

't Hooft Anomaly Matching for QCD

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Abstract

I present a set of theories which display non-trivial 't Hooft anomaly matching for QCD with F flavors. The matching theories are non-Abelian gauge theories with “dual” quarks and baryons, rather than the purely confining theories of baryons that 't Hooft originally searched for. The matching gauge groups are required to have an $F \pm 6$ dimensional representation. Such a correspondence is reminiscent of Seiberg’s duality for supersymmetric (SUSY) QCD, and these theories are candidates for non-SUSY duality. However anomaly matching by itself is not sufficiently restrictive, and duality for QCD cannot be established at present. At the very least, the existence of multiple anomaly matching solutions should provide a note of caution regarding conjectured non-SUSY dualities.

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1 Introduction

Many years ago, 't Hooft proposed searching for confining gauge theories with composite massless fermions [1]. The primary tool in such a search is the 't Hooft anomaly matching condition: the requirement that the global anomalies of a proposed low-energy effective theory equal those of the original ultraviolet theory. The requirement that no chiral symmetries are broken makes such a matching highly non-trivial. The further requirement that any number of fermion flavors can be decoupled (i.e. that adding mass terms and integrating out flavors can be accounted for in the low-energy effective description by integrating out all composites containing that flavor) puts another stringent constraint on proposed solutions to the 't Hooft problem. In his pioneering work [1], 't Hooft showed that for a vector-like $SU(3)$ gauge theory with fundamental quarks, a solution could only be found for the case of two flavors. He also showed that there were no solutions for $SU(5)$ theories with fundamental quarks. More general searches were performed [2] and a handful of possible solutions were found for chiral and vector-like theories.

More recently Seiberg [3] has revolutionized our understanding of supersymmetric (SUSY) gauge theories. In addition to finding confining SUSY gauge theories with massless composite fermions (and their SUSY partners), Seiberg also found theories with dual descriptions in terms of a different gauge group with different matter content. These dual theories can have trivial or non-trivial infrared fixed points. Seiberg's work obviously raises the question of whether such dual gauge descriptions persist in non-SUSY theories. Various authors [4] have considered the effects of adding soft-SUSY breaking mass terms and progress has been made when such masses are much smaller than the intrinsic scale of the gauge theory, however little is known about what happens when the SUSY breaking masses are increased to be larger than the intrinsic scale. Recently D-brane constructions [5] have also led to speculations about non-SUSY dualities. The main evidence for the conjectured duality is anomaly matching, but, as I will argue below, that by itself is insufficient to demonstrate duality.

Under what circumstances would it be reasonable for another gauge theory to provide an alternate description of the infrared physics of QCD? Certainly when the number of quark flavors, F , is sufficiently large so as to produce an infrared fixed point there is no a priori objection to such a duplicate description since such a conformal gauge theory has no particle interpretation. These fixed point theories cannot be said

to possess a low-energy effective theory in the conventional sense, but there may be other conformal gauge theories that describe the same fixed point. Banks and Zaks [6] have shown that by analytically continuing in F it is possible to establish such an infrared fixed point for QCD in perturbation theory for F below and sufficiently close to $33/2$. Presumably such fixed point behavior persists as F is reduced down to some critical value F^{crit} . Assuming that chiral symmetry breaking¹ marks the end of the fixed point regime, Appelquist, Wijewardhana, and I estimated [7] this critical value to be $F^{\text{crit}} \approx 4N_c = 12$. Lattice Monte Carlo studies [9] suggest that $F^{\text{crit}} > 8$. Whatever the precise value of F^{crit} is, for F in the range $F^{\text{crit}} < F < 33/2$ it seems worthwhile to consider a generalized ‘t Hooft problem: is there a gauge theory coupled to composite massless fermions that matches the anomalies of QCD? Such a matching theory would be a candidate for a dual description of QCD.

2 An Anomaly Matching Theory

The theory I wish to study is QCD with F flavors: there is an $SU(3)$ gauge group with F left handed quarks Q_L and F right-handed quarks Q_R . This theory has the anomaly-free global symmetry $SU(F)_L \times SU(F)_R \times U(1)_B$ i.e. a chiral flavor symmetry and vector-like baryon number. The fermion content (with global charges) is given in Table 1.

field	$SU(3)$	$SU(F)_L$	$SU(F)_R$	$U(1)_B$
Q_L	\square	\square	$\mathbf{1}$	$\frac{1}{3}$
Q_R	\square	$\mathbf{1}$	\square	$\frac{1}{3}$

Table 1: Fermion content of QCD with F flavors.

A solution of the generalized ‘t Hooft problem requires a gauge theory which matches all the global anomalies of QCD with F flavors. The fermion content of a model that accomplishes this is displayed in Table 2. The matching theory contains some left-handed and right-handed “dual” quarks which belong to an $F - 6$ dimensional representation of the gauge group G . There are also gauge singlet, flavor antisymmetric fermions $A_{L,R}$ and $B_{L,R}$ that have the correct quantum numbers to correspond to baryonic composites of the original quarks. Note that the baryons

¹Or equivalently [8] that the anomalous dimension of the quark mass operator equals one.

labeled B are the large flavor (chiral) analogs of the proton and neutron, while the A baryons are the analogs of the Λ baryon.

field	$G(F-6)$	$SU(F)_L$	$SU(F)_R$	$U(1)_B$
q_L	\square	\square	$\mathbf{1}$	$\frac{F-2}{F-6}$
q_R	\square	$\mathbf{1}$	\square	$\frac{F-2}{F-6}$
A_L	$\mathbf{1}$	$\begin{array}{c} \square \\ \square \end{array}$	$\mathbf{1}$	1
A_R	$\mathbf{1}$	$\mathbf{1}$	$\begin{array}{c} \square \\ \square \end{array}$	1
B_L	$\mathbf{1}$	\square	$\begin{array}{c} \square \\ \square \end{array}$	1
B_R	$\mathbf{1}$	$\begin{array}{c} \square \\ \square \end{array}$	\square	1

Table 2: Fermion content of the matching theory.

The anomaly matching equation for the $SU(F)_L^3$ and $SU(F)_R^3$ anomalies is:

$$3 = F - 6 + \frac{1}{2}(F-3)(F-6) + \frac{1}{2}F(F-1) - F(F-4), \quad (1)$$

while the equation for the $SU(F)_L^2 U(1)_B$ and $SU(F)_R^2 U(1)_B$ anomalies is:

$$1 = (F-6)\frac{F-2}{F-6} + \frac{1}{2}(F-2)(F-3) + \frac{1}{2}F(F-1) - F(F-2). \quad (2)$$

Note that the matching of the $U(1)_B^3$ and mixed gravitational anomalies is trivial since both theories are vector-like with respect to $U(1)_B$, while the matching of the $SU(F)_R^2 U(1)_B$ amounts to a definition of the $U(1)_B$ charge of the dual quark.

The requirement of anomaly matching by itself does not determine whether the gauge group $G(F-6)$ is $SU(F-6)$, $SO(F-6)$, or (for even F) $Sp(F-6)$. In fact all that is determined is that this group have an $F-6$ dimensional representation, so there are even more possibilities. Certainly the matching theory is not sensible for $F < 7$. For $F = 7$, the gauge representation is one dimensional, so the only possibility is a $U(1)$ gauge group. Since such a $U(1)$ theory is free in the infrared, such a description would not fulfill the theoretical prejudice mentioned above that duals are reasonable for the case of non-trivial infrared fixed points (also recall that lattice studies [9] suggest $F^{\text{crit}} > 8$). If the gauge group $G(F-6)$ is in fact $SU(F-6)$ then the matching theory is asymptotically free for $F \geq 8$, while if $G(F-6)$ is really $SO(F-6)$ then it is asymptotically free for $F \geq 10$. Thus if the matching theory is to provide a dual description of QCD, this gives a weak preference to the gauge group $SO(F-6)$.

3 Decoupling a Flavor

Since the anomaly matching is independent of F the decoupling of a flavor is straightforward. It is instructive however to consider what type of dynamics could produce the correct decoupling in the matching theory. A thorough understand of this decoupling would be required in order to establish that the matching theory is actually a dual description of the same physics. Thus I will consider in some detail one possible mechanism for the case that $G(F - 6)$ is identified with $SU(F - 6)$.

Adding a mass term to the QCD Lagrangian for the F -th flavor,

$$\mathcal{L} = m\bar{Q}_R^F Q_L^F + \text{h.c.} , \quad (3)$$

decouples one flavor, so the number of flavors F is reduced by one in the infrared. In the matching theory the gauge group must be broken to a group with a representation with dimension $(F - 7)$ and mass terms are needed for fermions carrying an index F . Such mass terms can be achieved with the introduction of the scalar fields displayed in Table 3.

field	$G(F - 6)$	$SU(F)_L$	$SU(F)_R$	$U(1)_B$
ϕ	\square	\square	\square	$\frac{-4}{F-6}$
M	$\mathbf{1}$	$\bar{\square}$	\square	0

Table 3: Possible scalar content for the matching theory.

If the matching theory has the following Yukawa interactions:

$$\mathcal{L} = \lambda_1 M \bar{q}_R q_L + \lambda_2 M (\bar{B}_R A_L + \bar{A}_R B_L) + \lambda_3 M \bar{B}_L B_R + \lambda_4 \phi (\bar{B}_R q_L + \bar{B}_L q_R) + \text{h.c.} , \quad (4)$$

then the correct mass terms are generated when M^{FF} and ϕ_{F-6}^{FF} have non-zero vevs. Note that if the gauge group of the matching theory is $SU(F - 6)$, then the vev of ϕ engenders the correct Higgs mechanism to break the gauge group to $SU(F - 7)$. It is also interesting to note that the gauge singlet field M has the correct quantum numbers to correspond to a mesonic bound state of the original quarks. Of course, in a non-SUSY theory there is no reason for scalars to be light in the absence of fine-tuning. If the scalars are required for a putative dual description of QCD, perhaps masses tuned to be of order Λ_{QCD} are sufficient for decoupling purposes rather than the more stringent requirement of masslessness.

4 A Second Anomaly Matching Theory

The fermion content of a second model that provides a solution of the generalized ‘t Hooft problem is displayed in Table 4. The matching theory contains some left-handed and right-handed “dual” quarks which belong to an $F + 6$ dimensional representation of the gauge group G . There are also gauge singlet, flavor symmetric fermions $S_{L,R}$ and $T_{L,R}$ which have the correct quantum numbers to correspond to baryonic composites of the original quarks.

field	$G(F + 6)$	$SU(F)_L$	$SU(F)_R$	$U(1)_B$
q_L	\square	$\bar{\square}$	$\mathbf{1}$	$-\frac{F+2}{F+6}$
q_R	\square	$\mathbf{1}$	$\bar{\square}$	$-\frac{F+2}{F+6}$
S_L	$\mathbf{1}$	$\square\square\square$	$\mathbf{1}$	1
S_R	$\mathbf{1}$	$\mathbf{1}$	$\square\square\square$	1
T_L	$\mathbf{1}$	\square	$\square\square$	1
T_R	$\mathbf{1}$	$\square\square$	\square	1

Table 4: Fermion content of the matching theory.

The anomaly matching equation for the $SU(F)_L^3$ and $SU(F)_R^3$ anomalies is:

$$3 = -(F + 6) + \frac{1}{2}(F + 3)(F + 6) + \frac{1}{2}F(F + 1) - F(F + 4) , \quad (5)$$

while the equation for the $SU(F)_L^2 U(1)_B$ and $SU(F)_R^2 U(1)_B$ anomalies is:

$$1 = -(F + 6)\frac{F + 2}{F + 6} + \frac{1}{2}(F + 2)(F + 3) + \frac{1}{2}F(F + 1) - F(F + 2) . \quad (6)$$

In contrast to the previous matching model, the gauge group $G(F + 6)$ is asymptotically free for any value of F , for any one of the three possible identifications: $SU(F + 6)$, $SO(F + 6)$, or $Sp(F + 6)$. The baryons labeled T are additional chiral, large flavor analogs of the proton and neutron, while the S baryons are the analogs of the (orbitally excited) spin- $\frac{1}{2}$ decuplet baryons.

The two matching theories I have presented here are the only solutions to the generalized ‘t Hooft problem (the search for a gauge theory coupled to composite massless fermions that matches the anomalies of QCD) subject to two additional simplicity conditions: that each distinct composite baryon appears at most once, and that the dimension of the gauge representation for the “dual” quark be linear in F with

coefficient 1. This statement can be proved simply by enumerating all the possible baryonic operators². These simplicity criterion may seem somewhat arbitrary, but they are motivated by the form of Seiberg’s SUSY dualities. Determining whether they are actually necessary conditions will require a deeper understanding of duality. I have also looked for anomaly matchings for larger gauge groups like $SU(5)$, but have not found any simple generalizations of the models presented here [10]. Loosening these criteria allows further solutions for QCD and other gauge theories. For example dropping the condition that the coefficient of F be equal to 1, matching solutions can be constructed for an $SU(5)$ gauge theory with F flavors involving both flavor symmetric and antisymmetric baryons and “dual” quarks in gauge representations of dimension $19F \pm 5$. Dropping the constraint of linearity in F completely allows a number of matching solutions for QCD where dimension of the gauge representation for the “dual” quark is quadratic in F .

5 Conclusions

I have presented two solutions to the generalized ‘t Hooft problem for QCD. These are the only solutions subject to two constraints of simplicity. It is clear that I have not established that the matching theories are dual in the sense of Seiberg. In the absence of more powerful consistency checks, it is impossible to tell whether both, either, or neither of the matching solutions gives a correct description of QCD physics for the range of flavors $F^{\text{crit}} < F < 33/2$ (i.e. the infrared fixed point phase). Thus, while the anomaly matching between QCD and the models I have described is intriguing, it may only be a mathematical curiosity rather than a consequence of a duality. More generally, one should be skeptical about any conjectured non-SUSY duality that relies only on anomaly matching for support.

What are the future prospects for establishing non-SUSY dualities? Further progress will almost certainly rely upon establishing new consistency checks. In the absence of SUSY, anomaly matching gives no information about the boson content of the theory. In SUSY theories there is generally a finite dimensional moduli space of inequivalent vacua that must match between dual pairs, however for non-SUSY theories there is a unique vacuum (up to global symmetry transformations), so the

²I have also assumed that a composite made completely of left-handed quarks is itself left-handed, for it to be a right-handed state would require a larger “orbital angular momentum”.

correspondence is trivial. Furthermore, in Seiberg's analysis he was able to require that the dual description provided a correct description of the physics for any number of flavors. Thus a powerful consistency check was provided by integrating out one flavor at a time and seeing that the mapping of the quark mass term to the dual description lead to the correct physics in each case. For non-SUSY theories however, phase transitions are to be expected as F is varied (since there are no constraints from holomorphy), so even if a dual description were established in the infrared fixed point phase it may not be possible to reduce F and produce the correct dual description (i.e. the chiral Lagrangian) for the chiral symmetry breaking phase. In the absence of new analytic consistency checks, the question of duality for QCD will probably require lattice Monte Carlo calculations for a definitive answer. Even with lattice calculations the analysis will not be straightforward. In the case of theories with non-trivial infrared fixed points, a candidate dual theory cannot be considered as a low-energy effective theory but rather as merely an equivalent description of the low-energy physics. Thus a test of duality for theories with non-trivial infrared fixed points will require two lattice computations, one for each of the dual pair, and detailed comparison of the infrared physics observed in each.

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