# Supergravity

# Gravity: on-shell

Einstein gravity $\leftrightarrow$ gauge theory of local Lorentz/translation symmetry
generators $M_{ab}$, $P_a$ $\leftrightarrow$ "gauge fields" $\omega_\mu^{ab}$, spin connection, $e_\mu^a$ *vierbein*
where $a, b = 0, \ldots, 3$ are Lorentz gauge group indices
$\mu, \nu = 0, \ldots, 3$ are spacetime indices
$e_\mu^a$ and $\omega_\mu^{ab}$ transform as collections of vectors
gauge fields $\leftrightarrow$ feld strengths $R_{\mu\nu}^{ab}$, (Riemann curvature), $C_{\mu\nu}^a$, (torsion)
$C_{\mu\nu}^a = 0$, solve for $\omega_\mu^{ab}$ in terms of $e_\mu^a$

counting: $e_\mu^a$ 16 components
subtract 4 (equations of motion)
subtract 4 (local translation invariance)
subtract 6 (local Lorentz invariance)

leaves 2 degrees of freedom: massless spin-2 particle

# Gravity: on-shell

Couplings to matter:

$$\nabla_\mu = \partial_\mu - e_\mu^a P_a - \omega_\mu^{ab} M_{ab}$$

feld strengths can be obtained from

$$\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu$$

Writing

$$e = |\mathrm{det} e_\mu^m|$$

invariant action with only two derivatives is linear in the field strength:

$$S_{\mathrm{GR}} = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x \, e \, \epsilon^{\mu\nu\rho\lambda} \epsilon_{abcd} \, e_\mu^a e_\nu^b \, R_{\rho\lambda}^{cd} = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x \, e \, R$$

where $R$ is the curvature scalar

# Supergravity: on-shell

$e_\mu^a \leftrightarrow$ helicity 2 particle, $\mathcal{N}=1$ SUSY requires helicity $3/2$ $\psi_\nu^\alpha$ (gravitino)
on-shell each has two degrees of freedom
gravitino is gauge field $\leftrightarrow Q_\alpha \leftrightarrow$ field strength $D_{\mu\nu\alpha}$
$C^a_{\mu\nu}=0$, solve for $\omega_\mu^{ab}$ find on-shell supergravity action:

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\, e\, R + \frac{i}{4}\int d^4x\, \epsilon^{\mu\nu\rho\sigma}\,\overline{\psi}_\mu\gamma_5\gamma_\nu D_{\rho\sigma}$$

call second term $S_{\rm gravitino}$
metric:

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

in terms of a local inertial coordinate system $\xi^a$ at the point $X$

$$e_\mu^a(X) = \frac{\partial \xi^a}{\partial x^\mu}$$

# Brans–Dicke Gravity

first consider toy example, scale-invariant Brans–Dicke theory:

$$S_{\rm BD} = \int d^4x \, \left[ \tfrac{e}{2}\, \sigma^2 R + \tfrac{e}{12} \partial^\mu \sigma \partial_\mu \sigma \right]$$

treat scalar $\sigma$ as a spurion field and set

$$\sigma = M_{\rm Pl}$$

break local conformal invariance to local Poincaré invariance
$\Rightarrow$ Einstein gravity

# Superconformal Gravity

in addition to the "gauge" fields $e_\mu^a$ and $\psi_{\nu\alpha}$ we have $A_\mu \leftrightarrow$ local $U(1)_R$ symmetry, and $b_\mu \leftrightarrow$ local conformal boosts
Counting degrees of freedom off-shell (subtracting gauge invariances):

| field | | | $d.o.f.$ |
|---|---|---|---|
| $e_\mu^a$ : | 16 | $-4-6-1$ | $=5$ |
| $\psi_\nu^\alpha$ : | 16 | $-4-4$ | $=8$ |
| $A_\mu$ : | 4 | $-1$ | $=3$ |
| $b_\mu$ : | 4 | $-4$ | $=0$ |

$e_\mu^a$ subtract 4 (translation), 6 (Lorentz), 1 (dilations)
$\psi_\nu^\alpha$ subtract 4 (SUSY $Q_\alpha$, $\overline{Q}_{\dot\alpha}$), 4 (conformal SUSY $S_\beta$ and $\overline{S}_{\dot\beta}$)
$A_\mu$ subtract 1(local $R$-symmetry)
$b_\mu$ subtract 4 (four conformal boost generators)

no auxiliary fields for the superconformal graviton multiplet

"gauge" fields, couple with gauge covariant derivatives

# Supergravity: off-shell

spurion chiral superfield to break the conformal symmetry:

$$\Sigma = (\sigma, \chi, \mathcal{F}_\Sigma)$$

in global $\mathcal{N} = 1$, $\Sigma$ is a chiral superfield
here it contains part of the off-shell graviton superfield
$\Sigma$ called conformal compensator

assign conformal weight 1 to the lowest component of $\Sigma$
($x^\mu$ and $\theta$ have conformal weight $-1$ and $-1/2$)
full superconformal gravity action is

$$S_{\rm scg} = \int d^4x \, \tfrac{e}{2}\sigma^*\sigma R + e \int d^4\,\theta \Sigma^\dagger \Sigma + S_{\rm gravitino}$$

derivatives are covariant in “gauge“ fields ($e^a_\mu$, $\psi_{\nu\alpha}$, $A_\mu, b_\mu$)
a superconformal Brans–Dicke theory

# Supergravity: off-shell

Treat $\sigma$, $\chi$, and $b_\mu$ as spurion fields

$$\sigma \quad = \quad M_{\rm Pl}\ , \quad \chi = 0\ , \quad b_\mu = 0$$

local superconformal invariance $\rightarrow$ local super-Poincaré invariance
resulting action is:

$$S_{\rm sg} = \int d^4x\, e \left[ \frac{M_{\rm Pl}^2}{2} R + \mathcal{F}_\Sigma \mathcal{F}_\Sigma^\dagger - \frac{2M_{\rm Pl}^2}{9} A_\mu A^\mu \right] + S_{\rm gravitino}$$

$\mathcal{F}_\Sigma$ and $A_\mu$ are auxiliary fields, counting:

| field | | | d.o.f. |
|---|---|---|---|
| $e_\mu^a$ : | 16 | $-4-6$ | $=6$ |
| $\psi_\nu^\alpha$ : | 16 | $-4$ | $=12$ |
| $A_\mu$ : | 4 | | $=4$ |
| $\mathcal{F}_\Sigma$ : | 2 | | $=2$ |

6 bosonic degrees of freedom from $\mathcal{F}_\Sigma$ and $A_\mu$ are just what is required to have $\mathcal{N}=1$ SUSY manifest off-shell

# Superspace

eight-dimensional space $z^M = (x^\mu, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$
require super-general coordinate invariance

$$z^M \to z'^M = z^M + \xi^M$$

where $\xi^M(z^M)$
Superspace scalars transform

$$\phi'(z') = \phi(z)$$

while fields with a superspace index

$$\psi_M = \frac{\partial \phi}{\partial z^M}$$

transform as

$$\psi'_M(z') = \frac{\partial z^N}{\partial z'^M} \psi_N(z)$$

# Superspace

construct a vielbein $E_M^A$
relates the superspace world coordinate to a locally Lorentz covariant (tangent space) coordinate
contains the off-shell multiplet $(e_\mu^a, \psi_{\nu\alpha}, A_\mu, \mathcal{F}_\Sigma)$

we can choose a coordinate system where, for $\theta = 0$,

$$E_\mu^a = e_\mu^a, \;\; E_\mu^\alpha = \frac{1}{2}\psi_\mu^\alpha, \;\; E_\mu^{\dot\alpha} = \frac{1}{2}\overline{\psi}_\mu^{\dot\alpha}$$

# Coupling to matter

arbitrary global SUSY theory:

$$\mathcal{L}_{\rm gl} = \int d^4\theta K(\Phi^\dagger, e^V\Phi) + \int d^2\theta \left( W(\Phi) - \tfrac{i\tau}{16\pi} W^\alpha W_\alpha \right) + h.c.$$

define conformal weight 0 fields and mass parameters by

$$\Phi' = \Sigma\,\Phi$$
$$m' = \Sigma\, m$$

dropping the primes, local superconformal-Poincaré invariant Lagrangian:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta f(\Phi^\dagger, e^V\Phi)\tfrac{\Sigma^\dagger\Sigma}{M_{\rm Pl}^2} + \int d^2\theta \tfrac{\Sigma^3}{M_{\rm Pl}^3} W(\Phi) - \int d^2\theta \tfrac{i\tau}{16\pi} W^\alpha W_\alpha + h.c. \\ &\quad -\tfrac{1}{6} f(\phi^\dagger,\phi)\sigma^*\sigma R + \mathcal{F}_\Sigma \mathcal{F}_\Sigma^\dagger - \tfrac{2M_{\rm Pl}^2}{9} A_\mu A^\mu + \mathcal{L}_{\rm gravitino}\end{aligned}$$

action:

$$S = \int d^4x\, e\, \mathcal{L}$$

# Coupling to matter

$M_{\rm Pl} \to \infty$ (global SUSY) limit, choose

$$f(\Phi^\dagger, e^V \Phi) = -3\, M_{\rm Pl}^2 e^{-K(\Phi^\dagger, e^V \Phi)/3M_{\rm Pl}^2}$$

rescaling the vierbein by a Weyl (local scale) transformation

$$e_\mu^a \to e^{-K/12M_{\rm Pl}^2} e_\mu^a$$

one finds bosonic piece of the action:

$$\begin{aligned} S_{\rm B} \quad &= \quad \int d^4x\, e \left[ \tfrac{M_{\rm Pl}^2}{2} R + K_j^i(\phi^\dagger, \phi)(\nabla^\mu \phi^i)^\dagger \nabla_\mu \phi_j \right. \\ & \qquad \left. -\mathcal{V}(\phi^\dagger, \phi) + \tfrac{i\tau}{16\pi}(F_{\mu\nu}F^{\mu\nu} + iF_{\mu\nu}\widetilde{F}^{\mu\nu}) + h.c. \right] \end{aligned}$$

where $K^i$ and $K_j^i$ (the Kähler metric) are given by

$$K^i(\phi^\dagger, \phi) = \tfrac{\partial K}{\partial \phi_i} \;, \quad K_j^i(\phi^\dagger, \phi) = \tfrac{\partial^2 K}{\partial \phi^{j\dagger} \partial \phi_i}$$

# Coupling to matter

scalar potential:

$$\mathcal{V}(\phi^\dagger,\phi) = e^{K/M_{\rm Pl}^2}\left[\left(K^{-1}\right)_i^j\left(W^i+\frac{WK^i}{M_{\rm Pl}^2}\right)\left(W_j^*+\frac{W^*K_j}{M_{\rm Pl}^2}\right)-\frac{3|W|^2}{M_{\rm Pl}^2}\right] + \frac{g^2}{2}\left(K^iT^a\phi_i\right)^2$$

last term is just the $D$-term potential

in supergravity the energy density can be negative
usually tune tree-level vacuum energy to zero by adding the appropriate constant to $W$

# Coupling to matter

auxiliary components of chiral superfields (no fermion bilinear VEVs):

$$\mathcal{F}_i = -e^{K/2M_{\rm Pl}^2} \left(K^{-1}\right)_i^j \left(W_j^* + \frac{W^* K_j}{M_{\rm Pl}^2}\right) \qquad (*)$$

from fermionic piece of Lagrangian, $\nabla_\mu \widetilde{\phi}_i$ contains a gravitino term

$$\frac{1}{M_{\rm Pl}} \psi_\mu^\alpha Q_\alpha \widetilde{\phi}_i = \frac{1}{M_{\rm Pl}} \psi_\mu^\alpha \mathcal{F}_i + \mathcal{O}(\sigma^\mu \partial_\mu \phi_i)$$

so the Kähler function contains a term:

$$i K_j^i \frac{1}{M_{\rm Pl}} \bar{\theta} \overline{\widetilde{\phi}}^j \theta^2 \psi_\mu \mathcal{F}_i \sigma^\mu \bar{\theta}$$

in analogy to the ordinary Higgs mechanism, that the gravitino eats the goldstino if there is a nonvanishing $\mathcal{F}$ component

in flat spacetime, goldstino adds right number of degrees of freedom to make a massive spin 3/2 particle

# Gravitino Mass

in flat spacetime

$$m_{3/2}^2 = \frac{\mathcal{F}^{*j} K_j^i \mathcal{F}_i}{3M_{\rm Pl}^2}$$

use (*) and $\mathcal{V} = 0 \Rightarrow$

$$m_{3/2}^2 = e^{K/M_{\rm Pl}^2} \frac{|W|^2}{M_{\rm Pl}^4}$$

taking a canonical Kähler function

$$K = Z\Phi^{i\dagger}\Phi_i$$

and $M_{\rm Pl} \to \infty$ reproduces usual global SUSY results

# Maximal Supergravity

massless supermultiplet with helicities $\leq 2$
SUSY charges change the helicity by $\frac{1}{2} \Rightarrow \mathcal{N} \leq 8$
arbitrary dimension cannot have more than $32 = 8 \times 4$ real SUSY charges
maximal dimension: spinor in 11 dimensions has 32 components

supergravity theory must have $e_\mu^a$ and $\psi_\mu^\alpha$ massless gauge fields
$D$ dimensions: "little" group $SO(D-2)$
graviton: symmetric tensor of $SO(D-2)$ has $(D-1)(D-2)/2-1$ dof
44 dof for $D = 11$
gravitino is a vector-spinor and a vector has $D-2$ dof

spinor of $SO(D)$ has $d_S$ components, where

$$d_S = 2^{(D-2)/2} \text{ (for } D \text{ even)}, \;\; d_S = 2^{(D-1)/2} \text{ (for } D \text{ odd)}$$

# 11 dimensions

Majorana spinor has $d_S = 32$ real components, 16 dof on-shell
tracelessness condition $\Gamma^\mu \psi_\mu^\alpha = 0$ leaves $(D-3)d_S/2$ degrees of freedom for the vector-spinor
gravitino has 128 real on-shell dof
gravitino - gaviton = 84 more fermionic dof than bosonic
difference made up by three index antisymmetric tensor $A_{\mu\nu\rho}$
antisymmetric tensor with $p$ indices (i.e. rank $p$) has

$$\tfrac{1}{p!}\,(D-2)\ldots(D-p-1)$$

dof on-shell, also called a $p$-form field

$$\tfrac{(11-2)(11-3)(11-4)}{6} = 3 \cdot 4 \cdot 7 = 84$$

# 11 dimensions: BPS solitons

The SUSY algebra of 11-D supergravity has two central charges
two Lorentz indices, five Lorentz indices $\leftrightarrow$ BPS solitons
central charge acts as a topological charge, spatial integral at fixed $t$
preserve index structure, solitons extend in two and five spatial directions
called $p$-branes for $p$ spatial directions
e.g. monopole is a 0-brane, couples to a 1-form gauge field $A_\mu$
$p$-brane couples to a $(p+1)$-form gauge field
2-brane couples to 3-form gauge field $A_{\mu\nu\rho}$
a $p$-form gauge field has a $(D-p-2)$-form dual gauge field
field strength $\leftrightarrow$ $A_{\mu\nu\rho}$ is a 4-form: $F_{\mu\nu\rho\lambda}$
contract with $\epsilon$ tensor gives dual 7-form $\leftrightarrow$ 6-form dual gauge field
couples to the 5-brane

# 10 dimensions

compactify 1 dimension on a circle
decompose $D = 11$ fields into massless $D = 10$ fields (constant on circle)

$$\begin{array}{rcl} e_\mu^a\,(44) & \rightarrow & e_\mu^a\,(35), B_\mu\,(8), \sigma\,(1) \\ A_{\mu\nu\rho}\,(84) & \rightarrow & A_{\mu\nu\rho}\,(56), A_{\mu\nu}\,(28) \\ \psi_\mu^\alpha\,(128) & \rightarrow & \psi_\mu^{+\alpha}\,(56), \psi_\mu^{-\alpha}\,(56), \lambda^{+\alpha}\,(8), \lambda^{-\alpha}\,(8) \end{array}$$

32 supercharges of $D = 11 \rightarrow$ two $D = 10$ spinors
spinors have opposite chirality
gravitino splits into states of opposite chirality, labeled by $+$ and $-$
this is Type IIA supergravity
two other supergravity theories in $D = 10$
Type I: single spinor of supercharges
Type IIB: supercharges are two spinors with the same chirality

# Low-Energy Effective Theories

Type IIA $\leftrightarrow$ Type IIA string theory
Type IIB $\leftrightarrow$ Type IIB string theory
Type I with $E_8 \times E_8$ or $SO(32) \leftrightarrow$ heterotic string theory
$D = 11$ supergravity $\leftrightarrow$ M-theory

# 4D helicities

massless vector $\rightarrow$ massless 4D vector and $D-4$ massless scalars
$\leftrightarrow$ two components with helicity 1 and $-1$ and $D-4$ helicity 0 states
$\Leftrightarrow$ $D-4$ lowering operators
e.g. 5D, the little group is $SO(3)$, one lowering operator $\sigma^- = \frac{1}{2}(\sigma^1 - i\sigma^2)$

traceless symmetric tensor field, $e_\mu^a$, $\Leftrightarrow$ symmetric product of two vectors:

| helicity | degeneracy $D=11$ | degeneracy $D=10$ |
|---|---|---|
| $2$ | $1$ | $1$ |
| $1$ | $7 = 1\cdot(11-4)$ | $6 = 1\cdot(10-4)$ |
| $0$ | $28 = 7\cdot 8/2 - 1 + 1$ | $21 = 6\cdot 7/2 - 1 + 1$ |
| $-1$ | $7 = 1\cdot(11-4)$ | $6 = 1\cdot(10-4)$ |
| $-2$ | $1$ | $1$ |

# 4D helicities: 2-form

2-form field $\Leftrightarrow$ antisymmetric product of two vectors:

| helicity | degeneracy $D = 11$ | degeneracy $D = 10$ |
|---|---|---|
| 1 | 7 | 6 |
| 0 | $7(7-1)/2+1=22$ | $6(6-1)/2+1=16$ |
| $-1$ | 7 | 6 |

where the $7(7-1)/2$ comes from antisymmetrizing the helicity 0 components, and the $+1$ corresponds to combining the helicity 1 and $-1$ components of the two vectors

# 4D helicities: 3-form

| helicity | degeneracy $D = 11$ | degeneracy $D = 10$ |
|---|---|---|
| $1$ | $21$ | $15$ |
| $0$ | $35 + 7 = 42$ | $20 + 6 = 26$ |
| $-1$ | $21$ | $15$ |

35 comes from antisymmetrizing three helicity 0 components, and the $+7$ corresponds to the combining helicity 1 and $-1$ components and one helicity 0 component of the three vectors

# 4D helicities: gravitino

$D = 11$ spinor has 8 helicity $\frac{1}{2}$ components and 8 helicity $-\frac{1}{2}$ components, while for $D = 10$ these components correspond to two opposite chirality spinors, we can reconstruct the gravitino by combining a vector and a spinor (remembering the tracelessness condition)

| helicity | degeneracy $D = 11$ | degeneracy $D = 10$ |
|---|---|---|
| $\frac{3}{2}$ | 8 | 8 |
| $\frac{1}{2}$ | $56 = 8 \cdot 7$ | $48 = 8 \cdot 6$ |
| $-\frac{1}{2}$ | $56 = 8 \cdot 7$ | $48 = 8 \cdot 6$ |
| $-\frac{3}{2}$ | 8 | 8 |

# $D = 11$ Supermultiplet

starting with a helicity $-2$ state and raising the helicity repeatedly by acting with 8 SUSY generators (and remembering to antisymmetrize)

| 11D sugra. state | helicity | degeneracy | $e_\mu^a$ | $A_{\mu\nu\rho}$ | $\psi_\mu^\alpha$ |
|---|---|---|---|---|---|
| $\overline{Q}^8 \vert\Omega_{-2}\rangle$ | $2$ | 1 | 1 | | |
| $\overline{Q}^7 \vert\Omega_{-2}\rangle$ | $\frac{3}{2}$ | 8 | | | 8 |
| $\overline{Q}^6 \vert\Omega_{-2}\rangle$ | $1$ | 28 | 7 | 21 | |
| $\overline{Q}^5 \vert\Omega_{-2}\rangle$ | $\frac{1}{2}$ | 56 | | | 56 |
| $\overline{Q}^4 \vert\Omega_{-2}\rangle$ | $0$ | 70 | 28 | 42 | |
| $\overline{Q}^3 \vert\Omega_{-2}\rangle$ | $-\frac{1}{2}$ | 56 | | | 56 |
| $\overline{Q}^2 \vert\Omega_{-2}\rangle$ | $-1$ | 28 | 7 | 21 | |
| $\overline{Q}\ \vert\Omega_{-2}\rangle$ | $-\frac{3}{2}$ | 8 | | | 8 |
| $\vert\Omega_{-2}\rangle$ | $-2$ | 1 | 1 | | |

# $D = 10$ Type IIA Supermultiplet

| IIA state | helicity | degen. | $e_\mu^a$ | $A_{\mu\nu\rho}$ | $A_{\mu\nu}$ | $B_\mu$ | $\sigma$ | $\psi_\mu^{\pm\alpha}$ | $\lambda^{\pm\alpha}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\overline{Q}^8 \vert\Omega_{-2}\rangle$ | $2$ | $1$ | $1$ | | | | | | |
| $\overline{Q}^7 \vert\Omega_{-2}\rangle$ | $\frac{3}{2}$ | $8$ | | | | | | $8$ | |
| $\overline{Q}^6 \vert\Omega_{-2}\rangle$ | $1$ | $28$ | $6$ | $15$ | $6$ | $1$ | | | |
| $\overline{Q}^5 \vert\Omega_{-2}\rangle$ | $\frac{1}{2}$ | $56$ | | | | | | $48$ | $8$ |
| $\overline{Q}^4 \vert\Omega_{-2}\rangle$ | $0$ | $70$ | $21$ | $26$ | $16$ | $6$ | $1$ | | |
| $\overline{Q}^3 \vert\Omega_{-2}\rangle$ | $-\frac{1}{2}$ | $56$ | | | | | | $48$ | $8$ |
| $\overline{Q}^2 \vert\Omega_{-2}\rangle$ | $-1$ | $28$ | $6$ | $15$ | $6$ | $1$ | | | |
| $\overline{Q} \vert\Omega_{-2}\rangle$ | $-\frac{3}{2}$ | $8$ | | | | | | $8$ | |
| $\vert\Omega_{-2}\rangle$ | $-2$ | $1$ | $1$ | | | | | | |

# 10D: BPS branes

SUSY algebra of Type IIA, central charges of rank 0, 1, 2, 4, 5, 6, 8
$\leftrightarrow$ p-branes
gauge fields of rank 1, 2, and 3
dual gauge fields of rank 5, 6, 7
1-brane $\leftrightarrow$ fundamental string of Type IIA string theory

Type IIA supergravity $\leftrightarrow$ compactified 11D supergravity
2-brane of 11D $\leftrightarrow$ 2-brane of Type IIA when $\perp$ circle
2-brane of 11D $\leftrightarrow$ 1-brane when wraps circle
5-brane of 11D supergravity $\leftrightarrow$ 5-brane and 4-brane of Type IIA

# Brane Tensions

11D supergravity has one coupling constant, $\kappa$, 11D Newton's constant

$$\mathcal{L} = \frac{1}{2\kappa^2}\, eR$$

11D Planck mass by $\kappa = M_{\rm Pl}^{-9/2}$
tension (energy per unit volume) of branes given powers of $M_{\rm Pl}$
energy per unit area of the 2-brane is $T_2 = M_{\rm Pl}^3$
5-brane we have $T_5 = M_{\rm Pl}^6$
2-brane and 5-brane of Type IIA have same tensions as 11D theory
1-brane and 4-brane have $T_1 = R_{10} M_{\rm Pl}^3$ and $T_4 = R_{10} M_{\rm Pl}^6$
1-brane is the fundamental string of Type IIA string theory
$\Rightarrow$ identify tension with string tension or string mass squared:

$$T_1 = R_{10} M_{\rm Pl}^3 \equiv \frac{1}{4\pi\alpha'} \equiv m_s^2$$

# String Coupling

express the tensions in terms of $m_s$ and Type IIA string coupling

$$g_s = (R_{10} M_{\rm Pl})^{3/2}$$

$$T_2 = \frac{m_s^3}{g_s} \ , \ \ T_4 = \frac{m_s^5}{g_s} \ , \ \ T_5 = \frac{m_s^6}{g_s^2}$$

branes are nonperturbative BPS solitons not surprising to see inverse powers of the coupling
$1/g_s$ dependence of the 2-brane and 4-brane significant
universal feature of what are now called D-branes

# $D = 10$ Type IIB Supermultiplet

SUSY algebra has central charges of rank 1, 3, 5, 7
expect the corresponding $p$-branes to couple to gauge fields of rank 2 and 4, $A_{\mu\nu}$ and $B_{\mu\nu\rho\lambda}$, and their duals
$e_\mu^a$, $\psi_\mu^\alpha$, $\lambda^\alpha$ have same dof as the Type IIA, difference being that $\psi_\mu^\alpha$ and $\lambda^\alpha$ have opposite chirality in the IIB theory
it turns out that $A_{\mu\nu}$ is complex, twice as many dof $= 56$
so far the fermions have 37 more dof than the $e_\mu^a$ and $A_{\mu\nu}$ combined, while an unconstrained 4-form field has 70 dof
5-form field strength corresponding to $B_{\mu\nu\rho\lambda}$ constrained to be self-dual, reduces dof to 35
need a complex scalar, $a$, to balance out the multiplet:

$$e_\mu^a\,(35), a\,(2), A_{\mu\nu}\,(56), B_{\mu\nu\rho\lambda}\,(35)$$
$$\psi_\mu^\alpha\,(112), \lambda^\alpha\,(16)$$

# $D = 10$ Type IIB Supermultiplet

two SUSY spinor charges of the Type IIB theory have same chirality
transform as vector under an $SO(2)$ group, i.e. $R$-charges $\pm 1$
single Clifford vacuum state with helicity $-2$ must have $SO(2)$ charge 0
gravitino splits into two parts with charges $\pm 1$
to antisymmetrize SUSY charges:
antisymmetrize $SO(2)$, symmetrize remaining four spinor indices
or
symmetrize $SO(2)$, antisymmetrize the remaining four spinor indices

# $D = 10$ Type IIB Supermultiplet

| IIB state | helicity | degeneracy | $e_\mu^a$ | $B_{\mu\nu\rho\lambda}$ | $A_{\mu\nu}$ | $a$ | $\psi_\mu^\alpha$ | $\lambda^\alpha$ |
|---|---|---|---|---|---|---|---|---|
| $\overline{Q}^8 \lvert\Omega_{-2}\rangle$ | $2$ | 1 | 1 | | | | | |
| $\overline{Q}^7 \lvert\Omega_{-2}\rangle$ | $\frac{3}{2}$ | 8 | | | | | 8 | |
| $\overline{Q}^6 \lvert\Omega_{-2}\rangle$ | $1$ | 28 | 6 | 10 | 12 | | | |
| $\overline{Q}^5 \lvert\Omega_{-2}\rangle$ | $\frac{1}{2}$ | 56 | | | | | 48 | 8 |
| $\overline{Q}^4 \lvert\Omega_{-2}\rangle$ | $0$ | 70 | 21 | 15 | 32 | 2 | | |
| $\overline{Q}^3 \lvert\Omega_{-2}\rangle$ | $-\frac{1}{2}$ | 56 | | | | | 48 | 8 |
| $\overline{Q}^2 \lvert\Omega_{-2}\rangle$ | $-1$ | 28 | 6 | 10 | 12 | | | |
| $\overline{Q} \lvert\Omega_{-2}\rangle$ | $-\frac{3}{2}$ | 8 | | | | | 8 | |
| $\lvert\Omega_{-2}\rangle$ | $-2$ | 1 | 1 | | | | | |

# $D = 10$ Type IIB Supermultiplet

symmetric combination of $4 \times 4$ is 10, antisymmetric under $SO(2)$
$\Rightarrow B_{\mu\nu\rho\lambda}$ has $SO(2)$ charge 0
antisymmetric combination of $4 \times 4$ is 6 and graviton has $SO(2)$ charge 0
$\Rightarrow$ two 6's corresponding to $A_{\mu\nu}$ must have charges $\pm 2$.
$\lambda^{\alpha}$ has $SO(2)$ charge $\pm 3$
scalar $a$ has $SO(2)$ charge $\pm 4$

# $D = 10$ Type I Supermultiplet

parity in Type IIB: 4-form, half of 2-form, half of scalar are odd
truncate by keeping only the even fields.
Majorana condition on the fermions reduces dof by one half

$$e_\mu^a\,(35), \sigma\,(1), A_{\mu\nu}\,(28)$$
$$\psi_\mu^\alpha\,(56), \lambda^\alpha\,(8)$$

construction of multiplet has a further complication: only four SUSY raising operators, starting with $|\Omega_{-2}\rangle$ yields a maximum helicity of 0
adding CPT conjugate $\rightarrow$ two helicity 0 components and four helicity $\frac{1}{2}$
graviton requires 21 helicity 0 components
gravitino and spinor require 28 helicity $\frac{1}{2}$ components
need to add 6 copies of a multiplet based on $|\Omega_{-1}\rangle$

# $D = 10$ Type I Supermultiplet

| Type I state | helicity | degen. | $e_\mu^a$ | $A_{\mu\nu}$ | $\sigma$ | $\psi_\mu^\alpha$ |
|---|---|---|---|---|---|---|
| $\overline{Q}^4 \vert\Omega_0\rangle$ | $2$ | 1 | 1 | | | |
| $\overline{Q}^3 \vert\Omega_0\rangle$ | $\frac{3}{2}$ | 4 | | | | 4 |
| $\overline{Q}^2 \vert\Omega_0\rangle + 6 \times \overline{Q}^4 \vert\Omega_{-1}\rangle$ | $1$ | 12 | 6 | 6 | | |
| $\overline{Q}\ \vert\Omega_0\rangle + 6 \times \overline{Q}^3 \vert\Omega_{-1}\rangle$ | $\frac{1}{2}$ | 28 | | | | 24 |
| $\overline{Q}^4 \vert\Omega_{-2}\rangle + 6 \times \overline{Q}^2 \vert\Omega_{-1}\rangle + \vert\Omega_0\rangle$ | $0$ | 38 | 21 | 16 | 1 | |
| $\overline{Q}^3 \vert\Omega_{-2}\rangle + 6 \times \overline{Q}\ \vert\Omega_{-1}\rangle$ | $-\frac{1}{2}$ | 28 | | | | 24 |
| $\overline{Q}^2 \vert\Omega_{-2}\rangle + 6 \times \vert\Omega_{-1}\rangle$ | $-1$ | 12 | 6 | 6 | | |
| $\overline{Q}\ \vert\Omega_{-2}\rangle$ | $-\frac{3}{2}$ | 4 | | | | 4 |
| $\vert\Omega_{-2}\rangle$ | $-2$ | 1 | 1 | | | |

# $D = 10$ Type I and Yang-Mills

$D = 10$, $\mathcal{N} = 1$ (16 real supercharges) Yang-Mills
contains a vector and spinor with eight dof each
couple to Type I supergravity
anomaly cancellation allows for the gauge group $E_8 \times E_8$ or $SO(32)$
two low-energy effective theories for the two heterotic string theories

$D = 10$, $\mathcal{N} = 1$ Yang-Mills is low-energy effective theory for Type I string theory

# $D = 5$, $\mathcal{N} = 8$, gauged supergravity

consider Type IIB supergravity compactified on $S^5$
integrate out nonzero modes on $S^5$
$SO(6) \sim SU(4)$ isometry $\rightarrow$ gauge symmetry in the effective theory
5D little group is $SO(3)$
each massless field has one component for each helicity
massless 5D have the same dof as the corresponding massive 4D

graviton has helicities: 2,1,0,-1, and -2: five dof
vector and 2-form have three helicity components 1, 0, and -1
gravitino has four helicity components: 3/2, 1/2, -1/2, and -3/2
spinor has helicity components 1/2 and -1/2

in addition to the $SU(4)$ gauge symmetry, there is $SO(2)$ $R$-symmetry from Type IIB theory
SUSY generators transform as $(\Box, +1) + (\overline{\Box}, -1)$

# 5D Graviton Supermultiplet

| 5D sugra. state | helicity | degeneracy | $e_\mu^a$ | $A_\mu$ | $B_{\mu\nu}$ | $\phi$ | $\psi_\mu^\alpha$ | $\lambda^\alpha$ |
|---|---|---|---|---|---|---|---|---|
| $\overline{Q}^8\|\Omega_{-2}\rangle$ | $2$ | 1 | 1 | | | | | |
| $\overline{Q}^7\|\Omega_{-2}\rangle$ | $\frac{3}{2}$ | 8 | | | | | 8 | |
| $\overline{Q}^6\|\Omega_{-2}\rangle$ | $1$ | 28 | 1 | 15 | 12 | | | |
| $\overline{Q}^5\|\Omega_{-2}\rangle$ | $\frac{1}{2}$ | 56 | | | | | 8 | 48 |
| $\overline{Q}^4\|\Omega_{-2}\rangle$ | $0$ | 70 | 1 | 15 | 12 | 42 | | |
| $\overline{Q}^3\|\Omega_{-2}\rangle$ | $-\frac{1}{2}$ | 56 | | | | | 8 | 48 |
| $\overline{Q}^2\|\Omega_{-2}\rangle$ | $-1$ | 28 | 1 | 15 | 12 | | | |
| $\overline{Q}\|\Omega_{-2}\rangle$ | $-\frac{3}{2}$ | 8 | | | | | 8 | |
| $\|\Omega_{-2}\rangle$ | $-2$ | 1 | 1 | | | | | |

# 5D Graviton Supermultiplet

representations of $SU(4) \times SO(2)$

| | | |
|---|---|---|
| graviton | $e_\mu^a$ | $(1, 0)$ |
| vector | $A_\mu$ | $\left(\ydiagram{2,1,1}, 0\right)$ |
| 2-form | $B_{\mu\nu}$ | $\left(\ydiagram{1,1}, 2\right) + \left(\ydiagram{1,1}, -2\right)$ |
| scalars | $\phi$ | $(1, \pm 1), \left(\ydiagram{2}, 2\right) + \left(\overline{\ydiagram{2}}, -2\right) + \left(\ydiagram{2,2}, 0\right)$ |
| gravitino | $\psi_\mu^\alpha$ | $(\square, 1) + (\overline{\square}, -1)$ |
| “gauginos“ | $\lambda^\alpha$ | $(\square, 3)\ , + (\overline{\square}, -3) + \left(\ydiagram{2,1}, 1\right) + \left(\overline{\ydiagram{2,1}}, -1\right)$ |