Intro to Supersymmetry
Part I: Basics

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Technion

This presentation contains material for my pre-susy 2015 (black-board) lectures. The ordering is somewhat different from the actual lectures.

There are many excellent references on supersymmetry, including the books by Wess and Bagger, Martin, Dine, Baer and Tata, Terning..

I will not have time to discuss supersymmetry breaking in detail here. For a very concise introduction see my Les Houches lectures hep-th/0601076.

Motivation

You have probably heard about the motivation for supersymmetry

Through these lectures, this will (hopefully) become clear and more concrete

But it's important to state at the outset:

There is no experimental evidence for supersymmetry

The amount of effort that has been invested in supersymmetry (theory and experiment) is thus somewhat surprising

[True: There is no experimental evidence for *any* underlying theory of electroweak symmetry breaking, which would give rise to the (fundamental scalar) Higgs mechanism as an effective description]

[There is experimental evidence for BSM:
dark matter, the baryon asymmetry—CP violation]

Why so much effort on supersymmetry?

It is a very beautiful and exciting idea

(I hope you'll see this in the lectures)

And it's something completely different from anything we know in Nature

supersymmetry is conceptually new: it relates bosons and fermions

And so is the Higgs: the first spin-zero (seemingly) fundamental particle

The herald of supersymmetry?

- The Higgs is the first scalar we see. Only scalars have quadratic divergences: supersymetry removes these divergences.
  We will see that in some sense: **Supersymmetry makes a scalar behave like a fermion**
- Given charged fermions: supersymmetry predicts spin-0 particles

Have we been wasting our time?

Supersymmetry is NOT a specific model (certainly not mSUGRA, cMSSM, minimal gauge mediation..)

There is a wide variety of supersymmetric extensions of the SM :

Different superpartner spectra, different signatures

In thinking about them: a whole toolbox:
triggers, searches, analysis
Even if it's not supersymmetry: may help discover something else

Supersymmetry supplies many concrete examples with:

- new scalars (same charges as SM fermions)
- new fermions (same charges as SM gauge bosons)

[and for discovery: spin is a secondary consideration]
potentially leading to

- missing energy
- displaced vertices
- (very) long lived particles
- disappearing tracks
- ...

Plan

Part I: Here we will see the basics through a few simple examples involving chiral super-multiplets. We will introduce supersymmetry, discuss the vacuum energy as an order parameter for supersymmetry breaking, and see how supersymmetry removes various divergences. Finally we will introduce auxiliary fields and superspace.

Part II: The Minimal Supersymmetric Standard Model: Here we will put to use what we learned in I.

- Motivation (now that you can appreciate it..)
- The field content
- The interactions: NO FREEDOM (almost)
- The supersymmetry-breaking terms: freedom + determine experimental signatures.

Part III: Here we will elaborate on supersymmetry breaking, and discuss various ways of mediating this breaking to the MSSM.

Spacetime symmetry:

The symmetry we are most familiar with:
Poincare:

- Translations $x^\mu \longrightarrow x^\mu + a^\mu$: generator P^μ
- Lorentz transformations: $x^\mu \longrightarrow x^\mu + w^\mu_\nu x^\nu$ generators: $J^{\mu\nu}$

(with $w^{\mu\nu}$ antisymmetric)

(Throughout consider global, infinitesimal transformations) contains:

rotations:
around axis k (with angle θ^k): $w^{ij} = \epsilon^{ijk}\theta^k$
eg for rotations around z:

$$x^0 \to x^0\,; \quad x^1 \to x^1 - \theta x^2\,; \quad x^2 \to x^2 + \theta x^1\,; \quad x^3 \to x^3$$

boosts:
along axis k (with velocity β^k): $-w^{0k} = w^{k0} = \beta^k$
eg for boost along z:

$$x^0 \to x^0 + \beta x^3\,; \quad ; \quad x^1 \to x^1\,; \quad x^2 \to x^2\,; \quad x^3 \to x^3 + \beta x^0$$

The algebra:

$$
\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[P^\mu, J^{\rho\sigma}] &= 0 \\
[J^{\mu\nu}, J^{\rho\sigma}] &= i(g^{\nu\rho} J^{\mu\sigma} - g^{\mu\rho} J^{\nu\sigma} - g^{\nu\sigma} J^{\mu\rho} + g^{\mu\sigma} J^{\nu\rho})
\end{aligned}
\qquad (1)
$$

Let's "discover" all the above in a simple field theory:
complex scalar field

$$\mathcal{L} = \partial^\mu \phi^* \, \partial_\mu \phi - m^2 \, |\phi|^2 \tag{2}$$

Symmetry: transformation of the fields which leaves the Equations Of Motion (EOMs) invariant
this is the case if action invariant, Lagrangian can change by a total derivative

$$\mathcal{L} \rightarrow \mathcal{L} + \alpha \partial_\mu \mathcal{J}^\mu \tag{3}$$

α=(small) parameter of the transformation

What's the symmetry of this theory?
U(1):

$$\phi(x) \rightarrow e^{i\alpha} \phi(x) \tag{4}$$

$\mathcal{L}$ is invariant   this U(1) is an Internal Symmetry (NOT space-time)

Spacetime:
Translations:

$$x^\mu \;\longrightarrow\; x^\mu + a^\mu \tag{5}$$

$$\phi(x) \to \phi(x-a) \;=\; \phi(x) - a^\mu \partial_\mu \phi(x) \tag{6}$$

or

$$\delta_a \phi(x) = a^\mu \partial_\mu \phi(x) \tag{7}$$

Lorentz transformations

$$x^\mu \;\longrightarrow\; x^\mu + w^{\mu\nu} x_\nu \tag{8}$$

$$\phi(x^\mu) \;\longrightarrow\; \phi(x^\mu - w^{\mu\nu} x_\nu) \tag{9}$$

so

$$\delta_w \phi(x) = w^{\mu\nu} x_\mu \partial_\nu \phi(x) = \frac{1}{2} w^{\mu\nu} (x_\mu \partial_\nu - x_\nu \partial_\mu)\, \phi(x) \tag{10}$$

action is invariant

Algebra:
2 translations (with a^μ, b^μ)

$$[\delta_a, \delta_b]\phi \equiv \delta_a(\delta_b\phi) - \delta_b(\delta_a\phi) = 0 \tag{11}$$

2 Lorentz: (with $w^{\mu\nu}$, $\lambda^{\rho\sigma}$)

$$[\delta_{w^{\mu\nu}}, \delta_{\lambda^{\rho\sigma}}]\phi = iw_{\mu\nu}\lambda_{\rho\sigma} \cdot i \, \{g^{\nu\rho}(x_\mu\partial_\sigma - x_\mu\partial_\sigma) + \text{ permutations}\} \tag{12}$$

let's move now to a supersymmetric theory

A simple supersymmetric field theory

Free theory with massive (Dirac) fermion ψ of mass m
2 complex scalars ϕ_+, ϕ_- of mass m

$$\mathcal{L} = \partial^\mu \phi_+^* \, \partial_\mu \phi_+ - m^2 \left|\phi_+\right|^2 + \partial^\mu \phi_-^* \, \partial_\mu \phi_- - m^2 \left|\phi_-\right|^2 + \bar{\psi}(i\partial\!\!\!/ - m)\psi \quad (13)$$

[the labels $+$, $-$ are just names, we'll see the reason for this choice soon]

[This isn't the most *minimal* supersymmetric 4d field theory. "Half of it" is: a 2-component (Weyl) fermion plus one complex scalar. But Dirac spinors are more familiar so start with this.]

spacetime symmetry:

translations, rotations, boosts: just as in our previous example

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only difference:
ψ itself is a spinor, so transforms:

$$\psi(x) \to \psi'(x') \tag{14}$$

actually, the L-handed and R-handed parts transform differently under Lorentz

$$\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{15}$$

with

$$\psi_L \quad \rightarrow \quad \psi'_L = (1 - i\theta^i \frac{\sigma^i}{2} - \beta^i \frac{\sigma^i}{2}) \tag{16}$$

$$\psi_R \quad \rightarrow \quad \psi'_R = (1 - i\theta^i \frac{\sigma^i}{2} + \beta^i \frac{\sigma^i}{2}) \tag{17}$$

so it will be useful to write everything in terms of 2-component spinors recall: we can write any R-handed spinor in terms of a L-handed one:

$$\psi_R = -\varepsilon\chi_L^* \tag{18}$$

where

$$\varepsilon \equiv -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{19}$$

Exercise: prove eq. (18)
so we can write our Dirac spinor in terms of two **L-handed spinors** ψ_+ and ψ_-: $\psi_L = \psi_+$, $\psi_R = -\varepsilon\psi_-^*$

$$\rightarrow \psi = \begin{pmatrix} \psi_+ \\ -\varepsilon\psi_-^* \end{pmatrix} \tag{20}$$

$$\}$$

let's write the Lagrangian in terms of these:

$$\begin{aligned} \mathcal{L} &= \partial^\mu \phi_+^* \, \partial_\mu \phi_+ + \psi_+^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_+ \\ &+ \partial^\mu \phi_-^* \, \partial_\mu \phi_- + \psi_-^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_- \\ &- m^2 \, |\phi_-|^2 - m^2 \, |\phi_+|^2 - m(\psi_+^T \varepsilon \psi_- + \mathrm{hc}) \end{aligned} \tag{21}$$

exercise: Derive this. Show also that $\psi_+^T \varepsilon \psi_- = \psi_-^T \varepsilon \psi_+$, where $\psi_\pm$ are any 2-component spinors.

Can the spacetime symmetry be extended?

Yes: there's more symmetry hiding in our theory:

take a constant (anti-commuting) (L) 2-component spinor ξ

$$\begin{aligned} \delta_\xi \phi_+ &= \sqrt{2}\, \xi^T \varepsilon \psi_+ \\ \delta_\xi \psi_+ &= \sqrt{2}\, i \sigma^\mu \varepsilon \xi^* \partial_\mu \phi_+ - m \xi \phi_-^* \end{aligned} \tag{22}$$

and similarly for $+ \to -$

the symmetry transformations take a boson into a fermion and vice versa: THIS IS SUPERSYMMETRY!

exercise: Check that this is indeed a symmetry:

1. Show this first for $m = 0$.
2. Repeat for $m \neq 0$. Note that this only holds if the masses of the fermion and scalars are the same.

Note: For $m = 0$, the $\phi_- - \psi_-$ and $\phi_+ - \chi_+$ parts decouple. each one is super-symmetric separately

so: this theory is not the most minimal supersymmetric theory (half of it is)
this is handy if we're to implement supersymmetry in the SM, because the SM is a *chiral theory*

is the symmetry we found indeed an extension of Poincare?

it's surely a spacetime symmetry since it takes a fermion into a boson (the transformation parameters carry spinor indices)

furthermore: consider the algebra:
take the commutator of 2 new transformations with parameters ξ, η:

$$[\delta_\xi, \delta_\eta]\phi_L = a^\mu \partial_\mu \phi_L \qquad \text{with} \qquad a^\mu = 2i\left(\xi^\dagger \bar{\sigma}^\mu \eta - \eta^\dagger \bar{\sigma}^\mu \xi\right) \tag{23}$$

a translation!
Exercise: Check eq. (23). You will have to use the EOMs.

Our simple theory is supersymmetric.
We have an extension of spacetime symmetry that involves anti-commuting generators.
The supersymmetry transformations relate bosons and fermions.
If the bosons and fermions had different masses: no supersymmetry.
And let's count the physical dof's: on-shell we have
fermions: $2 + 2 = 4$
bosons: $2 + 2 = 4$
(off shell: bosons same, but fermions: 2×4)

Global symmetries $\rightarrow$ Noether currents
j^μ with $\partial_\mu j^\mu = 0$ so that there is a conserved charge:

$$Q = \int d^3x \, j^0(x) \;\; \text{with} \;\; \frac{d}{dt} Q = 0 \tag{24}$$

For a susy transformation ξ_α: $j_{\mu\alpha}$ and the charge Q_α.
Found:

$$\left\{ Q_\alpha, Q^\dagger_\beta \right\} = 2\sigma^\mu_{\alpha\beta} P_\mu \tag{25}$$

The vacuum energy

In a supersymmetric theory:

$$Q_\alpha|0>=0 \tag{26}$$

so

$$0=\langle 0|\left\{\left\{Q_\alpha, Q_\beta^\dagger\right\}\right\}|0\rangle \propto \sigma^\mu_{\alpha\beta}\langle 0|P_\mu|0\rangle = \sigma^0_{\alpha\beta}\langle 0|H|0\rangle \tag{27}$$

so if SUSY unbroken:

$$\langle 0|H|0\rangle = 0 \tag{28}$$

The vacuum energy vanishes!!

The vacuum energy is an order parameter for susy breaking

Also note that for any state, the energy is proportional to

$$\left| Q_\alpha^\dagger | state \rangle \right|^2 \geq 0 \tag{29}$$

So looking for theories with spontaneous susy breaking is simple: one must find a potential with no zero-energy ground state, ie, $V > 0$.

The simplest supersymmetric theory with chiral supermultiplets that breaks supersymmetry spontaneously is the O'Raifeartaigh model.
it has 3 chiral supermultiplets:

$$(\phi, \psi)\,, \qquad (\phi_1, \psi_1)\,, \qquad (\phi_2, \psi_2) \tag{30}$$

and two mass parameters
I will only write now the scalar potential. We'll see the full Lagrangian later.

$$V = |y\phi_1^2 - f|^2 + m^2|\phi_1|^2 + |2\phi_1\phi + m\phi_2|^2 \tag{31}$$

here m is a mass, f has dimension mass^2, y is a dimensionless coupling
because of the special form of this potential, it is VERY easy to see that there is no supersymmetric minimum:
the first two terms cannot vanish simultaneously
supersymmetry is broken!

note that need $f \neq 0$ for that (must push some field away from origin) (and also $m \neq 0$)
let's assume $f < m^2/(2y)$
the ground state is at $\phi_1 = \phi_2 = 0$ with ϕ arbitrary (a flat direction of the potential)

$$V_0 = |f|^2 \tag{32}$$

when we write the full Lagrangian you'll show that the spectrum is:
fermions: one massless Weyl fermion
one Dirac fermion of mass m
(real) bosons: 2 massless, 2 with m^2, one with $m^2 + 2yf$, and one with $m^2 - 2yf$,
[for $f = 0$ susy restored: degeneracy restored]

why massless bosons?
because ϕ is arbitrary: flat direction (2 real dof's)

why a massless Weyl fermion?
normally a sponatneously broken global symmetry $\longrightarrow$ massless Goldstone boson
here: spontaneously broken *super*symmetry $\longrightarrow$ massless Goldstone fermion
generated by broken SUSY transformation

$$\mathrm{SUSY}|0\rangle \neq 0 \tag{33}$$

we will say more about this later.

note: needed a scale (we put it in by hand)
suppose we started with no scale in Lagrangian
susy unbroken
perturbation theory: no scale generated
so: susy unbroken *to all orders in perturbation theory!!*

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this is a very strong result
it's a consequence of the constrained structure of supersymmetry (holomorphy of superpotential)
it's true generally:
if susy is unbroken at tree level, it can only be broken by non-perturbative effects, with a scale that's generated dynamically:
just as in QCD:

$$\Lambda = M_{UV} \exp\left(\frac{-8\pi^2}{bg^2}\right) \tag{34}$$

exponentially suppressed compared to cutoff scale
this is called: **dynamical** supersymmetry breaking
we will come back to this when we discuss the standard model but leads to a beautiful scenario:
the supersymmetry breaking scale can naturally be 16 or so orders of magnitude below the Planck scale

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but remember: the vacuum energy usually diverges
and it's the worst divergence: quartic

(just like in the case of the harmonic oscillator: the infinite constant that we choose to be the zero energy in "Normal Ordering")

here: supersymmetry completely removes this divergence!

so this gives us hope that supersymmetry can help with other UV divergences

the next worst divergence: quadratic
where? scalar mass squared:

$$\delta m^2 \propto \Lambda^2 \tag{35}$$

This is why we are worried about fine tuning in the Higgs mass

But no one ever worries about the electron mass!
Why?
because fermion masses have no quadratic divergences!
only log divergences!
This is a very important result so we will see it in 3 ways
(words+math, just words, symmetry
dimensional analysis!)

why is there no quadratic divergence in the fermion mass? (1)
if start from some m_0 in the Lagrangian

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ - m_0)\psi \\ &= \bar{\psi}(i\partial\!\!\!/)\psi - m_0(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L) \end{aligned} \tag{36}$$

so if $m_0 = 0$ ψ_L, ψ_R don't talk to each other:
a mass (=L-R coupling) is never generated
then

$$\delta m \propto m_0 \tag{37}$$

with $m_0 = 0$ we have 2 different species: ψ_L—call it a "blue" fermion, and ψ_R, a red fermion, and they don't interact at all

Another way to see this (2):
consider $m_0 \neq 0$:
take a L fermion (spin along $\hat{p}$)   the blue fermion
we can run very fast alongside: $\hat{p} \longrightarrow -\hat{p}$, spin stays same:
L becomes R
the blue fermion turns into a red fermion
(helicity is not a good quantum number)
but if $m_0 = 0$: can never run fast enough..
$\longrightarrow$ L and R are distinct
the blue fermion and the red fermion are decoupled

we learn:

$$\delta m \propto m_0 \tag{38}$$

How can the cutoff Λ enter?
dimensional analyis:

$$\delta m \propto m_0 \log \frac{m_0}{\Lambda} \tag{39}$$

so

$$\delta m = 0 \cdot \Lambda + \# \, m_0 \log \frac{m_0}{\Lambda} \tag{40}$$

why is there no divergence in the fermion mass? (3)
CHIRAL SYMMETRY

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/)\psi - m_0(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \tag{41}$$

when $m_0 = 0$: $U(1)_L \times U(1)_R$
this forbids the mass term (would break to the diagonal $U(1)_V$)

supersymmetry: boson mass = fermion mass

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chiral symmetry: no quadratic divergence in fermion mass

so:

in a supersymmetric theory: no quadratic divergence in boson mass

but there's more:

You could say: we know there is no supersymmetry in Nature:
there is no scalar with mass $=$ electron mass
so why should we care?

supersymmetry is so powerful that even when it's broken by mass terms the quadratic divergence doesn't reappear!
all we need to see this is dimensional analysis:

what if we take a supersymmetric theory with a scalar and a fermion of mass m_0 and change the scalar mass to:

$$m_0^2 + \tilde{m}^2 \tag{42}$$

will there be a quadratic divergence in the scalar mass?

$$\delta m^2_{scalar} = \#\Lambda^2 + \# {m_0}^2_{scalar} \log \frac{{m_0}^2_{scalar}}{\Lambda^2} \qquad ?? \tag{43}$$

NO: again because of dimensional analysis:
for $\tilde{m}^2 = 0$: supersymmetry restored: there shouldn't be a quadratic divergence
so Λ^2 term must be proprtional to $\tilde{m}^2$
but there's nothing we can write in perturbation theory that would have the correct dimension

so: if supersymmetry is broken by

$$m_{0\,scalar}^{2} \neq m_{0\,fermion}^{2} \tag{44}$$

the scalar mass-squared has only log divergences

(the supersymmetry breaking does not spoil the cancellation of the quadratic divergence)

This type of breaking is called **soft**-supersymmetry breaking

[this is what we have in the MSSM]

[as opposed to **hard** breaking:

change some pure number:

take a supersymmetric theory and change the coupling of bosons compared to the coupling of a fermion

reintroduces divergences]

we derived all these results based on dimensional analysis
now let's see them concretely
for that we have to add interactions:
so go back to our simple theory

$$\begin{aligned} \mathcal{L} &= \partial^\mu \phi_+^* \, \partial_\mu \phi_+ + \psi_+^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_+ && (45) \\ &+ \partial^\mu \phi_-^* \, \partial_\mu \phi_- + \psi_-^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_- && (46) \\ &- m^2 \, |\phi_-|^2 - m^2 \, |\phi_+|^2 - m(\psi_+^T \varepsilon \psi_- + \mathrm{hc}) && (47) \end{aligned}$$

our 2 fermions look like the two pieces of an electron or a quark:
ψ_- (like the SM SU(2)-doublet quark)
ψ_+ (like the SM SU(2)-singlet quark)
so let's add a complex scalar h:
with the "Yukawa" interaction:

$$\delta\mathcal{L} = -y\, h\, \psi_+^T \varepsilon \psi_- + \mathrm{hc} \tag{48}$$

here y is the coupling
of course to make a supersymmetric theory we need also
a (L) fermion $\tilde{h}$ (their susy transformations are just like ϕ_+ and ψ_+)

for simplicity let's set $m = 0$

(if h and $\tilde{h}$ remind you of the Higgs and Higgsino that's great, but here they have nothing to do with mass generation, we are just interested in the interactions)

it's easy to see that if we just add this Yukawa interaction, the Lagrangian is not invariant under susy

so we must add more interactions:

$$\begin{aligned} \mathcal{L} &= \partial^\mu \phi_+^* \, \partial_\mu \phi_+ + \partial^\mu \phi_-^* \, \partial_\mu \phi_- + \partial^\mu h^* \, \partial_\mu h \\ &+ \psi_+^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_+ + \psi_-^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_- + \tilde{h}^\dagger i \bar{\sigma}^\mu \partial_\mu \tilde{h} \\ &+ \mathcal{L}_{int} \end{aligned} \tag{49}$$

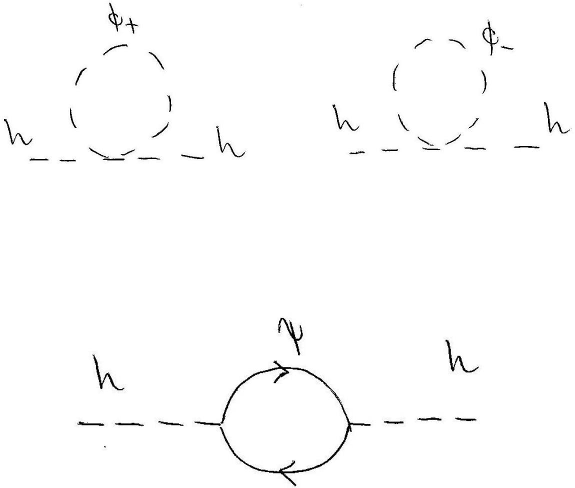
with

$$\begin{aligned} \mathcal{L}_{int} = & - y \left(h \psi_+^T \varepsilon \psi_- + \phi_+ \tilde{h}^T \varepsilon \psi_- + \phi_- \tilde{h}^T \varepsilon \psi_+ + \mathrm{hc} \right) \\ & - |y|^2 [|\phi_+|^2 \, |\phi_-|^2 + |h|^2 \, |\phi_-|^2 + |h|^2 \, |\phi_+|^2] \end{aligned} \tag{50}$$

now that we have an interacting supersymmetric theory
we are ready to consider the UV divergence in the scalar mass-squared

take δm_h^2

there's a ϕ_+ loop, a ϕ_- loop and a fermion loop



for the latter: let's convert to Dirac fermions:

$$y\, h\psi_+^T \varepsilon \psi_- + \mathrm{hc} = yh\bar{\psi} P_L \psi + \mathrm{hc} \tag{51}$$

so fermion loop: ("top contribution to Higgs mass")

$$-|y|^2 \int \frac{d^4 p}{(2\pi)^4} \mathrm{Tr} P_L \frac{i}{\not{p}} P_R \frac{i}{\not{p}} = 2|y|^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \tag{52}$$

boson loop: ("stop contribution to Higgs mass")

$$2 \times i|y|^2 \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2} = -2|y|^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \tag{53}$$

before we argued that the cancellation is not spoiled by soft supersymmetry breaking
let's see this in this example:
suppose we changed the $\phi_\pm$ masses-squared to $\tilde{m}^2_\pm$:
still no quadratic divergence

$$\begin{aligned} \delta m_h^2 &\propto |y|^2 \int \frac{d^4 p}{(2\pi)^4} \left[\frac{2}{p^2} - \frac{1}{p^2 - \tilde{m}_+^2} - \frac{1}{p^2 - \tilde{m}_-^2} \right] \\ &= |y|^2 \, \tilde{m}_1^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 (p^2 - \tilde{m}_+^2)} + (\tilde{m}_+^2 \to \tilde{m}_-^2) \end{aligned} \tag{54}$$

we see:
when supersymmetry is softly broken:
scalar mass squared is log divergent
the divergence is proportional to the susy breaking $\tilde{m}^2$

as opposed to a "hard breaking": if we changed one of the 4-scalar coupling from $|y|^2$: quadratic divergence we now know a lot of susy basics (more than half of what we need ..)

let's recap and add some language:

supersymmetry is an extension of Poincare: it's a spacetime symmetry

the basic supersymmetry "module" we know is

(a complex scalar + 2-component spinor) OF SAME MASS
eg

$$(\phi_+, \psi_+) \tag{55}$$

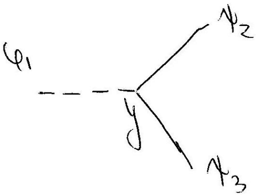
these transform into each other under supersymmetry

[together they form a representation, or multiplet of supersymmetry for obvious reasons, we call this the "chiral supermultiplet"]

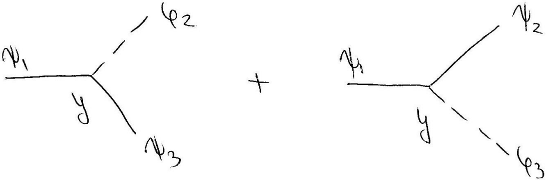
on-shell: number of fermionic dof's = number of bosonic dof's (true generally) we also saw:

supersymmetry dictates not just the field content but also the interactions:

the couplings of fermions, bosons of the same supermultiplets are related (true generally) starting from scalar1-fermion2-fermion3 vertex

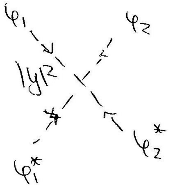


supersymmetry requires also
fermion1-scalar2-fermion3 + fermion1-fermion2-scalar3



all with same coupling

+ 4 scalar (**same** coupling squared)



+ . . .

you see that the structure of supersymmetric theories is very constrained
and that as a result it's less divergent

[
this is the *real* reason theorists like supersymmetry
it's easier..
(the more supersymmetry, the easier it gets
less divergences, more constraints, can calculate many things, even at strong coupling
by the time you get to maximal supersymmetry in 4d: a finite theory, scale invariance)
]

we also know a great deal about supersymmetry breaking:

unbroken supersymmetry: vacuum energy $= 0$

so: the vacuum energy is an order parameter for supersymmetry breaking

and therefore supersymmetry breaking $\leftrightarrow$ scale: $E_{\rm vac}$

supersymmetry (breaking) and UV divergences:

ubroken supersymmetry: only log divergences

soft breaking (=by dimensionful quantities): only log divergences

hard breaking (by pure numbers): reintroduces quadratic divergences
[so not that interesting]

Auxiliary fields

auxiliary fields are non-dynamical fields which we add to the theory to simplify calculations (we will see how this works now).
I avoided them so far in order to emphasize that these are really technical tools
in fact we could finish these lectures and understand everything we need about supersymmetry without them
remember that the susy algebra closed only on-shell. To show that the commutator of 2 susy transformations gives translations we had to use the EOMs
it would be nice to have a Lagrangian for which they hold off-shell too

so let's add an auxiliary field to each chiral multiplet, say

$$(\phi_+, \psi_+, F_+) \tag{56}$$

$$\begin{aligned}
\delta_\xi \phi_+ &= \sqrt{2}\xi^T \varepsilon \psi_+ & (57)\\
\delta_\xi \psi_+ &= \sqrt{2} i \sigma^\mu \varepsilon \xi^* \partial_\mu \phi_+ + \sqrt{2}\xi F & (58)\\
\delta_\xi F_+ &= -\sqrt{2} i \xi^\dagger \bar{\sigma}^\mu \partial_\mu \psi & (59)
\end{aligned}$$

F is a scalar of dimension 2
Note that its transformation is a TOTAL DERIVATIVE

The Lagrangian:

$$\begin{aligned}\mathcal{L} &= \Big[\partial^\mu \phi_+^* \, \partial_\mu \phi_+ + \psi_+^\dagger i\bar{\sigma}^\mu \partial_\mu \psi_+ + F_+^* F_+ + (m\phi_- F_+ + hc) \\ &+ (+ \leftrightarrow -)\Big] \\ &- m(\psi_+^T \varepsilon \psi_- + hc)\end{aligned} \tag{60}$$

F_+ is not dynamical so we can solve for it using its EOM:
$F_\pm^* = -m\phi_\mp$
(which restores the susy transformations we had before)

BUT:

$$\delta_\xi \mathcal{L} = 0 \tag{61}$$

and

$$[\delta_\xi, \delta_\eta] = a^\mu \partial_\mu \qquad \text{with} \qquad a^\mu = 2i\left(\xi^\dagger \bar{\sigma}^\mu \eta - \eta^\dagger \bar{\sigma}^\mu \xi\right) \tag{62}$$

even OFF SHELL

exercise: Check these 2 statements.

Now we can write the most general Lagrangian involving interacting chiral supermultiplets

$$(\phi_i, \psi_i, F_i) \tag{63}$$

choose an analytic function of ϕ_i:

$$W = W(\phi_i) \tag{64}$$

W is called the superpotential (because it determines the potential associated with the chiral fields in supersymmetric theories)
note W is analytic (or holomorphic): it does not depend on any ϕ_i^*

construct the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int} \tag{65}$$

where

$$\mathcal{L}_{\rm kin} = \partial^\mu \phi_i^* \, \partial_\mu \phi_i + \psi_i^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_i + F_i^* F_i \tag{66}$$

and

$$\mathcal{L}_{\rm int} = \frac{\partial W}{\partial \phi_i} F_i - \frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i^T \varepsilon \psi_j + {\rm hc} \tag{67}$$

This Lagrangian is supersymmetric!

exercise: Check. You only need to check this for the interacting part. We already know that the kinetic part is invariant.

indeed: supersymmetric interactions are very constrained
we can now derive many of the results we saw/quoted quite generally and elegantly:
first, let's solve for F_i:

$$F_i^* = -\frac{\partial W}{\partial \phi_i} \tag{68}$$

substituting this back into the Lagrangian:

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 \tag{69}$$

indeed $V \geq 0$!

our first example had $W = m\phi_+\phi_-$
our second example had $W = y\, h\, \phi_+\phi_-$
The O'Raifeartaigh model has $W = \phi(\phi_1^2 - f) + m\phi_1\phi_2$.
You can now re-derive all our results, as well as the spectrum of the O'Raifeartaigh model.

Supersymmetric Lagrangians

What are the Lagrangians we can write down?
With a theory of such a constrained structure, you expect to have many limitations
indeed: all the theories we can write down are encoded in 2 functions:
The Kähler potential (K): gives the kinetic and gauge interactions (as we will see, there is no freedom there at the level of 4d terms)
The superpotential (W): gives the non-gauge (Yukawa like) interactions of chiral fields
to see all this simply, we will introduce superspace.