$$W = W_{\text{hid}} + W_{\text{vis}}$$

$$f = c + f_{\text{hid}} + f_{\text{vis}}$$

$$\tau W^2_\alpha = \tau_{\text{hid}} W^2_{\alpha \text{hid}} + \tau_{\text{vis}} W^2_{\alpha \text{vis}}$$

all interactions between the two sectors due to supergravity form of $f$ implies a Kähler function of the form

$$K = -3M^2_{\text{Pl}} \ln \left(1 - \frac{f_{\text{hid}} + f_{\text{vis}}}{3M^2_{\text{Pl}}} \right)$$
Integrate out Hidden Sector

dropping Planck suppressed interactions in effective theory:

\[ \mathcal{L}_{\text{eff}} = \int d^4 \theta \psi^\dagger e^V \psi \frac{\Sigma^\dagger \Sigma}{M^2} + \int d^2 \theta \frac{\Sigma^3}{M^3} (m_0 \psi^2 + y \psi^3) \]

\[ - \frac{i}{16\pi} \int d^2 \theta \tau W^\alpha W_\alpha + h.c. \]

where the conformal weights of the fields determine the \( R \)-charges to be

\[ R[\Sigma] = \frac{2}{3}, \quad R[\psi] = 0 \]

only trace of the hidden sector is in compensator field \( \Sigma \), rescale

\[ \frac{\Sigma \psi}{M} \rightarrow \psi, \quad R[\psi] = \frac{2}{3} \]

\[ \mathcal{L}_{\text{eff}} = \int d^4 \theta \psi^\dagger e^V \psi + \int d^2 \theta (\frac{\Sigma}{M} m_0 \psi^2 + y \psi^3) \]

\[ - \frac{i}{16\pi} \int d^2 \theta \tau W^\alpha W_\alpha + h.c. \]

If \( m_0 = 0 \), then the theory is classically scale and conformally invariant
\( \Sigma \) decouples classically
Super-Weyl anomaly

quantum corrections break scale-invariance: couplings run
e.g. a two-point function has dependence on the cutoff Λ
⇒ dependence on Σ spurion of conformal symmetry:

\[ G = \frac{1}{p^2} h \left( \frac{p^2 M^2}{\Lambda^2 \Sigma \Sigma} \right) \]

\( h \) can only depend on the combination \( \Lambda \Sigma / M \) and conjugate because of
the classical conformal invariance
since \( \Lambda \) is real, only the combination \( \Lambda^2 \Sigma \Sigma \Sigma / M^2 \) appears
effects of the scaling anomaly determined by \( \beta \) functions and \( \gamma \)
cutoff dependence only occurs in the Kähler function and \( \tau \)
if we renormalize effective theory down to scale \( \mu \) we must have:

\[ \mathcal{L}_{\text{eff}} = \int d^4 \theta Z \left( \frac{\mu M}{\Lambda \Sigma}, \frac{\mu M}{\Lambda \Sigma} \right) \psi \psi^\dagger e^V \psi \\
+ \int d^2 \theta y \psi^3 - \frac{i}{16\pi} \int d^2 \theta \tau W^\alpha W_\alpha + h.c. \]
Compensator Dependence

$Z$ is real and $R$-symmetry-invariant, must have

$$Z = Z \left( \frac{\mu M}{\Lambda |\Sigma|} \right)$$

where

$$|\Sigma| = (\Sigma^\dagger \Sigma)^{1/2}$$

for global SUSY, with $\Sigma = M_{P1}$, axial symmetry is anomalous $\theta_{YM}$ shifts when the $\psi$s are re-phased due to the chiral anomaly in superconformal gravity, scale and axial anomalies vanish $\Sigma$ dynamical, re-phased $\Rightarrow$ shift in $\theta_{YM}$ is canceled since $\tau$ is holomorphic we have

$$\tau = i \frac{\tilde{b}}{2\pi} \ln \left( \frac{\mu M}{\Lambda \Sigma} \right)$$

$\mu$ dependence determines that $\tilde{b} = b$
SUSY breaking: gaugino mass

SUSY breaking will be communicated to auxiliary supergravity fields:

\[ \langle \Sigma \rangle = M + \mathcal{F}_\Sigma \theta^2 \]

induces a \( \theta^2 \) term in \( \tau \Rightarrow \) gaugino mass:

\[ M_\lambda = \left. \frac{i}{2\tau} \frac{\partial \tau}{\partial \Sigma} \right|_{\Sigma=M} \mathcal{F}_\Sigma = \frac{bg^2}{16\pi^2} \frac{\mathcal{F}_\Sigma}{M}. \]

this SUSY breaking mass arises through the one-loop anomaly this mechanism is known as anomaly mediation
SUSY breaking

We can also Taylor expand $Z$ in superspace:

$$Z = \left[ Z - \frac{1}{2} \frac{\partial Z}{\partial \ln \mu} \left( \frac{F_\Sigma}{M} \theta^2 + \frac{F_\Sigma^\dagger}{M} \bar{\theta}^2 \right) + \frac{1}{4} \frac{\partial^2 Z}{\partial (\ln \mu)^2} \frac{|F_\Sigma|^2}{M^2} \theta^2 \bar{\theta}^2 \right] \bigg|_{\Sigma=M}$$

canonically normalize kinetic terms by rescaling:

$$\psi' = Z^{1/2} \left( 1 - \frac{1}{2} \frac{\partial \ln Z}{\partial \ln \mu} \frac{F_\Sigma}{M} \theta^2 \right) \bigg|_{\Sigma=M}$$

Using

$$\gamma \equiv \frac{\partial \ln Z}{\partial \ln \mu} , \quad \beta_g \equiv \frac{\partial g}{\partial \ln \mu} , \quad \beta_y \equiv \frac{\partial y}{\partial \ln \mu}$$

we find

$$Z \psi'^\dagger e^V \psi = \left[ 1 + \frac{1}{4} \frac{\partial \gamma}{\partial \ln \mu} \frac{|F_\Sigma|^2}{M^2} \theta^2 \bar{\theta}^2 \right] \psi'^\dagger e^V \psi'$$

$$= \left[ 1 + \frac{1}{4} \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right) \frac{|F_\Sigma|^2}{M^2} \theta^2 \bar{\theta}^2 \right] \psi'^\dagger e^V \psi'$$
Squark and Slepton Masses

\[ M_{\tilde{\psi}}^2 = -\frac{1}{4} \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right) \frac{|F_\Sigma|^2}{M^2} \]

to leading order

\[ \gamma = \frac{1}{16\pi^2} \left( 4C_2(r)g^2 - ay^2 \right), \quad \beta_g = -\frac{bg^3}{16\pi^2}, \quad \beta_y = \frac{y}{16\pi^2} (ey^2 - fg^2) \]

so

\[ M_{\tilde{\psi}}^2 = \frac{1}{512\pi^4} \left[ 4C_2(r) b g^4 + ay^2(ey^2 - fg^2) \right] \frac{|F_\Sigma|^2}{M^2} \]

first term is positive for asymptotically free gauge theories

negative mass squared for sleptons since in the MSSM the \( U(1)_Y \) and \( SU(2)_L \) gauge couplings are not asymptotically free

\[ W(\psi) \] after rescaling gives trilinear interactions with coefficient

\[ A_{ijk} = \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) y_{ijk} \frac{F_\Sigma}{M} \]
Trilinear Terms

gauge mediation: messengers have masses

\[ \langle X \rangle = M_X \left( 1 + \frac{F_X}{M_X} \theta^2 \right) \]

anomaly mediation: the cutoff is

\[ \Lambda \frac{\Sigma}{M} = \Lambda \left( 1 + \frac{F_\Sigma}{M} \theta^2 \right) \]

mass of the regulator fields with anomaly mediation
the regulator is the messenger
after rescaling, the mass $m$ of a SUSY threshold becomes $m \Sigma / M$

low-energy $Z$ and $\tau$ have the following dependence:

$$Z \left( \frac{\mu M}{\Lambda |\Sigma|}, \frac{|m||\Sigma|}{\Lambda |\Sigma|} \right), \quad \tau \left( \frac{\mu M}{\Lambda \Sigma}, \frac{m \Sigma}{\Lambda \Sigma} \right)$$

gaugino and sfermion masses are independent of $m$ since $m/\Lambda$ has no
dependence on the spurion $\Sigma$

anomaly is insensitive to UV physics, completely determined
by the low-energy effective theory

threshold and regulator contribute with opposite signs and cancel
SUSY breaking in the mass term $\to$ cancellation would not complete

soft masses only depend on $\beta_g$, $\beta_y$, $\partial \gamma / \partial g$, $\partial \gamma / \partial y$ at weak scale, $M_W$
The $\mu$ problem

in order to get EWSB in the MSSM need $\mu$ and $b$ terms:

$$W = \mu H_u H_d, \quad V = b H_u H_d$$

with

$$b \sim \mu^2$$

need $\mu \sim$ soft masses, so in anomaly-mediation require

$$\mu \sim \frac{\alpha}{4\pi} \frac{\mathcal{F}_\Sigma}{M}$$

including a coupling to spurion field $\Sigma$ that directly gives a $\mu$ term:

$$W = \mu \frac{\Sigma^3}{M^3} H_u H_d$$

we also gives a tree-level $b$ term

$$b = 3 \frac{\mathcal{F}_\Sigma}{M} \mu \sim \frac{12\pi}{\alpha} \mu^2$$

which is much too large
The $\mu$ problem

more complicated possibility:

$$\mathcal{L}_{\text{int}} = \int d^4 \theta \delta \frac{X + X^\dagger}{M} H_u H_d \frac{\Sigma \Sigma^\dagger}{M^2} + h.c.$$  \hfill (*)

where $X$ is a SUSY breaking field, rescale

$$\frac{\Sigma H_i}{M} \rightarrow H_i$$

$$\mathcal{L}_{\text{eff int}} = \int d^4 \theta \delta \frac{X + X^\dagger}{M} H_u H_d \frac{\Sigma^\dagger}{\Sigma} + h.c.$$  

assuming $\langle X \rangle = \theta^2 \mathcal{F}_X$, picking out the $\overline{\theta}^2$ and $\theta^2 \overline{\theta}^2$ terms ⇒

$$\mu = \delta \left( \frac{\mathcal{F}_X^\dagger}{M} + \frac{\mathcal{F}_\Sigma^\dagger}{M} \right)$$

$$b = \delta \left( \frac{\mathcal{F}_X}{M} \frac{\mathcal{F}_\Sigma}{M} - \frac{\mathcal{F}_X^\dagger}{M} \frac{\mathcal{F}_\Sigma^\dagger}{M} \right)$$

$b$ vanishes at tree-level if $\mathcal{F}_\Sigma \propto \mathcal{F}_X$
The $\mu$ problem

$b$ term is generated at one-loop, canonically normalize the Higgs:

$$H'_i = Z_i^{1/2} (1 - \frac{1}{2} \gamma_i \frac{F_\Sigma}{M} \theta^2) \Bigg|_{\Sigma=M} H_i$$

if $\delta \sim \alpha/4\pi$, we find:

$$b = \frac{\delta}{2M} \left( \gamma_u \frac{F_\Sigma}{M} + \gamma_d \frac{F_\Sigma}{M} \right) = \mathcal{O}(\mu^2)$$

this relies on the coefficients of $X$ and $X^\dagger$ in (*) being equal seems fine-tuned

generated in 5D toy model without fine-tuning
fifth (extra) dimension has a compactification radius $r_c$
5D Gravity

for $r \ll r_c$ the gravitational potential is

$$\frac{1}{r^2 M^3}$$

rather than the 4D Newton potential

$$\frac{1}{r M_{P1}^2}$$

dynamic potential given by the spatial Fourier transform of the graviton propagator with zero energy exchange:

$$V(r) \sim \int d^{D-1}p \frac{e^{i\vec{p}.\vec{r}}}{p^2} \sim \frac{1}{r^{D-3}}$$

Matching the potentials at $r = r_c$ we have

$$M_{P1}^2 = r_c M^3$$
5D Vector Exchange

introduce a massive vector superfield $V$ which propagates in the 5D bulk (canonical dimension $3/2$)

Integrating over fifth dimension, assume the 4D effective theory has form:

$$\mathcal{L} = \int d^4 \theta r_c m^2 V^2 + aV(X + X^\dagger)M^{1/2} + \frac{bV}{M^{1/2}} H_u H_d \frac{\Sigma \Sigma^\dagger}{M^2} + h.c.$$  

first term is a mass term and $V$ is normalized to dimension $\frac{1}{2}$

Integrating out $V$ and performing the usual rescaling gives

$$\mathcal{L}_{int} \sim \int d^4 \theta \frac{ab}{r_c m^2} (X + X^\dagger)H_u H_d \frac{\Sigma \Sigma^\dagger}{M^2} + h.c. + \ldots$$

with

$$r_c m \sim O(1), \quad ab \sim O\left(\frac{\alpha}{4\pi}\right)$$

→ required interaction

existence proof that $\mu$ problem can be solved in anomaly-mediation
Slepton masses

squark and slepton masses:

\[ M_{\tilde{\psi}}^2 = \frac{1}{512\pi^4} \left[ 4C_2(r) b g^4 + a y^2(e y^2 - f g^2) \right] \left| \frac{F_\Sigma}{M^2} \right|^2 \]

\( b \) is negative for \( SU(2)_L \) and \( U(1)_Y \) \( \Rightarrow \) sleptons are tachyonic

possible solutions:

- new bulk fields which couple leptons and the SUSY breaking fields
- new Higgs fields with large Yukawa couplings
- new asymptotically free gauge interactions for sleptons, \( \Rightarrow \) leptons and sleptons are composite
- heavy SUSY violating threshold (messengers) with a light singlet

consider the last possibility, sometimes known as “anti-gauge mediation”
Anti-Gauge Mediation

consider a singlet $X$ and $N_m$ messengers $\phi$ and $\bar{\phi}$ in $\square$s and $\bar{\square}$s of $SU(5)$ GUT with a superpotential

$$W = \lambda X \phi \bar{\phi}$$

$X$ is pseudo-flat: it gets a mass through anomaly mediation when we renormalize down to a scale $\sim X$ we have a Kähler term

$$\int d^4 \theta Z \left( \frac{XX^\dagger M^2}{\Lambda^2 \Sigma \Sigma^\dagger} \right) X^\dagger X$$

scalar potential

$$V(X) = m_X^2(X)|X|^2$$

$$= \frac{N_m}{16 \pi^2} \lambda^2(X) \left[ A \lambda^2(X) - C^a g_a^2(X) \right] \frac{|F_\Sigma|^2}{M^2} |X|^2$$
If messengers have asymptotically free gauge interactions then $m_X^2 (X)$ can change sign, and $X$ is stabilized nearby (Coleman–Weinberg)

$$\langle X \rangle = m$$

then $F$ component of $X$ is proportional to $m F_\Sigma$:

$$F_X \sim \frac{N m \lambda^2}{16 \pi^2} \frac{m F_\Sigma}{M}$$

splitting in the messenger masses is a loop effect

threshold depends on light VEV $\to$ extra contribution to soft masses

low-energy couplings only depend on

$$\tilde{X} = X \frac{M}{\Sigma}, \quad \frac{F_{\tilde{X}}}{\langle \tilde{X} \rangle} = \frac{F_X}{m} - \frac{F_\Sigma}{M} \approx -\frac{F_\Sigma}{M}$$

because of the loop factor suppression
Taylor Expansion in Superspace

gaugino mass:

\[ M_\lambda = -\frac{1}{2\tau} \left. \frac{\partial \tau}{\partial \ln \Sigma} \right|_{\Sigma = M} \frac{\mathcal{F}_\Sigma}{M_{Pl}} \]

\[ = \frac{1}{2\tau} \left( \frac{\partial \tau}{\partial \ln \mu} + \frac{\partial \tau}{\partial \ln X} \right) \frac{\mathcal{F}_\Sigma}{M_{Pl}} \]

\[ = \frac{\alpha(\mu)}{4\pi} (b - N_m) \frac{\mathcal{F}_\Sigma}{M_{Pl}} \]

first term is usual anomaly mediation
second term is minus the gauge mediation answer

hence the name Anti-Gauge Mediation
Taylor Expansion in Superspace

squark or slepton mass squared:

\[
M_{\tilde{\psi}}^2 = - \left( \frac{\partial}{\partial \ln \mu} + \frac{\partial}{\partial \ln |X|} \right)^2 \ln Z(\mu, |X|) \frac{|\mathcal{F}_\Sigma|^2}{4M_{Pl}^2}
\]

\[
= \frac{2C_2(r)b}{(4\pi)^2} \left[ \alpha^2(\mu) - \alpha^2(\mu) \frac{N_m}{b} + (\alpha^2(\mu) - \alpha^2(m)) \frac{N_m^2}{b^2} \right] \frac{|\mathcal{F}_\Sigma|^2}{M_{Pl}^2}
\]

first term is anomaly mediation term
second term is minus the gauge mediation term
final term is RG running induced by gaugino mass
Slepton Masses

\[
M_{\tilde{\psi}}^2 = - \left( \frac{\partial}{\partial \ln \mu} + \frac{\partial}{\partial \ln |X|} \right)^2 \ln Z(\mu, |X|) \frac{|\mathcal{F}_\Sigma|^2}{4M_{Pl}^2} \\
= \frac{2C_2(r)b}{(4\pi)^2} \left[ \alpha^2(\mu) - \alpha^2(\mu) \frac{N_m}{b} + \left( \alpha^2(\mu) - \alpha^2(m) \right) \frac{N_m^2}{b^2} \right] \frac{|\mathcal{F}_\Sigma|^2}{M_{Pl}^2}
\]

for the sleptons \( M_{\tilde{\psi}}^2 > 0 \Rightarrow \) RG term dominates \( \Leftrightarrow N_m \) sufficiently large cannot \( m \) too large, higher dimension operators dominate, e.g.

\[
\int d^4\theta \frac{X^\dagger X}{M_{Pl}^2} \psi^\dagger e^V \psi
\]

would give

\[
M_{\tilde{\psi}}^2 = - \frac{|\mathcal{F}_X|^2}{M_{Pl}^2}
\]

for \( m \sim M_{GUT} \) we need \( N_m \geq 4 \)
Problem

adding singlet $S$ can generate $\mu$ and $b$ terms

$$\int d^2 \theta \lambda' SH_u H_d + \frac{k}{3} S^3 + \frac{y}{2} S^2 X$$

one-loop a kinetic mixing:

$$\int d^4 \theta \tilde{Z} S X^\dagger + \text{h.c.}$$

for $\langle X \rangle \neq 0$, $S$ is massive and can be integrated out:

$$S \sim -\frac{\lambda'}{y} \frac{H_u H_d}{X}$$

$$\rightarrow \mathcal{L}_{\text{eff}} = -\frac{\lambda'}{y} \int d^4 \theta \frac{X^\dagger}{X} H_u H_d \tilde{Z} \left( \frac{|X|M_{Pl}}{\Lambda|\Sigma|} \right) + \text{h.c.}$$

produces $\mu$ term at one-loop, $b$ term at two-loops:

$$\mu = -\frac{\lambda'}{y} \frac{1}{2} \frac{\partial \tilde{Z}}{\partial \ln |X|} , \quad b = -\frac{\lambda'}{y} \frac{1}{4} \frac{\partial^2 \tilde{Z}}{\partial (\ln |X|)^2}$$
Gaugino mediation

RG running from gaugino mass $\rightarrow + \text{ mass}^2$ for squarks and sleptons
consider models where only gauginos get masses at leading order
squarks have strong gauge coupling $\Rightarrow$ heavier than sleptons
large top Yukawa coupling and heavy stops $\Rightarrow$ radiative EWSB
simple set up: compact extra dimension with radius

$$r_c \sim \frac{1}{M_{\text{GUT}}}$$

gauge fields propagate in bulk
SUSY breaking on brane at the other end of the fifth dimension
Yukawa couplings only source of flavor violation
GIM mechanism suppresses FCNCs
Gaugino mediation

gaugino propagating in bulk mediates SUSY breaking
Gaugino mediation

4D gauge coupling related to the 5D coupling by

\[
\frac{1}{g_4^2} F_{\mu \nu}^a F^{a\mu \nu} = \frac{1}{g_5^2} \int dx^5 F_{\mu \nu}^a F^{a\mu \nu}, \quad g_4^2 = \frac{g_5^2}{r_c}
\]

no chirality in 5D, minimal SUSY theory has \( \mathcal{N} = 2 \)

vector supermultiplet → 4D vector + adjoint chiral:

\[
(A_N, \lambda_L, \lambda_R, \phi) \rightarrow (A_\nu, \lambda_L) + (\phi + i A_5, \lambda_R)
\]

fifth component of gauge field is a scalar

choose boundary conditions so:
adjoint chiral supermultiplet vanishes on one 3-brane
vector multiplet does not
only vector supermultiplet has massless mode (independent of \( x_5 \))
⇒ breaks SUSY → \( \mathcal{N} = 1 \)
Gaugino mediation

SUSY breaking on one brane communicated by local interactions

\[ \mathcal{L} \propto \int dx^5 \int d^2 \theta \left( 1 + \delta (x_5 - r_c) \frac{X}{M^2} \right) W^\alpha W_\alpha + \text{h.c.} \]
\[ \propto r_c \lambda^\dagger \bar{\sigma}^\mu D_\mu \lambda + \frac{F_X}{M^2} \lambda^\dagger \lambda + \ldots \]

gaugino mass generated by auxiliary fields on SUSY breaking brane

\[ M_\lambda = \frac{1}{r_c M} \frac{F_X}{M} \]
Gaugino mediation

bulk gluino loops with two mass insertions give the largest contribution to the squark/slepton masses:

$$M_{\tilde{\psi}}^2 \sim \frac{g_5^2}{16\pi^2} \left( \frac{F_X}{M^2} \right)^2 \frac{1}{r_c^3} = \frac{g_4^2}{16\pi^2} M_{\lambda}^2,$$

suppressed relative to gluino mass squared
for $r_c \ll M_W^{-1}$ 4D RG running

$$\mu \frac{d}{d\mu} m_Q^2 \propto -g^2 M_{\lambda}^2 + cg^4 \text{Tr} \left( (-1)^2 F m_i^2 \right)$$

dominates by a large logarithm, $\ln r_c M_W$, over the 5D loop contribution

all the soft masses are determined by gaugino masses and $r_c$
very predictive scenario