Composite Higgs

Tony Gherghetta : Pre SUSY 2015 Aug. 21-22, 2015

[Reviews Contino : 1005.4296; Bellazzini, Csaki, Serra : 1401.2457; Panico, Wulzer 1506.01961]

EWSB

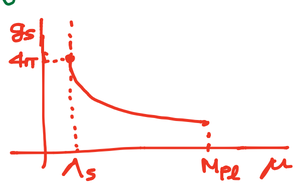
Higgs potential : $V(H) = -m_H^2 |H|^2 + \lambda_H |H|^4$ where $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

Higgs discovery: $m_h^2 = 2\lambda_H v^2 \simeq (125\,\text{GeV})^2$ & $v^2 = \frac{m_H^2}{\lambda_H} = (246\,\text{GeV})^2$

$\Rightarrow \boxed{m_H^2 = (89\,\text{GeV})^2}$ and $\boxed{\lambda_H \simeq 0.13}$

Why is $m_H \ll \underbrace{M_{Pl}}_{\text{Planck scale} \simeq 10^{19}\,\text{GeV}}$? Possible answer : strong dynamics!

New strong force with coupling g_s :



$$\frac{d}{d\ln\mu}\left(\frac{1}{g_s^2}\right) = \frac{-b_s}{16\pi^2}$$

$$\Rightarrow \Lambda_s = M_{Pl}\, e^{-\frac{8\pi^2}{g_s^2(M_{Pl})|b_s|}} \quad (b_s < 0)$$

$$\Rightarrow m_H^2 \sim \Lambda_s^2 \ll M_{Pl}^2$$

Similar to QCD! $\Lambda_{QCD} \sim 250\,\text{MeV} \ll M_{Pl}$

Idea: Higgs boson $\Leftrightarrow$ bound state of new strong dynamics

But strong dynamics will also produce other bound states.

Question Can Higgs "bound state" be naturally lighter?

Yes! Higgs can be a pseudo Nambu-Goldstone boson
[Georgi, Kaplan 1984]

<u>Analogy</u> Pions in QCD In limit $m_u, m_d \to 0$

$$\text{global symmetry}: \mathcal{G} = \underbrace{SU(2)_L}_{\binom{u}{d}_L \to g_L \binom{u}{d}_L} \times \underbrace{SU(2)_R}_{\binom{u}{d}_R \to g_R \binom{u}{d}_R}$$

("chiral" symmetry)
$g_{L,R} \in SU(2)_{L,R}$

<u>Spontaneously broken</u> $\downarrow$ $\langle \bar{q}_L q_R \rangle \neq 0$ QCD interactions cause quark-antiquark pairs to condense

$$\text{unbroken global symmetry}: \mathcal{H} = SU(2)_V$$

(vector "isospin" symmetry
$g_L = g_R$ preserved)

<u>Goldstone's theorem</u> For every spontaneously broken continuous symmetry, there is a <u>massless</u> particle (<u>Nambu-Goldstone boson</u>)

If global symmetry $\mathcal{G}$ broken to $\mathcal{H}$, then the number of NG bosons = dimension of coset space $\underbrace{\mathcal{G}/\mathcal{H}}$. i.e. $\dim \mathcal{G}/\mathcal{H} = \dim \mathcal{G} - \dim \mathcal{H}$

equivalence classes of elements in $\mathcal{G}$
consider $g_1, g_2 \in \mathcal{G}$ then $g_1 \sim g_2$ if $\exists\, h \in \mathcal{H}$ s.t. $g_1 = g_2 h$

<u>QCD</u> $\mathcal{G} = SU(2)_L \times SU(2)_R \longrightarrow \mathcal{H} = SU(2)_V \Rightarrow \dim \mathcal{G}/\mathcal{H} = \dim \mathcal{G} - \dim \mathcal{H} = 6 - 3 = 3$

$\Rightarrow$ 3 <u>Nambu-Goldstone bosons</u> (pions - $\underbrace{\pi^a}_{\text{isospin triplet}}$)

Strong dynamics can be described by "linear σ-model" (mimics effect of $\bar{q}q$ condensate)

$$\mathcal{L} = |\partial_\mu \Sigma|^2 + m^2 |\Sigma|^2 - \frac{\lambda}{4}|\Sigma|^4$$

where $\Sigma =$ complex scalar
$\Sigma \to g_L \Sigma g_R^\dagger$
$g_{L,R} \in SU(2)_{L,R}$

Potential minimum: $\langle \Sigma \rangle = \frac{V}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ where $V^2 = \frac{2m}{\sqrt{\lambda}}$

Fluctuations: $$\Sigma(x) = \frac{1}{\sqrt{2}}(V+\sigma(x))\,\underbrace{e^{i\frac{2}{f_\pi}\pi^a \tau^a}}_{\equiv U(x)} = \frac{1}{\sqrt{2}}(V+\sigma(x))\exp\left[\frac{i}{f_\pi}\begin{pmatrix} \pi^0 & \sqrt{2}\pi^- \\ \sqrt{2}\pi^+ & \pi^0 \end{pmatrix}\right]$$

τ^a: broken G/H generators

Note Symmetry is not really "broken", instead is realized nonlinearly

$$\Sigma \to g_L \Sigma g_R^\dagger \quad (g_{L,R} = e^{i\theta_{L,R}\tau^a}) \Rightarrow \pi^a \to \pi^a + \underbrace{\frac{f_\pi}{2}(\theta_L - \theta_R)}_{\text{constant } (\theta_L \neq \theta_R)} + \ldots$$ "shift symmetry"

$\Rightarrow$ Pion potential: $V(\pi) = 0$ $(m_\pi = 0)$ (at quantum level)

Shift symmetry forbids a pion mass term!

However:

Quark masses explicitly break chiral symmetry by small amount

$$\mathcal{L}_m = \bar{q}Mq \quad \text{where } M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

Treat M as a spurion where $M \to g_L M g_R^\dagger$ $\Rightarrow$

$$\mathcal{L} = \frac{V^3}{2}\mathrm{Tr}(MU + M^\dagger U^\dagger)$$
$$= \underbrace{-\frac{1}{2}\frac{V^3}{f_\pi^2}(m_u+m_d)}_{m_\pi^2}\vec{\pi}^2 + \ldots$$

$\Rightarrow$ $m_\pi \neq 0$ but $m_\pi \ll m_\rho$ since $m_{u,d}$ are small

m_ρ: generic resonance mass scale

$\Rightarrow$ pion = pseudo Nambu-Goldstone boson

Substitute Σ into $\mathcal{L}$ $\Rightarrow$ chiral Lagrangian
↳ describes pion interactions

Do something similar for the Higgs boson!

Consider global symmetry groups with coset $\mathcal{G}/\mathcal{H} \supset$ Higgs doublet!

Generate Higgs potential? Break EW symmetry?

Note Strong dynamics does not break EW symmetry
$\longrightarrow$ differs from technicolor (scaled-up version of QCD)!

Suppose new strong dynamics spontaneously breaks global symmetry $\mathcal{G}$ at scale f:

$$\mathcal{G} \xrightarrow{f} \underbrace{\mathcal{H}}_{\text{unbroken global group}}$$

Require:

- $\mathcal{H} \supset SU(2)_L \times U(1)_Y$ = SM electroweak gauge group
- Coset $\mathcal{G}/\mathcal{H}$ must contain a Higgs doublet (4 real scalar fields)

Note The underlying strong dynamics is not specified and not needed to determine Higgs properties.

SO(5)/SO(4) model

[Agashe, Contino, Pomarol 2004]

Consider $SO(5) \xrightarrow{f} SO(4)$

Recall $SO(n)$ = group of $n\times n$ orthogonal matrices with unit determinant
Lie alegbra = $n\times n$, traceless, imaginary Hermitian matrices $\Rightarrow$ dim = $\frac{n(n-1)}{2}$

$\langle H\rangle \neq 0$ $\rightarrow SO(3)_c$ = custodial symmetry (required to ensure $\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$)

- $SO(4) \cong SU(2)_L \times SU(2)_R \supset SU(2)_L \times U(1)_Y$
- number NG bosons = dim $SO(5)/SO(4)$ = dim $SO(5)$ - dim $SO(4)$
  = $10 - 6$
  = $\boxed{4}$ $\leftarrow$ Higgs doublet
  $\Rightarrow$ $SO(5)/SO(4)$ = minimal model

Model $SO(5) \to SO(4)$ breaking with scalar fields $\vec{\Phi} = \underline{5}$ of $SO(5)$ and effective Lagrangian: "linear σ-model"

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\Phi}^T \partial^\mu \vec{\Phi} - \frac{g_\rho^2}{8}\left(\vec{\Phi}^T\vec{\Phi} - f^2\right)^2$$

where g_ρ = composite sector coupling ($1 \lesssim g_\rho < 4\pi$)

Fluctuations about potential minimum:

$$\vec{\Phi} = e^{i\frac{\sqrt{2}}{f}\overbrace{\pi_i(x)}^{\text{4 real scalar fields parametrize coset}}\hat{T}^i} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f+\sigma(x) \end{bmatrix} = (f+\sigma)\begin{bmatrix} \sin\frac{\pi}{f}\cdot \vec{\pi} \\ \cos\frac{\pi}{f} \end{bmatrix}$$

where $\hat{T}^i$ = broken $SO(5)/SO(4)$ generators ($i = 1, \dots 4$)

$$\underbrace{\vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{pmatrix}}_{SO(4)\text{ fourplet}} \qquad \pi = \sqrt{\vec{\pi}^T\vec{\pi}} \qquad \text{with Higgs doublet } H = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_2 + i\pi_1 \\ \pi_4 - i\pi_3 \end{pmatrix}$$

Substituting $\vec{\Phi}$ into $\mathcal{L}$ gives:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}(g_\rho f)^2\sigma^2 - \frac{1}{2}g_\rho^2 f\sigma^3 - \frac{1}{8}g_\rho^2\sigma^4$$
$$+ \frac{1}{2}\left(1+\frac{\sigma}{f}\right)^2\left[\frac{f^2}{|H|^2}\sin^2\frac{\sqrt{2}|H|}{f}\,\partial_\mu H^\dagger\partial^\mu H + \frac{f^2}{4|H|^4}\left(\frac{2|H|^2}{f^2} - \sin^2\frac{\sqrt{2}|H|}{f}\right)\partial_\mu|H|^2\partial^\mu|H|^2\right]$$

<u>Note</u>

1. σ is a massive resonance $m_\sigma = g_\rho f$
2. H^i fields are <u>massless</u> (since $V(H) = 0$) and derivatively coupled (like QCD chiral Lagrangian) i.e. H^i are <u>NG bosons</u>

③ Electroweak gauge-Higgs interactions obtained by:

$$\partial_\mu H \longrightarrow D_\mu H = (\partial_\mu \underbrace{- i g W_\mu^a \frac{\sigma^a}{2} - i g' B_\mu \frac{\mathbb{1}}{2}}_{\text{SM: } SU(2)_L \times U(1)_Y \,\subset\, SU(2)_L \times SU(2)_R \,\cong\, SO(4)}) H$$

to be justified later

Assume $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \underset{\text{constant}}{V} + h(x) \end{pmatrix}$ (unitary gauge) then obtain:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2 \frac{V+h}{f} \left(|W|^2 + \frac{1}{2\cos^2\theta_W} Z^2 \right) + \cdots$$

$$\Rightarrow \quad m_W = \cos\theta_W \, m_Z = \frac{1}{2} g f \sin\frac{V}{f} \equiv \frac{1}{2} g v \quad \Rightarrow \quad \boxed{f \sin\frac{V}{f} = v}$$

where v = electroweak VEV
= 246 GeV

Higgs couplings to gauge bosons:

$$\mathcal{L} \supset \frac{g^2 v^2}{4} \left(|W|^2 + \frac{1}{2\cos^2\theta_W} Z^2 \right) \left[2\sqrt{1-\xi}\, \frac{h}{v} + (1-2\xi) \frac{h^2}{v^2} + \cdots \right] \qquad \text{where } \xi = \frac{v^2}{f^2}$$

$$\Rightarrow \quad \boxed{\frac{g_{hVV}}{g_{hVV}^{SM}} = \sqrt{1-\xi} < 1} \qquad \boxed{\frac{g_{hhVV}}{g_{hhVV}^{SM}} = 1 - 2\xi}$$

due to compact group $\mathcal{G}$

Expansion of $\mathcal{L}$ for large f:

$$\underbrace{D_\mu H^\dagger D^\mu H}_{\text{dim 4}} \underbrace{- \frac{2}{3f^2} |H|^2 D_\mu H^\dagger D^\mu H + \frac{1}{6f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H)}_{\text{dim 6}} + \cdots$$

Note ① $O_T = \frac{1}{2f^2} (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \overleftrightarrow{D}^\mu H)$ is absent! $\Rightarrow \rho = \frac{m_W^2}{(\cos\theta_W m_Z)^2} = 1$

As expected since $\mathcal{H} = SO(4) \cong SU(2)_L \times SU(2)_R \longrightarrow$ custodial symmetry

② Linear σ-model is an effective description of the (unknown) strong dynamics

$\longrightarrow$ more general formalism based on non-linearly realized symmetry

Callan, Coleman, Wess, Zumino (CCWZ)

(see Panico, Wulzer review)

How to generate $V(H) \neq 0$?

$\longrightarrow$ need an <u>explicit</u> violation of global symmetry $\mathcal{G}$

Early attempts introduced new interactions [see Georgi-Kaplan 1984, Banks 1984]

Instead, assume that global symmetry of composite sector is broken by <u>mixing</u> with an elementary sector: [Agashe, Contino, Pomarol 2004]

Elementary
Ψ_i, A_μ
SM matter, gauge

$$\mathcal{L}_{mix} = \underbrace{\lambda_{L,R}}_{\text{related to SM Yukawa coupling}} \bar{\Psi}_{L,R} \overbrace{\mathcal{O}_\Psi}^{\text{fermionic operator}} + \underbrace{g_V}_{\text{related to SM gauge coupling}} A^\mu \overbrace{J^\mu}^{\text{vector current}}$$

Composite
strongly-coupled
$\mathcal{O}_i$

Elementary fields are <u>not</u> complete $\mathcal{G}$ multiplets $\Longrightarrow$ explicitly breaks $\mathcal{G}$
- parametrised by SM couplings g_{SM}

<u>Note</u> ① This is motivated by 5D models and AdS/CFT correspondence

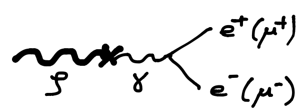
② Explicit breaking could also be due to constituent masses (like quark mass in QCD) $\longrightarrow$ assume strong sector does not have this contribution.

$\mathcal{L}_{mix} \Rightarrow$ mass eigenstates are <u>mixtures</u> of elementary and composite states

i.e. $|\text{Physical}\rangle_i = \cos\theta_i \, |\text{elem.}_i\rangle + \sin\theta_i \, |\text{comp.}_i\rangle$

Known as <u>partial compositeness</u>

<u>Note</u> Similar to γ-ρ mixing in QCD:
- explains $\rho^0 \longrightarrow e^+e^- \, (\mu^+\mu^-)$



Explains fermion mass hierarchy : [Kaplan 1991; TG, Pomarol 2000]

Consider : $\mathcal{L} = \frac{\lambda_L}{\Lambda_{UV}^{d_L - 5/2}} \bar{\Psi}_L \mathcal{O}_L + \frac{\lambda_R}{\Lambda_{UV}^{d_L - 5/2}} \bar{\Psi}_R \mathcal{O}_R$

$\mathcal{O}_{L,R}$ = fermionic op.
$\lambda_{L,R} \simeq \mathcal{O}(1)$ constants
$\Psi_{L,R}$ = elementary fermions

Now $\langle 0 | \mathcal{O}_{L,R} | \underbrace{F^{(c)}_{L,R}}_{\text{fermionic resonances } F_{L,R},\, F^c_{L,R}} \rangle \neq 0$

↳ excites single-particle state from vacuum

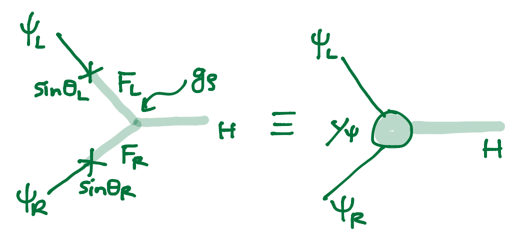
$$\Rightarrow \quad \mathcal{L}_{mass} = -\left(\bar{\Psi}_{L,R},\, \bar{F}_{L,R},\, \bar{F}^c_{L,R}\right) \begin{pmatrix} 0 & 0 & \lambda_{L,R} \\ 0 & 0 & g_\rho \\ \lambda_{L,R} & g_\rho & 0 \end{pmatrix} \begin{pmatrix} \Psi_{L,R} \\ F_{L,R} \\ F^c_{L,R} \end{pmatrix}$$

Mass eigenstates:

$$|phys_i\rangle = \cos\theta_i \,|elem_i\rangle + \sin\theta_i \,|comp_i\rangle$$

where $\sin\theta_{L,R} = \frac{\lambda_{L,R}}{\sqrt{g_\rho^2 + \lambda_{L,R}^2}} \simeq \frac{\lambda_{L,R}}{g_\rho^2}$ assuming $\lambda_{L,R} \ll g_\rho$ $\quad (1 \lesssim g_\rho < 4\pi)$

Yukawa interaction:



$$\Rightarrow \quad y_\Psi = g_\rho \sin\theta_L \sin\theta_R \simeq \frac{\lambda_L(\mu)\lambda_R(\mu)}{g_\rho} \quad \text{where} \quad \lambda_{L,R}(\mu) = \lambda_{L,R}\left(\frac{\mu}{\Lambda_{UV}}\right)^{d_{L,R} - \frac{5}{2}}$$

Light fermions : $y_\Psi \ll 1 \Rightarrow d_{L,R} > \frac{5}{2} \Rightarrow$ mostly elementary! $(\sin\theta_{L,R} \ll 1)$

Top quark : $y_t \sim 1 \Rightarrow d_{L,R} \sim \frac{5}{2} \Rightarrow$ mostly composite! $(\sin\theta_{L,R} \lesssim 1)$

Note: Requiring $\underbrace{\delta g_{b_L}}_{\text{corrections to } Z\bar{b}_L b_L \text{ coupling}} \simeq \left(\frac{\lambda_{t_L}}{g_*}\right)^2 \frac{v^2}{f^2}$ to satisfy exptl constraints $(\delta g_{b_L} \lesssim 10^{-3})$ $\Rightarrow \begin{cases} \lambda_{t_L} \simeq 1, \; \lambda_{t_R} \simeq g_\rho \Rightarrow \text{composite } t_R! \\ \lambda_{t_L} \simeq \lambda_{t_R} \simeq \sqrt{y_t\, g_\rho} \end{cases}$

Fermion embeddings

Recall: $\mathcal{L} = \lambda_L f_L \mathcal{O}_L + \lambda_R f_R \mathcal{O}_R$ What are possible reps of $\mathcal{O}_{L,R}$?

Convenient to embed $f_{L,R}$ into reps. of SO(5):

$$\Rightarrow \mathcal{L} = \lambda_L \underbrace{(f_L)_\alpha}_{\equiv (\Psi_L)^{I_L}} (\mathcal{O}_L)_{I_L} + \lambda_R \underbrace{f_R}_{\equiv (\Psi_R)^{I_R}} (\mathcal{O}_R)_{I_R}$$

α = SM group index

$I_{L,R}$ = $\mathcal{G}$ group index

SM fermions: $2_{1/6}, 1_{2/3}, \mathbb{1}_{-1/3}$ cannot be embedded into SO(5)!

Instead, consider $SO(5) \times \underbrace{U(1)_X}_{\text{unbroken}} \longrightarrow SO(4) \times \underbrace{U(1)_X}_{\text{unbroken}}$

Note The global group must also include QCD color $SU(3)_c$ which we will ignore.

$\Rightarrow$ hypercharge $\boxed{Y = T_{3R} + X}$

SO(5) fermion representations

Consider fundamental $\mathbf{5}$ rep. with $U(1)_X$ charge $+\frac{2}{3}$ then

$$\mathbf{5}_{2/3} \longrightarrow \overset{SO(4)\times U(1)_X}{\mathbf{4}_{2/3} \oplus \mathbf{1}_{2/3}} \longrightarrow \overset{SU(2)_L \times U(1)_Y}{\mathbf{2}_{7/6} \oplus \underbrace{\mathbf{2}_{1/6}}_{q_L} \oplus \underbrace{\mathbf{1}_{2/3}}_{t_R}}$$

Thus have the following embeddings:

$$Q_L = \frac{1}{\sqrt{2}} \begin{pmatrix} -ib_L \\ -b_L \\ -it_L \\ t_L \\ 0 \end{pmatrix} \qquad T_R = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}$$

Partial compositeness $\Rightarrow$ $\mathcal{L} = \lambda_R \underbrace{(\bar{T}_R)^I (\mathcal{O}_F^R)_I}_{\mathcal{G}\text{ invariant}} + \lambda_L \underbrace{(\bar{Q}_L)^I (\mathcal{O}_F^L)_I}_{\mathcal{G}\text{ invariant}} + \text{h.c}$ $\quad I = 1, \ldots 5$

Integrate out strong dynamics

$\Rightarrow \mathcal{L}_{eff}[Q_L, T_R]$ must be $\mathcal{G}$ invariant

Given $U(\Pi) = e^{i\frac{\sqrt{2}}{f_\pi}\Pi_i \hat{T}^i}$ transformations under $\mathcal{G}$

(Implicitly defined by $U(\Pi^{(g)}) = g \cdot U(\Pi) \cdot h^{-1}[\Pi; g]$ where $g \in \mathcal{G}$, $h \in \mathcal{H}$)

$\Rightarrow$ Yukawa interactions are fixed by symmetry!

Higgs couplings to fermions

With $q_L \subset \mathbf{5} = Q_L$, $t_R \subset \mathbf{5} = T_R$ where $\mathcal{U}(\pi) = \begin{bmatrix} \mathbb{1} - (1-\cos\frac{\pi}{f})\frac{\vec{\pi}\vec{\pi}^T}{\pi^2} & \sin\frac{\pi}{f}\frac{\vec{\pi}}{\pi} \\ -\sin\frac{\pi}{f}\frac{\vec{\pi}^T}{\pi} & \cos\frac{\pi}{f} \end{bmatrix}$

obtain :

$$\mathcal{L}_{top} = -m_t \bar{t}t - k_t \frac{m_t}{v} h\bar{t}t - c_2 \frac{m_t}{v^2} \underbrace{h^2\bar{t}t}_{\text{new dim5 interaction!}}$$

where $\boxed{k_t^5 \equiv \frac{g^{CH}_{htt}}{g^{SM}_{htt}} = \frac{1-2\xi}{\sqrt{1-\xi}}}$ $\quad \boxed{c_2^5 = -2\xi}$ $\quad \xi \equiv \frac{v^2}{f^2}$ $\quad$ MCHM$_5$ (5+5 model)

Note As $\xi \to 0$ ($f \to \infty$) obtain $k_t^5 \to 1$, $c_2^5 \to 0$ (SM limit)

Other possibilities

① spinor rep $\mathbf{4}$: $\mathbf{4}_{1/6} \longrightarrow \underbrace{2_{1/6}}_{q_L} \oplus \underbrace{1_{2/3}}_{t_R} \oplus \underbrace{1_{-1/3}}_{b_R}$

$q_L \subset \mathbf{4}$; $t_R \subset \mathbf{4}$ $\Longrightarrow$ $\boxed{k_t^4 = k_b^4 = \sqrt{1-\xi}}$ $\quad \boxed{c_2^4 = -\frac{\xi}{2}}$ $\quad$ MCHM$_4$ (4+4 model)

Note: This model is ruled out because $Z\bar{b}_L b_L$ coupling correction is too large

② symmetric traceless $\mathbf{14}$ rep : $\mathbf{14}_{2/3} \longrightarrow 3_{5/3} \oplus 3_{2/3} \oplus 3_{-1/3} \oplus 2_{7/6} \oplus \underbrace{2_{1/6}}_{q_L} \oplus \underbrace{1_{2/3}}_{t_R}$

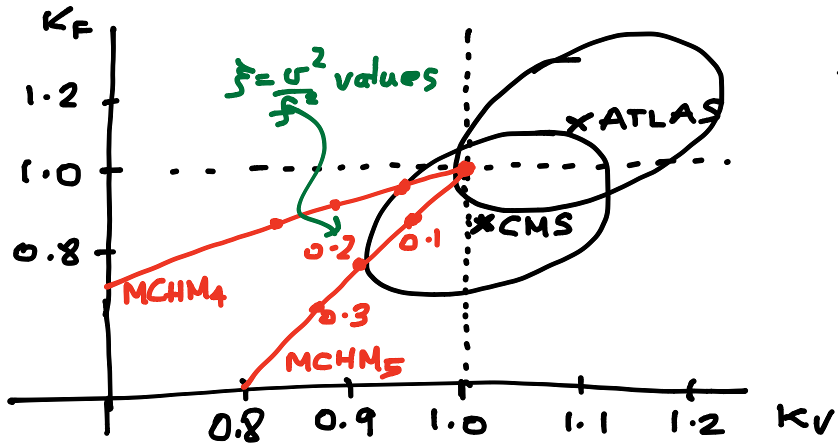
(i) 14+14 model : $q_L \subset \mathbf{14}_{2/3}$; $t_R \subset \mathbf{14}_{2/3}$

$$\Rightarrow k_t^{14} = \frac{5(1-8\xi+8\xi^2)Y_1 - 2(4-23\xi+20\xi^2)Y_2}{2\xi(1-\xi)\left[5(2\xi-1)Y_1 + 2(4-5\xi)Y_2\right]}$$

Y_1, Y_2 = Yukawa couplings

(ii) 14+1 model $\quad q_L \subset \mathbf{14}_{2/3}$ $\quad \underbrace{t_R \subset \mathbf{1}_{2/3}}_{\text{allows for the possibility of fully composite } t_R}$ $\Longrightarrow$ same as MCHM$_5$

LHC limits on Higgs couplings



LHC (7+8 TeV)

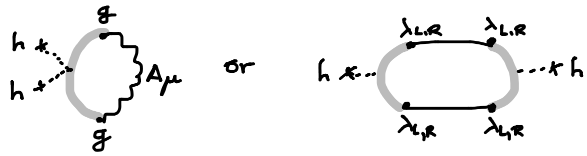
$$\kappa_{V,F} \equiv \frac{g_{V,F}}{g^{SM}_{V,F}}$$

Note: Coupling corrections < 1 (due to compact global group $\mathcal{G}$)

Higgs potential

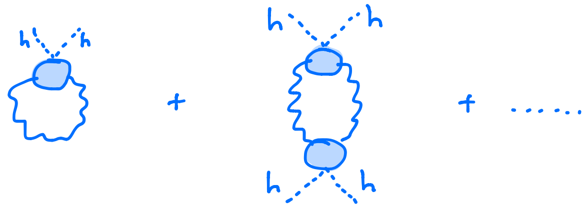
$$\mathcal{L} = \lambda_L q_L \mathcal{O}_L + \lambda_R t_R \mathcal{O}_R + g A_\mu J^\mu$$

e.g.



where $\langle \mathcal{O}\mathcal{O} \rangle \propto \frac{1}{g_\rho^2} = \frac{N}{16\pi^2}$

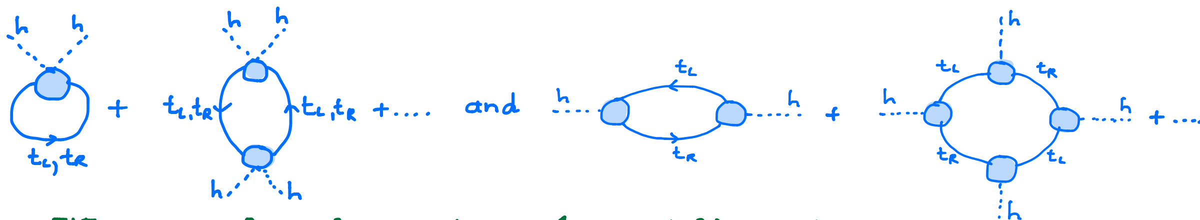
Gauge boson contribution :



$$V_{gauge} = \frac{A}{16\pi^2}\left(3g^2 + g'^2\right) \sin^2 \frac{H}{f}$$

[Assumes $SO(4)$ enlarged to $O(4)$ to suppress corrections to $Z\bar{b}_L b_L$ coupling]

Fermion contribution :



$$V^{5+5}_{fermion} \propto \left(\frac{c_L}{2}\lambda_L^2 - c_R \lambda_R^2 + c_{LL}\lambda_L^4 + c_{RR}\lambda_R^4 + c_{LR}\lambda_L^2\lambda_R^2\right)\sin^2\frac{H}{f} + \left(c'_{LL}\lambda_L^4 + c'_{RR}\lambda_R^4 + c'_{LR}\lambda_L^2\lambda_R^2\right)\sin^4\frac{H}{f}$$

$$V^{14+1}_{fermion} \propto \lambda_L^2\left(c_1 \sin^2\frac{H}{f} + c_2 \sin^4\frac{H}{f}\right)$$

Generically :

$$\boxed{V(H) = -\alpha f^2 \sin^2\frac{H}{f} + \beta f^4 \sin^4\frac{H}{f}}$$

where

due to fermion loop

QCD : $N_c = 3$

$$-\alpha = \underbrace{-a\frac{N_c}{16\pi^2}\lambda_t^2 g_\rho^2 f^2}_{\text{top}} + \underbrace{b\frac{1}{16\pi^2} g^2 g_\rho^2 f^2}_{\text{gauge boson}} \;;\quad \beta = \begin{cases} c\,\frac{N_c}{16\pi^2}\lambda_t^2 g_\rho^2 & \underline{5+5} \\ c'\,\frac{N_c}{16\pi^2}\lambda_t^4 g_\rho^2 & \underline{14+1} \end{cases}$$

Since $\lambda_t > g \Rightarrow$ top quark contribution breaks EW symmetry!

EWSB Recall $\frac{v}{f} = \sin\frac{\langle H\rangle}{f} \Rightarrow V(v) = -\underbrace{\hat{\alpha} f^2}_{\alpha} v^2 + \beta v^4$

$$\frac{dV(v)}{dv} = 0 \Rightarrow \boxed{v^2 = \frac{\hat{\alpha} f^2}{2\beta}}$$

For $\hat{\alpha} \sim \beta \sim O(1)$ require $\boxed{f \sim v}$

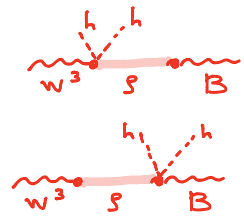
But resonance masses $m_\rho \simeq g_\rho f$ where $1 \lesssim g_\rho \lesssim 4\pi$

$\Rightarrow$ contributions to EW precision observables

S-parameter $\quad \frac{s}{16\pi^2 v^2} H^\dagger \tau^a H B^{\mu\nu} W_{a\mu\nu} \qquad S = \frac{s}{2\pi}$

Vector resonances (ρ) contribute to S parameter

$\Rightarrow S \sim \frac{m_W^2}{m_\rho^2} \Rightarrow \boxed{m_\rho \gtrsim 2.5\text{ TeV}}$ or $f \gtrsim \frac{2.5\text{ TeV}}{g_\rho}$



T-parameter $\quad \frac{-t}{16\pi^2 v^2} ((D^\mu H)^\dagger H)(H^\dagger D_\mu H) \qquad T = \frac{t}{8\pi e^2}$

Provided $\mathcal{H} \supset SU(2)_L \times SU(2)_R \to$ custodial symmetry

$\Rightarrow T$ parameter ✓ OK

Thus EWPT $\Rightarrow f > v$

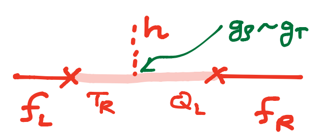
$\Rightarrow$ tuning in Higgs potential $\boxed{\sim \frac{v^2}{f^2}}$
(in $\hat{\alpha}, \beta$ coefficients)

Higgs mass $\quad m_h \simeq 125$ GeV

$$V(h) = -\alpha f^2 \sin^2\left(\frac{\langle H\rangle + h}{f}\right) + \beta f^4 \sin^4\left(\frac{\langle H\rangle + h}{f}\right)$$

$$\Rightarrow \boxed{m_h^2 = 8\left(1 - \frac{v^2}{f^2}\right)\beta v^2}$$

Typically $\quad \beta \sim \frac{N_c}{16\pi^2} y_t^2 g_\rho^2$ QCD: $N_c = 3$



$$\Rightarrow m_h^2 \sim \frac{N_c}{\pi^2} m_t^2 \frac{m_T^2}{f^2} \qquad \left(\frac{m_T^2}{f^2} \equiv g_T^2\right)$$

where m_T = fermionic top partner mass

Assuming $g_T \sim g_\rho$ then LHC limit $m_\rho \gtrsim 2.5$ TeV or $g_\rho \gtrsim 3$
for $\xi = 0.1$

$\Rightarrow \quad m_h \gtrsim m_t$!

However no need to have $g_T \sim g_\rho$!

Choose $g_T < g_\rho$ to obtain Higgs mass ~ 125 GeV

$\Rightarrow$ $m_T < m_\rho$ $\Rightarrow$ light top-partners!

5+5 model $\mathcal{O}_L = \mathbf{5}$; $\mathcal{O}_R = \mathbf{5}$

$$\mathbf{5}_{2/3} \longrightarrow \underbrace{\mathbf{2}_{7/6} \oplus \mathbf{2}_{1/6} \oplus \mathbf{1}_{2/3}}_{\text{possible top partner reps}} \qquad SO(4)\times U(1)_X \text{ reps.}$$

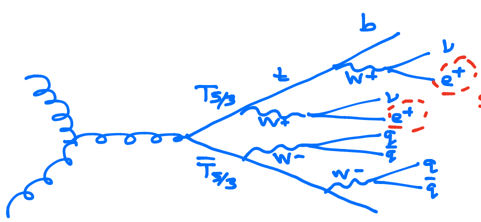
Includes exotic fermions
$$\begin{cases} 2_{7/6} \Rightarrow Q_{EM} = \frac{7}{6} \pm \frac{1}{2} = \overbrace{\frac{5}{3}}^{+\frac{5}{3}\text{ charge!}} \text{ or } \frac{2}{3} ! \\ 2_{1/6} \Rightarrow Q_{EM} = \frac{1}{6} \pm \frac{1}{2} = \frac{2}{3} \text{ or } -\frac{1}{3} \end{cases}$$

14+1 model $\mathcal{O}_L = \mathbf{14}$; $\mathcal{O}_R = \mathbf{1}$

$$\mathbf{14}_{2/3} \longrightarrow \underbrace{3_{5/3} \oplus 3_{2/3} \oplus 3_{-1/3} \oplus 2_{7/6}}_{\text{exotic fermions}} \oplus 2_{1/6} \oplus 1_{2/3}$$

Top partner limits :

e.g. $T_{5/3}$



same sign dileptons

LHC (Run 1) : $\boxed{m_T \gtrsim 700-800 \text{ GeV}}$

Note g_T constrained by Higgs mass

As lower limit on $m_T = g_T f$ increases $\Rightarrow$ f increases

$\Rightarrow$ tuning increases
(similar to stop mass limits in SUSY)

Higgs decays

Composite Higgs models can generate large $h \to Z\gamma$ rate

Dim 6 operators: [Giudice, Grojean, Pomarol, Rattazzi hep-ph/0703164]

invariant under shift symmetry
$$\left\{\begin{array}{l} \mathcal{O}_{HB} = \frac{ig'}{m_W^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\ \mathcal{O}_{HW} = \frac{ig}{m_W^2}(D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu} \end{array}\right.$$

not invariant under shift
$$\left\{\begin{array}{l} \mathcal{O}_g = \frac{g_3^2}{m_W^2} H^\dagger H\, G^a_{\mu\nu} G^{a\mu\nu} \\ \mathcal{O}_\gamma = \frac{g'^2}{m_W^2} H^\dagger H\, B_{\mu\nu} B^{\mu\nu} \end{array}\right.$$

generated by explicit breaking of shift symmetry $\to$ suppression $\frac{g_{SM}^2}{g_\rho^2}$ $\leftarrow$

$\mathcal{O}_g \Rightarrow h \to gg$

$\mathcal{O}_\gamma \Rightarrow h \to \gamma\gamma$

$\mathcal{O}_\gamma,\ \mathcal{O}_{HW} - \mathcal{O}_{HB} \Rightarrow h \to Z\gamma$

$\mathcal{O}_{HW} - \mathcal{O}_{HB}$: invariant under shift symmetry $\Rightarrow$ can be enhanced due to strong dynamics

Ex. $SO(5)/SO(4)$: $\mathcal{O}_{HW} - \mathcal{O}_{HB}$ $\left\{\begin{array}{l} \text{tree-level (spin-1 resonances)} \\ \text{loop-level (composite fermions)} \end{array}\right.$ [Azatov et al 1308.2676]

Note
- requires P_{LR} breaking without contributing to $Z \to b\bar{b}$
- consistent with EWPT (since modified Higgs couplings contributes to S)

Dark matter

Can be identified with a singlet scalar

$\longrightarrow$ requires nonminimal coset

e.g. $SO(6)/SO(5)$ has 5 NGBs $= \underbrace{4}_{\text{Higgs doublet}} + \overbrace{1}^{\text{singlet, } \eta}$

Parity: $\eta \to -\eta$ from $O(6)/O(5)$

$\Rightarrow$ η stable (see Frigerio et al 1204.2808)

Gauge Coupling Unification [Ref: Agashe, Contino, Sundrum 2005]

Assume coset space $\mathcal{G}/\mathcal{H}$ and composite t_R.

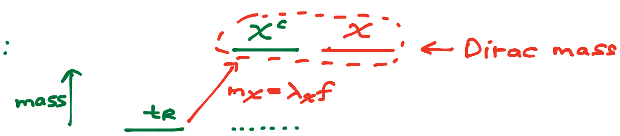
Strong dynamics $\Rightarrow$ $(t_R, \underbrace{\chi^c}_{\text{extra fermion states in } \mathcal{H} \text{ multiplet}})$ = complete $\mathcal{H}$ multiplet

To decouple χ^c, introduce new top "companions" χ = elementary fermions

Partial compositeness $\Rightarrow$ $\mathcal{L} = \lambda_\chi \chi \mathcal{O}_\chi$

$\Rightarrow$ $\mathcal{L} \supset m_\chi \chi \chi^c$ where $m_\chi = \lambda_\chi f$ - Dirac mass term

Top companions split $\mathcal{H}$ multiplet:



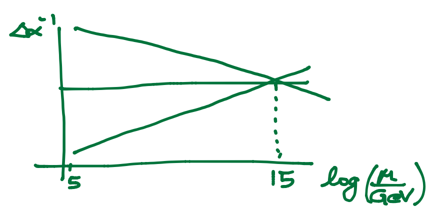
Elementary fermions χ provide new contribution to the running of SM gauge couplings:

$$\alpha_i(\mu) - \alpha_j(\mu) = SM - \{\underbrace{H, t^c}_{\text{composite Higgs, top}}, \overbrace{\tilde{t}^c}^{\text{top "companions" contribution}}\}$$

1-loop β function coefficients:

$$b_1 - b_2 = \frac{94}{15} \quad ; \quad b_2 - b_3 = \frac{13}{3} \quad \Rightarrow \quad \frac{b_2 - b_3}{b_1 - b_2} \simeq 0.69$$

(c.f. MSSM value $\simeq 0.71$)



Note ① $\mathcal{H}$ must contain $SU(5)$ $\Rightarrow$ composite sector contributes universally to running

② Two-loop corrections can only be estimated

$$\frac{d\alpha_i^{-1}}{d\ln\mu} = \frac{b_i}{2\pi} + \frac{B_{ij}}{2\pi}\frac{\alpha_j}{2\pi} + \frac{C_{i\chi}}{2\pi}\frac{\lambda_\chi^2}{16\pi^2}$$

$B \sim 9\, b_{\text{strong}}$

$C \sim 3\lambda_\chi\, b_{\text{strong}}$

Flavor

Partial compositeness: $\mathcal{L} = \lambda_L^{ij} f_L^i \mathcal{O}_L^j + \lambda_R^{ij} f_R^i \mathcal{O}_R^j$ $\quad$ ij = flavor indices

Assume $\lambda_L^{ij} \sim \lambda_R^{ij} \sim \mathcal{O}(1)$ $\quad$ i.e. no hierarchies in the couplings ("anarchic")

ΔF = 1

$b \to s\gamma$ $\quad \mathcal{L} \sim \frac{\lambda_{q_L}^2 \lambda_{d_R}^3}{g_\rho} \frac{v}{m_\rho^2} \bar{s}_L \sigma^{\mu\nu} b_R F^{\mu\nu} \quad \Rightarrow \quad f \gtrsim \frac{3-5}{g_\rho}$ TeV (depending on Re, Im parts)

$$\mathcal{L} \sim \frac{\lambda_{q_L}^3 \lambda_{d_R}^2}{g_\rho} \frac{v}{m_\rho^2} \bar{s}_R \sigma^{\mu\nu} b_L F^{\mu\nu} \quad \Rightarrow \quad f \gtrsim \frac{10}{g_\rho}\ \text{TeV}$$

[Note: Assumes tree-level violation; Limits reduce by factor 10 for loop level]

$\mathrm{Re}(\epsilon_K'/\epsilon_K)$ $\quad$ CP violation in $K^0 \to 2\pi$

$$\mathcal{L} \sim \frac{1}{\lambda_c} \frac{m_d}{m_\rho^2} \bar{s}_L \sigma^{\mu\nu} d_R G^{\mu\nu} \quad \Rightarrow \quad f \gtrsim \frac{13}{g_\rho}\ \text{TeV}$$

$\sin\theta_c \simeq 0.22$

ΔF = 2 $\quad \mathcal{L} \sim \frac{\lambda_i \lambda_j \lambda_k \lambda_\ell}{g_\rho^2} \frac{1}{m_\rho^2} (\bar{f}_i \gamma^\mu f_j)(\bar{f}_k \gamma_\mu f_L)$

Neutron EDM $\quad \mathcal{O}_{ff\gamma} = C_{ff\gamma} \frac{e m_f}{16\pi^2} \bar{f} \sigma^{\mu\nu} F_{\mu\nu} \gamma^5 f \quad ; \quad \mathcal{O}_{ffg} = C_{ffg} \frac{g_s m_f}{16\pi^2} \bar{f} \sigma^{\mu\nu} G_{\mu\nu} \gamma^5 f$

Assuming order-one complex phases ($\mathrm{Im}\, C_{ff\gamma} \neq 0$, $\mathrm{Im}\, C_{ffg} \neq 0$) $\Rightarrow$ $f \gtrsim \frac{20-60}{g_\rho}$ TeV $(\mathcal{O}_{uu\gamma}, \mathcal{O}_{dd\gamma})$

$f \gtrsim \frac{25-65}{g_\rho}$ TeV $(\mathcal{O}_{uug}, \mathcal{O}_{ddg})$

Lepton sector

Electron EDM $\quad \mathcal{O}_{ee\gamma} = C_{ee\gamma} \frac{e m_e}{16\pi^2} \bar{e} \sigma^{\mu\nu} F_{\mu\nu} \gamma^5 e \quad \Rightarrow \quad f \gtrsim \frac{140\ \text{TeV}}{g_\rho}$

$\mu \to e\gamma$ $\quad \mathcal{L} = m_\mu e F_{\mu\nu} \left(\frac{\bar{\mu}_L \sigma^{\mu\nu} e_R}{\Lambda_L^2} + \frac{\bar{\mu}_R \sigma^{\mu\nu} e_L}{\Lambda_R^2} \right) \quad \Rightarrow \quad f \gtrsim \frac{300\ \text{TeV}}{g_\rho}$

To satisfy bounds clearly need additional structure

e.g. flavor symmetry or heavy resonances (i.e. tuned)

Comparison with SUSY models:

	Composite Higgs	SUSY
Higgs boson	composite	elementary
symmetry	global symmetry of new strong force → protects Higgs mass via shift symmetry	Fermionic extension of spacetime Poincaré → protects Higgs mass via supersymmetry
Higgs potential	Generated by partial compositeness → EWSB triggered by top Yukawa	Soft susy-breaking contributions → EWSB triggered by top Yukawa
Higgs mass	Requires light top partners	Requires large A_t terms or heavy stops
Dark matter	pNGB	LSP
Gauge coupling unification	due to composite t_R + top companions	due to Higgsinos, gauginos
Fermion mass hierarchy	Explained by partial compositeness	Not explained
Flavor constraints	addressed by flavor symmetries or heavy resonances	addressed by heavy sfermions, alignment or degeneracy
Tuning	$\lesssim 5\%$	$\lesssim 0.5\%$ (MSSM) 3-5% (low-scale SUSY breaking + NMSSM)

Summary

- Composite Higgs framework is an alternative solution to the hierarchy problem
  - $\longrightarrow$ Higgs is a pseudo Nambu-Goldstone boson
- Partial compositeness (via top quark) generates Higgs potential
  - 125 GeV Higgs $\Rightarrow$ light (exotic) top partners $\gtrsim$ 700 GeV
  - Deviations in gauge boson/fermion couplings
  - Possibly large $h \to Z\gamma$
- Incorporates dark matter & gauge coupling unification
- LHC Run 1 limits $\Rightarrow$ tuning $\lesssim$ 5%

Future Directions/Questions

- Flavor bounds $\Rightarrow$ $f \gtrsim$ 10-100 TeV
  - $\longrightarrow$ partial compositeness only for 3rd generation?
    or "split" composite Higgs (e.g. Barnard, TG, Sankar Ray, Spray: 1409.7391)
- Composite twin Higgs $\longrightarrow$ noncolored top partners (ameliorates tuning)
  (e.g. Barbieri et al 1501.07803; Low, Tesi, Wang 1501.07890)
- What is the underlying UV description?
  (for recent proposals see Barnard, TG, Sankar Ray 1311.6562
  Ferretti, Karateev 1312.5330)