

1 Feynman Parameters

$$\frac{1}{A_1^{m_1} \dots A_n^{m_n}} = \int_0^\infty dx_1 \dots dx_n \delta(x_1 + \dots + x_n - 1) \frac{x_1^{m_1-1} \dots x_n^{m_n-1}}{(x_1 A_1 + \dots + x_n A_n)^{m_1+\dots+m_n}} \frac{\Gamma(m_1 + \dots + m_n)}{\Gamma(m_1) \dots \Gamma(m_n)}$$

$$\frac{1}{A^n} - \frac{1}{B^n} = \int_0^\infty dx dy \delta(x + y - 1) \frac{n(B - A)}{(xA + yB)^{n+1}}$$

2 Dim. Reg.

$$\int \frac{d^{2\omega} k}{(2\pi)^{2\omega}} \frac{1}{(k^2 - \Delta)^r} = i (-1)^r \frac{\Gamma(r - \omega)}{(4\pi)^\omega \Gamma(r)} \Delta^{\omega-r}$$

$$\int \frac{d^{2\omega} k}{(2\pi)^{2\omega}} \frac{k^\mu k^\nu}{(k^2 - \Delta)^r} = i (-1)^{1+r} \frac{\Gamma(r - \omega - 1)}{2(4\pi)^\omega \Gamma(r)} \Delta^{\omega+1-r} g^{\mu\nu}$$

3 Shifting

$$\int \frac{d^4 p}{(2\pi)^4} \frac{p^\mu}{((p+q)^2 - m^2)^2} = \int \frac{d^4 p}{(2\pi)^4} \frac{(p-q)^\mu}{(p^2 - m^2)^2} + \frac{i q^\mu}{32\pi^2}$$

$$\begin{aligned} \int \frac{d^4 p}{(2\pi)^4} \frac{p^\mu p^\nu p^\tau}{((p+q)^2 - m^2)^3} &= \int \frac{d^4 p}{(2\pi)^4} \frac{(p-q)^\mu (p-q)^\nu (p-q)^\tau}{(p^2 - m^2)^3} \\ &\quad + \frac{i}{192\pi^2} (g^{\mu\nu} q^\tau + g^{\nu\tau} q^\mu + g^{\mu\tau} q^\nu) \end{aligned}$$

$$\begin{aligned} \int \frac{d^4 p}{(2\pi)^4} \frac{p^\mu p^\nu}{((p+q)^2 - m^2)^2} &= \int \frac{d^4 p}{(2\pi)^4} \frac{(p-q)^\mu (p-q)^\nu}{(p^2 - m^2)^2} \\ &\quad - \frac{i q^\mu q^\nu}{32\pi^2} - \frac{i}{96\pi^2} (g^{\mu\nu} q^2 + 2q^\mu q^\nu) \end{aligned}$$

4 Miscellaneous

$$\frac{1}{4\pi} \int d\Omega_k \mathcal{J}(\mathbf{v}, \mathbf{v}') = \int_0^1 dx \frac{q^2 (2x(1-x) - 1)}{m^2 - q^2 x(1-x)}$$

where

$$\mathcal{J}(\mathbf{v}, \mathbf{v}') \equiv \frac{2p.p'}{(E - \mathbf{p} \cdot \hat{\mathbf{k}})(E' - \mathbf{p}' \cdot \hat{\mathbf{k}})} - \frac{m^2}{(E - \mathbf{p} \cdot \hat{\mathbf{k}})(E - \mathbf{p} \cdot \hat{\mathbf{k}})} - \frac{m^2}{(E' - \mathbf{p}' \cdot \hat{\mathbf{k}})(E' - \mathbf{p}' \cdot \hat{\mathbf{k}})}$$

and $p' = p + q$, $p = E(1, \mathbf{v})$, $p' = E'(1, \mathbf{v}')$