

Additional Exercises for Chapter 1

1. Construct the massive supermultiplet of $\mathcal{N} = 3$ SUSY for the lowest weight state (Clifford “vacuum” $|\Omega_0\rangle$) having spin 0. (Use the notation where the raising operator on this state produces a state $(\square, 2)$ where the \square indicates a 3 of the $SU(3)$ R -symmetry and 2 denotes a spin half doublet. You can use the following $SU(3)$ group theory results (keep in mind that $\bar{\square} = \square \leftrightarrow \bar{3}$)

$$\square \times \square = \square + \square \leftrightarrow 3 \times 3 = \bar{3}_A + 6_S, \quad (1)$$

$$\bar{\square} \times \square = 1 + \square \leftrightarrow \bar{3} \times 3 = 1 + 8. \quad (2)$$

Check that there are an equal number of bosonic and fermionic states in the supermultiplet. Is this state equivalent to a massive supermultiplet of $\mathcal{N} = 4$?

2. Consider $\mathcal{N} = 4$ SUSY with a 4×4 central charge matrix \mathbf{Z} . In a skew diagonal basis we can write

$$Z = \begin{pmatrix} Z_1 \epsilon^{ab} & 0 \\ 0 & Z_2 \epsilon^{ab} \end{pmatrix} \quad (3)$$

where $a = 1, 2$ and $b = 1, 2$. In this basis the SUSY algebra can be written as

$$\{Q_\alpha^{aL}, Q_{\dot{\alpha}bN}^\dagger\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_b^a \delta_N^L, \quad (4)$$

$$\{Q_\alpha^{aL}, Q_\beta^{bN}\} = 2\sqrt{2}\epsilon_{\alpha\beta}\epsilon^{ab}\delta^{LN}Z_N, \quad (5)$$

$$\{Q_{\dot{\alpha}aL}^\dagger, Q_{\dot{\beta}bN}^\dagger\} = 2\sqrt{2}\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{ab}\delta_{LN}Z_N, \quad (6)$$

where $L = 1, 2$; $N = 1, 2$; and the repeated index N is not summed over. Defining

$$A_\alpha^L = \frac{1}{2} \left[Q_\alpha^{1L} + \epsilon_{\alpha\beta} (Q_\beta^{2L})^\dagger \right], \quad (7)$$

$$B_\alpha^L = \frac{1}{2} \left[Q_\alpha^{1L} - \epsilon_{\alpha\beta} (Q_\beta^{2L})^\dagger \right], \quad (8)$$

reduces the algebra in the rest frame to

$$\{A_\alpha^L, A_{\beta N}^\dagger\} = \delta_{\alpha\beta}\delta_N^L(M + \sqrt{2}Z_L), \quad (9)$$

$$\{B_\alpha^L, B_{\beta N}^\dagger\} = \delta_{\alpha\beta}\delta_N^L(M - \sqrt{2}Z_L), \quad (10)$$

Consider a massive state with $M = \sqrt{2}Z_1 = \sqrt{2}Z_2$ and construct the short multiplet starting with the spin 0 Clifford “vacuum” $|\Omega_0\rangle$. How many raising operators are there? Label the elements of the multiplet with dimension of the R -symmetry representation, d_R , and the spin degeneracy $2j + 1$.