

Additional Exercises for Chapter 2

- Consider a Wess-Zumino-like model with the superpotential

$$W = \frac{y}{3}\phi^3 + \frac{\lambda}{4M}\phi^4 . \quad (1)$$

What are the off-shell SUSY transformations of the the scalar ϕ and its superpartner fermion ψ expressed only in terms of ϕ and ψ ?

- For the superpotential given in (1) what is the corresponding Lagrangian in terms of ϕ and ψ ?
- Schematically (include the parametric dependence on the couplings) what are the Feynman rules for the cubic and quartic interactions?
- Check that the SUSY transformation of the gauge field A_μ^a in

$$\mathcal{L}_{\psi \text{ gauge int.}} = -\psi^\dagger \bar{\sigma}^\mu g A_\mu^a T^a \psi , \quad (2)$$

cancels against the SUSY transformation of ϕ and ϕ^* in

$$\mathcal{L}_{Yukawa} = -\sqrt{2}g \left[(\phi^* T^a \psi) \lambda^a + \lambda^{\dagger a} (\psi^\dagger T^a \phi) \right] . \quad (3)$$

You will need to use the generalized Pauli identity (A.27). Note that the cancellation relates two terms where two fields are replaced by superpartners.

- For the superpotential

$$W = m \phi_2 \phi_3 + \frac{y}{2} \phi_1 \phi_3^2 ; \quad (4)$$

- calculate the scalar potential;
- show that the SUSY vacua are given by $\langle \phi_1 \rangle = v$; $\langle \phi_2 \rangle = 0$, $\langle \phi_3 \rangle = 0$, for arbitrary v ;
- expanding around the vacua $\phi_1 = v + \tilde{\phi}_1$, and writing the scalar potential out to quadratic order, find the mass squared matrix, M_ϕ^2 , for the scalars;
- find the fermion mass matrix M_ψ and verify that $M_\psi M_\psi^\dagger = M_\phi^2$.

- Using

$$V^a = \theta \sigma^\mu \bar{\theta} A_\mu^a + \theta^2 \bar{\theta} \lambda^{\dagger a} + \bar{\theta}^2 \theta \lambda^a + \frac{1}{2} \theta^2 \bar{\theta}^2 D^a , \quad (5)$$

perform the superspace integration for the Lagrangian

$$\mathcal{L} = \int d^4\theta \Phi^\dagger e^{2gT^a V^a} \Phi \quad (6)$$

keeping only terms of order g or higher. (We did the g independent terms in class.)

- For a general SUSY gauge theory with renormalizable interactions find the soft SUSY breaking terms that are produced by giving θ^2 spurion components to the background chiral superfields corresponding to the mass and Yukawa coupling, and the coefficient of $W_\alpha W^\alpha$ as well as the wavefunction renormalization:

$$Z = 1 + b\theta^2 + b^*\bar{\theta}^2 + c\theta^2\bar{\theta}^2 . \tag{7}$$