Dynamical SUSY breaking

A rule of thumb for SUSY breaking

theory with no flat directions that spontaneously breaks a continuous global symmetry generally breaks SUSY
$\Rightarrow$ Goldstone boson with a scalar partner (a modulus), but if there are no flat directions this is impossible

rule gives a handful of dynamical SUSY breaking theories

With duality we can find many examples of dynamical SUSY breaking

The 3-2 model

Affleck, Dine, and Seiberg found the simplest known model of dynamical SUSY breaking:

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\Box$	$\Box$	$1/3$	1
L	$\mathbf{1}$	$\Box$	-1	-3
$\overline{U}$	$\overline{\Box}$	$\mathbf{1}$	$-4/3$	-8
$\overline{D}$	$\overline{\Box}$	$\mathbf{1}$	$2/3$	4

For $\Lambda_3 \gg \Lambda_2$ instantons give the standard ADS superpotential:

$$W_{\rm dyn} = \frac{\Lambda_3^7}{\det(\overline{Q}Q)}$$

which has a runaway vacuum. Adding a tree-level trilinear term

$$W = \frac{\Lambda_3^7}{\det(\overline{Q}Q)} + \lambda\, Q\bar{D}L \ ,$$

removes the classical flat directions and produces a stable minimum

The 3-2 model

$U(1)$ is broken and we expect (rule of thumb) that SUSY is broken

$$\frac{\partial W}{\partial L_\alpha} = \lambda \epsilon^{\alpha\beta} Q_{m\alpha} \overline{D}^m = 0$$

tries to set $\det \overline{Q}Q$ to zero since

$$\begin{aligned} \det \overline{Q}Q &= \det \begin{pmatrix} \overline{U}Q_1 & \overline{U}Q_2 \\ \overline{D}Q_1 & \overline{D}Q_2 \end{pmatrix} \\ &= \overline{U}^m Q_{m\alpha} \overline{D}^n Q_{n\beta} \epsilon^{\alpha\beta} \ . \end{aligned}$$

potential cannot have a zero-energy minimum since the dynamical term blows up at $\det \overline{Q}Q = 0$

SUSY is indeed broken

The 3-2 model

estimate the vacuum energy by taking all the VEVs to $\sim \phi$
For $\phi \gg \Lambda_3$ and $\lambda \ll 1$ in a perturbative regime

$$\begin{aligned} V &= \left|\frac{\partial W}{\partial Q}\right|^2 + \left|\frac{\partial W}{\partial \overline{U}}\right|^2 + \left|\frac{\partial W}{\partial \overline{D}}\right|^2 + \left|\frac{\partial W}{\partial L}\right|^2 \\ &\approx \frac{\Lambda_3^{14}}{\phi^{10}} + \lambda \frac{\Lambda_3^7}{\phi^3} + \lambda^2 \phi^4 \end{aligned}$$

minimum near

$$\langle \phi \rangle \approx \frac{\Lambda_3}{\lambda^{1/7}}$$

solution is self-consistent

$$V \approx \lambda^{10/7} \Lambda_3^4$$

goes to 0 as $\lambda \to 0$, $\Lambda_3 \to 0$

Duality and the 3-2 model

Using duality can also understand the case where $\Lambda_2 \gg \Lambda_3$

SUSY broken nonperturbatively

$SU(3)$ gauge group has two flavors, completely broken for generic VEVs

$SU(2)$ gauge group has four $\Box$'s $\equiv$ two flavors

$\Rightarrow$ confinement with chiral symmetry breaking

mesons and baryons:

$$\begin{array}{rcl} M & \sim & \left(\begin{array}{cc} LQ_1 & LQ_2 \\ Q_3Q_1 & Q_3Q_2 \end{array} \right) \\ B & \sim & Q_1Q_2 \\ \bar{B} & \sim & Q_3L \end{array}$$

effective superpotential is

$$W = X\, \left(\det M - B\overline{B} - \Lambda_2^4\right) + \lambda \left(\sum_{i=1}^{2} M_{1\mathrm{i}}\overline{D}^{\mathrm{i}} + \overline{B}\,\overline{D}^3 \right)$$

where X is a Lagrange multiplier field

Duality and the 3-2 model

$$W = X\,\left(\det M - B\overline{B} - \Lambda_2^4\right) + \lambda\left(\sum_{i=1}^{2} M_{1\mathrm{i}}\overline{D}^{\mathrm{i}} + \overline{B}\,\overline{D}^{3}\right)$$

$\overline{D}$ eqm tries to force M_{1i} and $\overline{B}$ to zero
constraint means that at least one of M_{11}, M_{12}, or $\overline{B}$ is nonzero
$\Rightarrow$ SUSY is broken at tree-level in the dual description

$$V \approx \lambda^2\Lambda_2^4$$

Comparing the vacuum energies we see that the $SU(3)$ interactions dominate when $\Lambda_3 \gg \lambda^{1/7}\Lambda_2$

for $\Lambda_2 \sim \Lambda_3$ consider the full superpotential

$$W = X\,\left(\det M - B\bar{B} - \Lambda_2^4\right) + \frac{\Lambda_3^7}{\det(\overline{Q}Q)} + \lambda\,Q\bar{D}L$$

which still breaks SUSY, analysis more complicated

SU(5) with $\overline{\square} + \begin{array}{c}\square\\[-4pt]\square\end{array}$

chiral gauge theory has no classical flat directions
ADS tried to match anomalies in a confined description
only “bizarre,” “implausible” solutions
assume broken $U(1) \Rightarrow$ broken SUSY (using the rule of thumb)

Adding flavors $(\square + \overline{\square})$ with masses Murayama showed that SUSY is broken, but masses $\rightarrow \infty$ strong coupling

With duality Pouliot showed that SUSY is broken at strong coupling

SU(5) with $\overline{\Box} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$

with 4 flavors theory s-confines

	$SU(5)$	$SU(4)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	1	0	9	0
$\overline{Q}$	$\overline{\Box}$	1	$\Box$	4	-3	0
Q	$\Box$	$\Box$	1	-5	-3	$\frac{1}{2}$

denote composite meson by $(Q\overline{Q})$, spectrum of massless composites is:

	$SU(4)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$(Q\overline{Q})$	$\Box$	$\Box$	-1	-6	$\frac{1}{2}$
$(A\overline{Q}^2)$	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	8	3	0
(A^2Q)	$\Box$	1	-5	15	$\frac{1}{2}$
(AQ^3)	$\overline{\Box}$	1	-15	0	$\frac{3}{2}$
$(\overline{Q}^5)$	1	1	20	-15	0

SU(5) with $\overline{\square} + \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

with a superpotential

$$\begin{aligned} W_{\rm dyn} &= \tfrac{1}{\Lambda^9}\Big[(A^2Q)(Q\overline{Q})^3(A\overline{Q}^2) + (AQ^3)(Q\overline{Q})(A\overline{Q}^2)^2 \\ &\qquad + (\overline{Q}^5)(A^2Q)(AQ^3)\Big] \end{aligned}$$

first term antisymmetrized in $SU(5)$ and $SU(4)$ indices
second term antisymmetrized in just $SU(5)$ indices

add mass terms and Yukawa couplings for the extra flavors:

$$\Delta W = \textstyle\sum_{i=1}^{4} mQ_i\overline{Q}_i + \sum_{i,j\le 4}\lambda_{ij}A\overline{Q}_i\overline{Q}_j \ ,$$

which lift all the flat directions

eqm give

$$\begin{aligned} \frac{\partial W}{\partial(\overline{Q}^5)} &= (A^2Q)(AQ^3) = 0 \\ \frac{\partial W}{\partial(Q\overline{Q})} &= 3(A^2Q)(Q\overline{Q})^2(A\overline{Q}^2) + (AQ^3)(A\overline{Q}^2)^2 + m = 0 \end{aligned}$$

SU(5) with □ + ⊟

$$\begin{aligned}
\frac{\partial W}{\partial(\overline{Q}^5)} &= (A^2Q)(AQ^3) = 0 && (*) \\
\frac{\partial W}{\partial(Q\overline{Q})} &= 3(A^2Q)(Q\overline{Q})^2(A\overline{Q}^2) + (AQ^3)(A\overline{Q}^2)^2 + m = 0 && (**)
\end{aligned}$$

Assuming $(A^2Q) \neq 0$ then the first equation of motion (*) requires $(AQ^3) = 0$ and multiplying (**) by (A^2Q) we see that because of the antisymmetrizations the first term vanishes ⇒

$$(AQ^3)(A\overline{Q}^2)^2 = -m \qquad (***)$$

contradiction!
Assuming that $(AQ^3) \neq 0$ then (*) requires $(A^2Q) = 0$, and plugging into (**) we find eqn (***) directly Multiplying eqn (***) by (AQ^3) we find that the left-hand side vanishes again due to antisymmetrizations, so $(AQ^3) = 0$, contradiction!

SUSY is broken at tree-level in dual description

Intriligator–Thomas–Izawa–Yanagida

	$SU(2)$	$SU(4)$
Q	$\square$	$\square$
S	$\mathbf{1}$	$\begin{array}{c}\square\\[-4pt]\square\end{array}$

$$W = \lambda S^{ij} Q_i Q_j$$

strong $SU(2)$ enforces a constraint.

$$\mathrm{Pf}(QQ) = \Lambda^4$$

eqm for S:

$$\frac{\partial W}{\partial S^{ij}} = \lambda Q_i Q_j = 0$$

equations incompatible
SUSY is broken

Intriligator–Thomas–Izawa–Yanagida

for large λS, we can integrate out the quarks, no flavors ⇒ gaugino condensation:

$$\begin{aligned} \Lambda_{\rm eff}^{3N} &= \Lambda^{3N-2}\,(\lambda S)^2 \\ W_{\rm eff} = 2\Lambda_{\rm eff}^3 &= 2\Lambda^2\lambda S \\ \tfrac{\partial W_{\rm eff}}{\partial S^{ij}} &= 2\lambda\Lambda^2 \end{aligned}$$

again vacuum energy is nonzero

theory is vector-like, Witten index $\mathrm{Tr}(-1)^{\mathbf{F}}$ is nonzero with mass terms turned on so there is at least one supersymmetric vacuum
index is topological, does not change under variations of the mass

loop-hole potential for large field values are very different with $\Delta W = m_s S^2$ from the theory with $m_s \to 0$, in this limit vacua can come in from or go out to ∞

Pseudo-Flat Direction

S appears to be a flat direction but with SUSY breaking theories becomes pseudo-flat due to corrections from the Kähler function

For large values of λS wavefunction renormalization:

$$Z_S = 1 + c\lambda\lambda^\dagger \ln\left(\frac{\mu_0^2}{\lambda^2 S^2}\right)$$

vacuum energy:

$$V = \frac{4|\lambda|^2}{|Z_S|}\Lambda^4 \approx |\lambda|^2\Lambda^4\left[1 + c\lambda\lambda^\dagger \ln\left(\frac{\lambda^2 S^2}{\mu_0^2}\right)\right]$$

potential slopes towards the origin
can be stabilized by gauging a subgroup of $SU(4)$. Otherwise low-energy effective theory with local minimum at $S = 0$
effective theory non-calculable near $\lambda S \approx \Lambda$

Baryon Runaways

Consider a generalization of the 3-2 model:

	$SU(2N-1)$	$Sp(2N)$	$SU(2N-1)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	1
L	$\mathbf{1}$	$\square$	$\square$	-1	$-\frac{3}{2N-1}$
$\overline{U}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	0	$\frac{2N+2}{2N-1}$
$\overline{D}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	-6	$-4N$

with a tree-level superpotential

$$W = \lambda Q L \overline{U}$$

turn off $SU(2N-1)$ and λ, $Sp(2N)$	non-Abelian Coulomb phase for weakly coupled dual description s-confines for $N=3$ confines with χSB for $N=2$
turn off the $Sp(2N)$ and λ, $SU(2N-1)$	s-confines for $N \geq 2$

Baryon Runaways

consider the case that $\Lambda_{SU} \gg \Lambda_{Sp}$

classical moduli space that can be parameterized by:

	$SU(2N-1)$	$U(1)$	$U(1)_R$
$M = (LL)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-2	$-\frac{6}{2N-1}$
$B = (\overline{U}^{2N-2}\overline{D})$	$\square$	-6	$-\frac{4(N^2-N+1)}{2N-1}$
$b = (\overline{U}^{2N-1})$	$\mathbf{1}$	0	$2N+2$

subject to the constraints

$$M_{jk}B_l\epsilon^{klm_1\cdots m_{2N-3}} = 0 \qquad M_{jk}b = 0$$

two branches:
$M = 0$ and $B, b \neq 0$
$M \neq 0$ and $B, b = 0$

Baryon Runaways

branch where $M = 0$ (true vacuum ends up here)

$$\langle \overline{U} \rangle = \begin{pmatrix} v\cos\theta & \\ & v\mathbf{1}_{2N-2} \end{pmatrix}, \qquad \langle \overline{D} \rangle = \begin{pmatrix} v\sin\theta \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

For $v > \Lambda_{SU}$, $SU(2N-1)$ is generically broken and the superpotential gives masses to Q and L or order λv. The low-energy effective theory is pure $Sp(2N)$ $\Rightarrow$ gaugino condensation

$$\Lambda_{\text{eff}}^{3(2N+2)} = \Lambda_{Sp}^{3(2N+2)-2(2N-1)} \left(\lambda \overline{U}\right)^{2(2N-1)}$$

$$W_{\text{eff}} \propto \Lambda_{\text{eff}}^3 \sim \Lambda_{Sp}^3 \left(\frac{\lambda \overline{U}}{\Lambda_{Sp}}\right)^{(2N-1)/(N+1)}$$

For $N > 2$ this forces $\langle \overline{U} \rangle$ towards zero

Baryon Runaways

For $v < \Lambda_{SU}$, then $SU(2N-1)$ s-confines: effective theory

	$Sp(2N)$	$SU(2N-1)$
L	$\square$	$\square$
$(Q\overline{U})$	$\square$	$\overline{\square}$
$(Q\overline{D})$	$\square$	$\mathbf{1}$
(Q^{2N-1})	$\square$	$\mathbf{1}$
B	$\mathbf{1}$	$\square$
b	$\mathbf{1}$	$\mathbf{1}$

with a superpotential

$$\begin{aligned} W_{\rm sc} \quad = \quad & \tfrac{1}{\Lambda_{SU}^{4N-3}}\left[(Q^{2N-1})(Q\overline{U})B + (Q^{2N-1})(Q\overline{D})b - \mathrm{det}\overline{Q}Q\right] \\ & +\lambda(Q\overline{U})L \ . \end{aligned}$$

integrated out $(Q\overline{U})$ and $L$with $(Q\overline{U}) = 0$,

$$W_{\rm le} = \tfrac{1}{\Lambda_{SU}^{4N-3}}(Q^{2N-1})(Q\overline{D})b$$

Baryon Runaways

On this branch $\langle b\rangle = \langle \overline{U}^{2N-1}\rangle \neq 0$, gives a mass to (Q^{2N-1}) and $(Q\overline{D})$ leaves pure $Sp(2N)$ as the low-energy effective theory. So we again find gaugino condensation

$$\Lambda_{\text{eff}}^{3(2N+2)} = \Lambda_{Sp}^{3(2N+2)-2(2N-1)} \left(\lambda\Lambda_{SU}\right)^{2(2N-1)} \left(\frac{b}{\Lambda_{SU}}\right)^{2}$$

$$W_{\text{eff}} \propto \Lambda_{\text{eff}}^{3} \sim b^{1/(N+1)} \left(\Lambda_{Sp}^{N+4}\lambda^{2N-1}\Lambda_{SU}^{(2N-2)}\right)^{1/(N+1)}$$

which forces $b \rightarrow \infty$ (this is a baryon runaway vacuum)
effective theory only valid for scales below Λ_{SU}
already seen that beyond this point the potential starts to rise again
vacuum is around

$$\langle b\rangle = \langle \overline{U}^{2N-1}\rangle \sim \Lambda_{SU}^{2N-1}$$

With more work one can also see that SUSY is broken when $\Lambda_{Sp} \gg \Lambda_{SU}$

Baryon Runaways: $N = 3$

$Sp(2N)$ s-confines

	$SU(5)$	$SU(5)$
(QQ)	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$
(LL)	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$
(QL)	$\square$	$\square$
$\overline{U}$	$\overline{\square}$	$\overline{\square}$
$\overline{D}$	$\overline{\square}$	$\mathbf{1}$

with

$$W = \lambda(QL)\overline{U} + Q^{2N-1}L^{2N-1}$$

global $SU(5) \supset$ SM gauge groups, candidate for gauge mediation
integrate (QL) and $\overline{U}$ to find $SU(5)$ with an antisymmetric tensor, an antifundamental, and some gauge singlets, which we have already seen breaks SUSY

Baryon Runaways

other branch $M = (LL) \neq 0$
D-flat directions for L break $Sp(2N)$ to $SU(2)$, effective theory is:

	$SU(2N-1)$	$SU(2)$
Q'	$\Box$	$\Box$
L'	$\mathbf{1}$	$\Box$
$\overline{U}'$	$\overline{\Box}$	$\mathbf{1}$
$\overline{D}$	$\overline{\Box}$	$\mathbf{1}$

and some gauge singlets with a superpotential

$$W = \lambda Q' \overline{U}' L'$$

This is a generalized 3-2 model

Baryon Runaways

For $\langle L\rangle \gg \Lambda_{SU}$ the vacuum energy is independent of the $SU(2)$ scale and proportional to $\Lambda^4_{SU(2N-1)}$ which itself is proportional to a positive power of $\langle L\rangle$, thus the effective potential in this region drives $\langle L\rangle$ smaller.

For $\langle L\rangle \ll \Lambda_{SU}$ use the s-confined description, and find the baryon b runs away. For $\langle L\rangle \approx \Lambda_{SU}$, the vacuum energy is

$$V \sim \Lambda^4_{SU} \ ,$$

which is larger than the vacuum energy on the other branch

global minimum is on the baryon branch with $b = (\overline{U}^{2N-1}) \neq 0$

Direct gauge mediation

suppose fields that break SUSY have SM gauge couplings
only need two sectors rather than three

	$SU(5)_1$	$SU(5)_2$	$SU(5)$
Y	$\mathbf{1}$	$\square$	$\overline{\square}$
ϕ	$\overline{\square}$	$\mathbf{1}$	$\square$
$\overline{\phi}$	$\square$	$\overline{\square}$	$\mathbf{1}$

with a superpotential

$$W = \lambda Y^i_j \overline{\phi}^j \phi_i$$

weakly gauge global $SU(5)$ with the SM gauge groups
$Y \gg \Lambda_1, \Lambda_2$, ϕ and $\overline{\phi}$ get a mass, matching gives

$$\Lambda_{\rm eff}^{3\cdot 5} = \Lambda_1^{3\cdot 5-5} \left(\lambda X\right)^5$$

where $X = (\det Y)^{1/5}$

Direct gauge mediation

effective gauge theory has gaugino condensation

$$W_{\text{eff}} = \Lambda_{\text{eff}}^3 \sim \lambda X \Lambda_1^2$$

SUSY broken a la the Intriligator–Thomas–Izawa–Yanagida
vacuum energy given by

$$V \approx \tfrac{|\lambda \Lambda_1^2|^2}{Z_X}$$

where Z_X is the wavefunction renormalization for X
for large X the vacuum energy grows monotonically
local minimum occurs where anomalous dimension $\gamma = 0$
for $\langle X \rangle > 10^{14}$ GeV, the Landau pole for λ is above the Planck scale

Direct gauge mediation

problem: for small values of X, SUSY minimum along a baryonic direction
look at the constrained mesons and baryons of $SU(5)_1$

$$W = A(\det M - B\overline{B} - \Lambda_1^{10}) + \lambda Y M \ .$$

SUSY minimum at $B\overline{B} = -\Lambda_1^{10}$, $Y = 0$, $M = 0$
SUSY minimum would have to be removed, or the non-supersymmetric minimum made sufficiently metastable by adding appropriate terms to the superpotential that force $B\overline{B} = 0$.

Direct gauge mediation

phenomenological problem: heavy gauge boson messengers can give negative contributions to squark and slepton squared masses. Consider the general case where a VEV

$$\langle X \rangle \quad = \quad M + \theta^2 \mathcal{F}$$

breaks SUSY and

$$G \times H \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

with

$$\frac{1}{\alpha(M)} \quad = \quad \frac{1}{\alpha_G(M)} + \frac{1}{\alpha_H(M)}$$

Direct gauge mediation

Analytic continuation in superspace gives

$$M_\lambda = \frac{\alpha(\mu)}{4\pi}(b - b_H - b_G)\frac{\mathcal{F}}{M}$$

and

$$\begin{aligned} m_Q^2 &= 2C_2(r)\frac{\alpha(\mu)^2}{(16\pi^2)^2}\left(\frac{F}{M}\right)^2 \\ &\quad \left[(b + (R^2 - 2)b_H - 2b_G)\xi^2 + \frac{b - b_H - b_G}{b}(1 - \xi^2)\right] \end{aligned}$$

where

$$\xi = \frac{\alpha(M)}{\alpha(\mu)} \quad R = \frac{\alpha_H(M)}{\alpha(M)}$$

typically gives a negative mass squared for right-handed sleptons

Direct gauge mediation

if not all the messengers are heavy, then two-loop RG gives:

$$\mu \frac{d}{d\mu} m_Q^2 \propto -g^2 M_\lambda^2 + c g^4 \mathrm{Tr}\left((-1)^{2F} m_i^2\right)$$

the one-loop term proportional to the gaugino mass squared drives the scalar mass positive as the renormalization scale is run down
two-loop term can drive the mass squared negative
effect is maximized when the gaugino is light
when gluino is the heaviest gaugino, sleptons get dangerous negative contributions

also dangerous in models where the squarks and sleptons of the first two generations are much heavier that 1 TeV

Single sector models

suppose the strong dynamics that breaks SUSY also produce composite MSSM particles

rather than having three sectors, there is really just one sector.

	$SU(k)$	$SO(10)$	$SU(10)$	$SU(2)$
Q	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
L	$\overline{\square}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
$\overline{U}$	$\mathbf{1}$	$\square$	$\overline{\square}$	$\mathbf{1}$
S	$\mathbf{1}$	$\mathbf{16}$	$\mathbf{1}$	$\square$

$$W = \lambda Q L \overline{U}$$

global $SU(10) \supset$ SM or GUT

Single sector models

This is a baryon runaway model
for large $\det\overline{U} \gg \Lambda_{10}$

$$W_{\text{eff}} \sim \overline{U}^{10/k}$$

for small $\det\overline{U} \ll \Lambda_{10}$:

$$W_{\text{eff}} \sim \overline{U}^{10(1-\gamma)/k} \ ,$$

γ is the anomalous dimension of $\overline{U}$

for $10 \geq k > 10(1-\gamma)$ SUSY is broken

Single sector models

two composite generations corresponding to spinor S
composite squarks and sleptons have masses of order

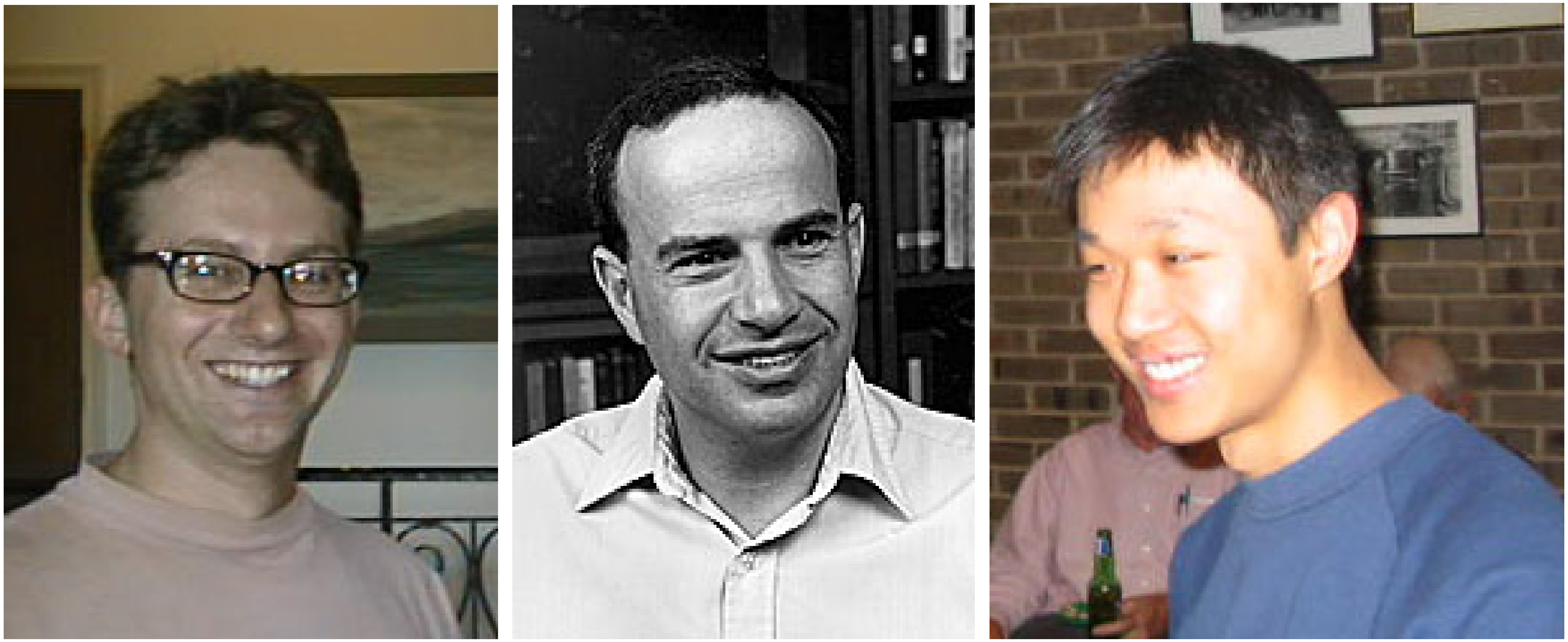
$$m_{\text{comp}} \approx \frac{\mathcal{F}}{\overline{\overline{U}}}$$

gauge mediation via the strong $SO(10)$ interactions
global $SU(2)$ enforces a degeneracy that suppresses FCNCs

composite fermions only get couplings to Higgs from higher dim. ops
gaugino and third-gen. scalars masses from gauge mediation

superpartners of the first two (composite) generations are much heavier than the superpartners of the third generation
similar to "more minimal" SUSY SM spectrum

Intriligator—Seiberg—Shih



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Intriligator—Seiberg—Shih

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)'$	$U(1)_R$
ϕ	$\square$	$\overline{\square}$	$\mathbf{1}$	1	1	0
$\overline{\phi}$	$\overline{\square}$	$\mathbf{1}$	$\square$	-1	1	0
M	$\mathbf{1}$	$\square$	$\overline{\square}$	0	-2	2

with the superpotential

$$W = \bar{\phi} M \phi - f^2 \mathrm{Tr} M$$

unbroken $SU(N) \times SU(F) \times U(1) \times U(1)' \times U(1)_R$

SUSY Breaking

$$\frac{\partial W}{\partial M_i^j} = \bar{\phi}_j \phi^i - f^2 \delta_j^i \neq 0$$

$\bar{\phi}_j \phi^i$ gets VEV $\Rightarrow SU(N)$ completely broken

but $\bar{\phi}_j \phi^i$ has rank $N < F$
$\Rightarrow M$ has non-zero $\mathcal{F}$ components

Wait a Minute

This is just a dual of SUSY $SU(F-N)$ QCD, quark masses $\propto f^2/\mu$

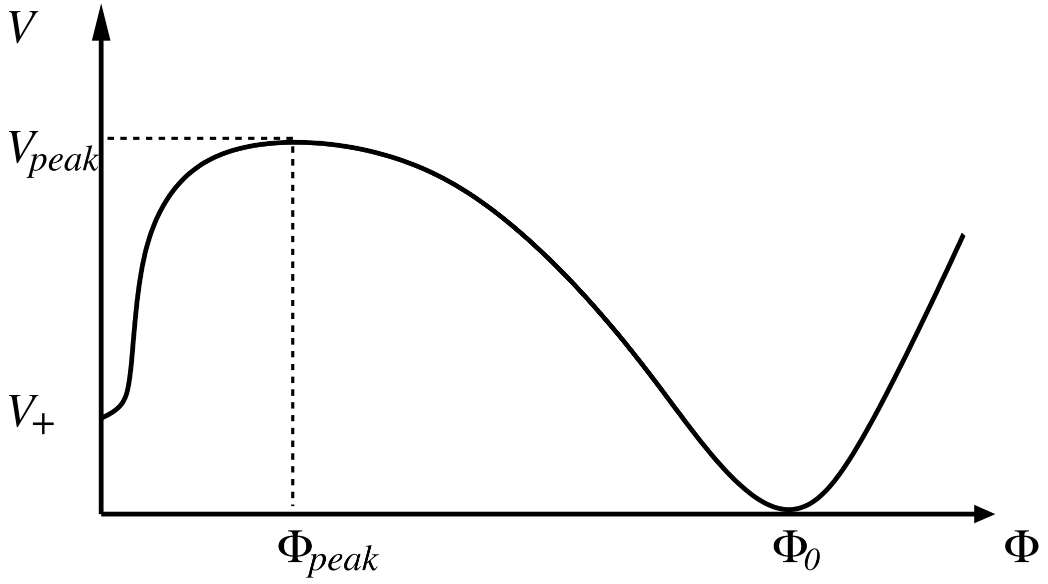
SUSY vacuum at

$$\langle M\rangle \propto f^{-2}\left(f^{2F}\Lambda^{3(F-N)-F}\right)^{1/(F-N)}$$

$$\langle M\rangle \gg f \;\text{ if }\; F > 3N$$

dual is IR free

Intriligator—Seiberg—Shih



tunnelling $\propto e^{-S}$

$S \gg 1$ if $F > 3N$