

Constraints on the SU(3) Electroweak Model

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ABSTRACT: We consider a recent proposal by Dimopoulos and Kaplan to embed the electroweak $SU(2)_L \times U(1)_Y$ into a larger group $SU(3)_W \times SU(2) \times U(1)$ at a scale above a TeV. This idea is motivated by the prediction for the weak mixing angle $\sin^2 \theta_W = 1/4$, which naturally appears in these models so long as the gauge couplings of the high energy $SU(2)$ and $U(1)$ groups are moderately large. The extended gauge dynamics results in new effective operators that contribute to four-fermion interactions and Z pole observables. We calculate the corrections to these electroweak precision observables and carry out a global fit of the new physics to the data. For $SU(2)$ and $U(1)$ gauge couplings larger than 1, we find that the 95% C.L. lower bound on the matching (heavy gauge boson mass) scale is 11 TeV. We comment on the fine-tuning of the high energy gauge couplings needed to allow matching scales above our bounds. The remnants of $SU(3)_W$ breaking include multi-TeV $SU(2)_L$ doublets with electric charge $(\pm 2, \pm 1)$. The lightest charged gauge boson is stable, leading to cosmological difficulties.

1. Introduction

Unification of the standard model forces into a larger gauge symmetry is perhaps the most elegant physics beyond the Standard Model. Full unification of $SU(3)_c \times SU(2)_L \times U(1)_Y$ into a grand unified theory such as $SU(5)$ simplifies the matter content of the model, determines the hypercharge normalization, and predicts gauge coupling unification. For the standard model, the combination of precision measurements of the low energy gauge couplings plus the non-observation of proton decay rule out all of the simplest non-supersymmetric grand unified theories. Given this disappointing result, it is natural to consider other alternatives, such as the partial unification of $SU(2)_L \times U(1)_Y$ into a larger group. Weinberg first considered unifying the electroweak gauge symmetries into $SU(3)$ back in 1972 [1]. The left-handed doublet plus the right-handed singlet leptons fit neatly into triplets of $SU(3)$ [2]. The $U(1)_Y$ normalization is fixed by the embedding into $SU(3)$, which results in the famous prediction $\sin^2 \theta_W = 1/4$ at the scale of $SU(3)$ breaking. However, quarks cannot be simply embedded into $SU(3)$ representations due to their fractional hypercharge.

Recently there has been renewed interest in predicting $\sin^2 \theta_W = 1/4$ through new gauge dynamics that appears not too far above the electroweak scale [3,4]. The proposal by Dimopoulos and Kaplan [3] embeds the electroweak $SU(2)_L \times U(1)_Y$ into a larger product gauge symmetry $SU(3)_W \times SU(2) \times U(1)$ in which quarks and leptons are charged under just the high energy $SU(2) \times U(1)$. The larger gauge structure is spontaneously broken down to the electroweak model by the vev of a scalar field Σ transforming under all of the gauge groups. By choosing a larger product gauge group that includes $SU(2) \times U(1)$, there is no difficulty in accommodating all quark and lepton hypercharges (since the quark and lepton hypercharges simply corresponds to their charges under the high energy $U(1)$). The price to be paid for this freedom is twofold: The normalization of hypercharge is unexplained, and there is no simplification of the matter content of the standard model. However, if the gauge couplings of the high energy $SU(2)$ and $U(1)$ groups are at least moderately large, the electroweak gauge couplings are determined primarily by $SU(3)_W$ with the Weinberg $\sin^2 \theta_W = 1/4$ prediction of the weak-to-hypercharge coupling ratio.

Since the minimal gauge extension to $SU(3)_W \times SU(2) \times U(1)$ has nothing to do with the $U(1)_Y$ normalization, the intersection of the $SU(2)_L$ and $U(1)_Y$ gauge couplings to any value at any scale is possible. Only when the high energy $U(1)$ is embedded into a simple group at an even higher scale is the normalization fixed. The simplest possibility is that the $U(1)$ embedded into an $SU(2)$ group [3]. This determines the $U(1)$ normalization and fixes the Σ field's $U(1)$ charge to be $1/2$. From this point on, we tacitly assume this assignment of $U(1)$ charge to the Σ field.

The breaking of $SU(3)_W \times SU(2) \times U(1)$ to the electroweak model results in eight new gauge bosons that obtain a mass of order $\langle \Sigma \rangle$ times gauge couplings, and four that become the electroweak gauge bosons. Integrating out the heavy gauge bosons results in new effective operators suppressed by squares of gauge boson masses that contribute to four-fermion processes and Z pole

observables [5, 6]. In this paper we calculate the mass spectrum and the low-energy effective Lagrangian of the $SU(3)_W \times SU(2) \times U(1)$ model and the resulting corrections to electroweak precision observables due to the heavy gauge bosons.

We may gain some intuition as to what we might expect by remembering an older idea in which the left-handed quarks and left-handed leptons are charged under two different $SU(2)$ gauge symmetries [7]. In this “ununified standard model”, the scale of the $SU(2)_q \times SU(2)_l$ breaking was found to be significantly constrained by electroweak precision corrections [8]. The limit on the matching scale rises dramatically for the case in which the leptonic coupling is taken to be much larger than the quark coupling $g_l \gg g_q$. This is because the matching scale for the new gauge dynamics corresponds to the heaviest gauge boson, of order $g_l u$ where u is the vev of a bidoublet scalar field that breaks $SU(2)_q \times SU(2)_l$ to $SU(2)_L$.

It is this general observation suitably applied to the $SU(3)_W \times SU(2) \times U(1)$ model that significantly constrains the new gauge dynamics. We emphasize that the matching scale is by definition the scale at which the high energy theory including the full $SU(3)_W$ gauge symmetry is decomposed into the standard model $SU(2)_L \times U(1)_Y$. Effective theory demands that this scale coincide with the scale of the heaviest gauge boson, since the full product gauge theory cannot be realized until all of the gauge bosons are propagating degrees of freedom. Broken gauge multiplets that are split in mass due to a hierarchy in gauge couplings are accounted for through threshold corrections to the renormalization group evolution of the electroweak gauge couplings.

The organization of this paper is as follows. In Sec. 2 we calculate the mass spectrum of the $SU(3)_W \times SU(2) \times U(1)$ broken to $SU(2)_L \times U(1)_Y$. After integrating out the heavy gauge bosons, we calculate their tree-level effects on the masses and couplings between the electroweak gauge bosons and matter in Sec. 3. The resulting modifications to the electroweak theory are used to calculate the corrections to precision electroweak observables in Sec. 4, which are determined by just two parameters. We then perform a global fit of the new physics contributions to the experimental observables. The constraints on the new physics extracted from the fit imply constraints on the region of allowed $SU(2) \times U(1)$ gauge couplings. This requires evolving the electroweak gauge couplings using the renormalization group equations with threshold corrections that we calculate in Sec. 5. In Sec. 6 we discuss the phenomenology of the heavy gauge bosons. We focus on the electrically charged gauge bosons that do not couple to quarks and leptons, pointing out that they are only produced in pairs and that the lightest one does not decay. We estimate the cosmological abundance of these charged, stable relics and find that it is of order the critical density or larger. In addition to resulting cosmological difficulties, this is problematic in light of the strong experimental bounds on the abundance of charged stable particles through searches for heavy isotopes of ordinary nuclei. Finally, we conclude in Sec. 7.

2. Spectrum

The gauge group of the model is $SU(3)_W \times SU(2) \times U(1)$ with gauge couplings g_3 , \tilde{g} , and \tilde{g}' respectively. The quarks and leptons are uncharged under $SU(3)_W$, having the same quantum numbers under $SU(2) \times U(1)$ as they do under $SU(2)_L \times U(1)_Y$. The Higgs field is replaced by two scalars Φ and Σ . Φ is uncharged under $SU(3)_W$, but with the same quantum numbers as the Higgs under $SU(2) \times U(1)$, while Σ transforms as $(3, 2, -1/2)$. At some high scale $SU(3)_W \times SU(2) \times U(1)$ is broken down to the electroweak group $SU(2)_L \times U(1)_Y$ by the vev

$$\langle \Sigma \rangle = \begin{pmatrix} M & 0 \\ 0 & M \\ 0 & 0 \end{pmatrix}. \quad (2.1)$$

The gauge structure of this model is similar to the recently proposed deconstructed models where the Higgs is a pseudo-Goldstone boson [9].

The gauge bosons of the three groups mix to form the following representations of $SU(2)_L \times U(1)_Y$: $(3, 0) \oplus (3, 0) \oplus (1, 0) \oplus (1, 0) \oplus (2, 3/2) \oplus (2, -3/2)$. We first consider the $(3, 0)$ sector. These fields arise from the $SU(2)$ gauge bosons \tilde{W}^a and the $A^{1,2,3}$ gauge bosons of the $SU(2)$ subgroup of $SU(3)_W$. In the (A^a, \tilde{W}^a) basis (for $a = 1, 2, 3$) the mass matrix is:

$$M^2 \begin{pmatrix} g_3^2 & g_3 \tilde{g} \\ g_3 \tilde{g} & \tilde{g}^2 \end{pmatrix} \quad (2.2)$$

Thus the light and heavy mass eigenstates are:

$$W_L^a = c_\phi A^a - s_\phi \tilde{W}^a \quad (2.3)$$

$$W_H^a = s_\phi A^a + c_\phi \tilde{W}^a \quad (2.4)$$

with masses

$$M_{W_L} = 0 \quad (2.5)$$

$$M_{W_H} = \sqrt{\tilde{g}^2 + g_3^2} M \quad (2.6)$$

where

$$s_\phi = \frac{g_3}{\sqrt{\tilde{g}^2 + g_3^2}}, \quad c_\phi = \frac{\tilde{g}}{\sqrt{\tilde{g}^2 + g_3^2}}. \quad (2.7)$$

The $SU(2)$ singlets arise from the $U(1)$ gauge boson \tilde{B} and the A^8 component of the $SU(3)_W$ gauge bosons. These will constitute the $(1, 0)$ sector. The mass matrix in the (A^8, \tilde{B}) basis at the high scale is

$$M^2 \begin{pmatrix} \frac{1}{3} g_3^2 & \frac{1}{\sqrt{3}} g_3 \tilde{g}' \\ \frac{1}{\sqrt{3}} g_3 \tilde{g}' & \tilde{g}'^2 \end{pmatrix} \quad (2.8)$$

Thus the light and heavy mass eigenstates are:

$$B_L = c_\psi A^8 - s_\psi \tilde{B} \quad (2.9)$$

$$B_H = s_\psi A^8 + c_\psi \tilde{B} \quad (2.10)$$

with masses

$$M_{B_L} = 0 \quad (2.11)$$

$$M_{B_H} = \sqrt{\tilde{g}'^2 + \frac{g_3^2}{3}} M \quad (2.12)$$

where

$$s_\psi = \frac{g_3}{\sqrt{3\tilde{g}'^2 + g_3^2}}, \quad c_\psi = \frac{\sqrt{3}\tilde{g}'}{\sqrt{3\tilde{g}'^2 + g_3^2}}. \quad (2.13)$$

Finally we consider the $(2, \pm 3/2)$ sector that comes from the $A^{4,5,6,7}$ gauge bosons of $SU(3)_W$. These fields have no $SU(2) \times U(1)$ partners to mix with, and thus their mass is simply

$$M_i = \frac{g_3}{\sqrt{2}} M. \quad (2.14)$$

Since g_3 is expected to be smaller than \tilde{g} and \tilde{g}' , these gauge bosons will be substantially lighter than W_H^a and B_H . These intermediate scale gauge bosons, however, do not have any direct couplings to quarks and leptons. Thus they would only be seen in processes involving virtual SM gauge bosons.

The effective gauge couplings of the $SU(2)_L \times U(1)_Y$ groups are:

$$g = \tilde{g}s_\phi \quad (2.15)$$

$$g' = \tilde{g}'s_\psi. \quad (2.16)$$

The coupling of W_H^a (B_H) to quarks and leptons is $-\tilde{g}c_\phi$ ($-\tilde{g}'c_\psi$).

3. The Low-energy Effective Action

We now construct the effective theory below the mass scale of the heavy gauge bosons. Integrating out W_H^a and B_H induces additional operators in the effective theory. These operators modify the usual relations between the standard model parameters, and therefore their coefficients can be constrained from electroweak precision measurements. There are three types of operators that will be relevant for us: corrections of the coupling of the $SU(2)_L \times U(1)_Y$ gauge bosons to their corresponding currents, operators that are quadratic in the $SU(2)_L \times U(1)_Y$ gauge fields and quartic in the ordinary Higgs field Φ , and four-fermion operators.

Exchanges of the heavy W_H^a and B_H gauge bosons give the following operators which are quadratic in the light gauge fields:

$$\begin{aligned} \mathcal{L}_2 = & -\frac{g^2 c_\phi^4}{8M^2} W_L^{a\mu} W_{\mu L}^a (\Phi^\dagger \Phi)^2 - \frac{g'^2 c_\psi^4}{8M^2} B_L^\mu B_{\mu L} (\Phi^\dagger \Phi)^2 - \frac{g^2 c_\psi^4}{8M^2} W_L^{a\mu} W_{\mu L}^b (\Phi^\dagger \sigma^a \Phi) (\Phi^\dagger \sigma^b \Phi) \\ & - \frac{g'^2 c_\phi^4}{8M^2} B_L^\mu B_{\mu L} (\Phi^\dagger \sigma^a \Phi)^2 - \frac{gg'(c_\phi^4 + c_\psi^4)}{4M^2} W_L^{a\mu} B_{\mu L} (\Phi^\dagger \sigma^a \Phi) (\Phi^\dagger \Phi), \end{aligned} \quad (3.1)$$

where σ^a are the Pauli σ matrices. For example, the first term arises in the following way. The kinetic term of the Higgs field $(D_\mu \Phi)^\dagger D^\mu \Phi$ contains the coupling

$$\mathcal{L}_{\tilde{W}^2 \Phi^2} = \frac{\tilde{g}^2}{4} \tilde{W}_\mu^a \tilde{W}^{b\mu} (\Phi^\dagger \sigma^a \sigma^b \Phi). \quad (3.2)$$

Expressing $\tilde{W}^a = c_\phi W_H^a - s_\phi W_L^a$ we obtain a coupling between the heavy and light gauge bosons of the form

$$\mathcal{L}_{W_L W_H \Phi^2} = -\frac{\tilde{g}^2 s_\phi c_\phi}{4} (W_{\mu L}^a W_H^{b\mu} + W_{\mu L}^b W_H^{a\mu}) (\Phi^\dagger \sigma^a \sigma^b \Phi) = -\frac{\tilde{g}^2 s_\phi c_\phi}{2} W_{\mu L}^a W_H^{a\mu} (\Phi^\dagger \Phi). \quad (3.3)$$

The first term in \mathcal{L}_2 then arises by integrating out the heavy gauge boson $W_H^{a\mu}$ by taking its equation of motion and expressing it in terms of the light fields. The operators in (3.1) are the ones that give corrections to the light gauge boson masses after Φ gets a vev. Thus after Φ gets the usual vev:

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad (3.4)$$

and including the effects of the higher dimension operators (3.1), we find that the mass of the W is

$$M_W^2 = g^2 \frac{v^2}{4} \left(1 - \frac{c_\phi^4 v^2}{4M^2} \right) \quad (3.5)$$

The mass matrix in the (W_L^3, B_L) basis is:

$$\frac{v^2}{4} \left(1 - \frac{(c_\psi^4 + c_\phi^4) v^2}{4M^2} \right) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \quad (3.6)$$

So the mass of the Z is

$$M_Z^2 = (g^2 + g'^2) \frac{v^2}{4} \left(1 - \frac{(c_\phi^4 + c_\psi^4) v^2}{4M^2} \right) \quad (3.7)$$

In addition, exchanges of W_H^a and B_H give corrections to the coupling of the $SU(2)_L \times U(1)_Y$ gauge bosons to their corresponding currents and additional four-fermion operators:

$$\begin{aligned} \mathcal{L}_c = & g W_{L\mu}^a J^{a\mu} \left(1 - (\Phi^\dagger \Phi) \frac{c_\phi^4}{2M^2} \right) + g' B_{L\mu} J_Y^\mu \left(1 - (\Phi^\dagger \Phi) \frac{c_\psi^4}{2M^2} \right) \\ & - g W_{L\mu}^a J_Y^\mu (\Phi^\dagger \sigma^a \Phi) \frac{c_\psi^4}{2M^2} - g' B_{L\mu} J^{a\mu} (\Phi^\dagger \sigma^a \Phi) \frac{c_\phi^4}{2M^2} - J_\mu^a J^{a\mu} \frac{c_\phi^4}{2M^2} - J_\mu^Y J^{Y\mu} \frac{c_\psi^4}{2M^2} \end{aligned} \quad (3.8)$$

Using this expression we can now evaluate the effective Fermi coupling G_F in this theory. The simplest way to obtain the answer for this is by integrating out the W_L bosons from the theory by adding the W mass term to (3.8). The expression we obtain for the effective four-fermion operator is

$$-\frac{g^2}{2M_W^2} J^{+\mu} J_\mu^- \left(1 - \frac{c_\phi^4 v^2}{2M^2}\right) - J^{+\mu} J_\mu^- \frac{c_\phi^4}{2M^4} = -2\sqrt{2}G_F J^{+\mu} J_\mu^-, \quad (3.9)$$

where $J^\pm = \frac{1}{2}(J^1 \pm iJ^2)$. Plugging in the correction to the W mass we obtain that G_F in this model is uncorrected, that is

$$G_F = \frac{1}{\sqrt{2}v^2}. \quad (3.10)$$

This is in fact not a coincidence, but a general result in such models which was first derived in [7]. The (A^a, \tilde{W}^a) mass matrix can be written as a product of the coupling matrix G and the matrix of vevs V :

$$\mathcal{M}^2 = G V^2 G, \quad (3.11)$$

where $G = \text{diag}(g_3, \tilde{g})$ and the matrix of squared vevs is,

$$V^2 = M^2 \begin{pmatrix} 1 & 1 \\ 1 & (1 + \frac{v^2}{4M^2}) \end{pmatrix}. \quad (3.12)$$

Then the charged current interactions at zero momentum transfer are given by,

$$\frac{1}{2} J_\mu^\dagger G \mathcal{M}^{-2} G J^\mu = \frac{1}{2} J_\mu^\dagger V^{-2} J^\mu, \quad (3.13)$$

where $J^\mu = (0, j_q^\mu + j_l^\mu)$ is the charged quark (j_q^μ) and lepton (j_l^μ) current vector in the $(\text{SU}(3)_W, \text{SU}(2))$ basis. Evaluating,

$$V^{-2} = \frac{4}{v^2} \begin{pmatrix} (1 + \frac{v^2}{4M^2}) & -1 \\ -1 & 1 \end{pmatrix}. \quad (3.14)$$

We find that the charged current four-fermion interaction is given by,

$$\frac{2}{v^2} (j_q^\mu + j_l^\mu)^2. \quad (3.15)$$

From this we read off that G_F is given by (3.10) and does not receive tree level corrections from the $\text{SU}(3)_W$ interactions.

Finally, to fix all SM parameters we need to identify the photon and the neutral-current couplings from (3.8):

$$\begin{aligned} \mathcal{L}_{\text{nc}} = & e A_\mu J_Q^\mu + \frac{e}{s c} Z_\mu \left[J^{3\mu} \left(1 - \frac{(c_\phi^4 + c_\psi^4) v^2}{4M^2}\right) - J_Q^\mu \left(s^2 - \frac{c_\psi^4 v^2}{4M^2}\right) \right] \\ & - J^{3\mu} J_\mu^3 \frac{c_\phi^4}{2M^2} - (J^3 - J_Q)^\mu (J^3 - J_Q)_\mu \frac{c_\psi^4}{2M^2}. \end{aligned} \quad (3.16)$$

Here $e = gg'/\sqrt{g^2 + g'^2}$ as in the standard model, thus there is no correction to the expression of the electric charge e compared to the SM. Similarly to the evaluation of the effective G_F , we can calculate the low-energy effective four-fermion interactions from the neutral currents. The result we obtain is

$$\mathcal{L}_{NC}^{(4f)} = -\frac{2}{v^2}(J^3 - s^2 J_Q)^2 - \frac{1}{2M^2}(J_Q)^2 \left(s^4(c_\phi^4 + c_\psi^4) - 2s^2 c_\psi^4 + c_\psi^4 \right), \quad (3.17)$$

where the first term is just the SM result, while the second term is the correction. Note that the correction term to the four-fermion interactions contains only the charged currents, and so it does not contribute to neutrino scattering processes or atomic parity violation.

4. The Contributions to Electroweak Observables

To relate our parameters to observables we use the standard definition of $\sin \theta_0$ from the Z pole [10],

$$\sin^2 \theta_0 \cos^2 \theta_0 = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_F M_Z^2}, \quad (4.1)$$

$$\sin^2 \theta_0 = 0.23105 \pm 0.00008 \quad (4.2)$$

where $\alpha(M_Z^2)^{-1} = 128.92 \pm 0.03$ is the running SM fine-structure constant evaluated at M_Z [11]. We can relate this measured value with the bare value in this class of models,

$$\sin^2 \theta_0 = s^2 + \frac{s^2 c^2}{c^2 - s^2} \frac{(c_\phi^4 + c_\psi^4) v^2}{4M^2}, \quad (4.3)$$

which is obtained by considering all corrections to (4.1) in the usual way (see [10]). Also, we have the simple result that the running couplings defined by Kennedy and Lynn [12] which appear in Z -pole asymmetries are the same as the bare couplings:

$$s_*^2(q^2) = s^2, \quad e_*^2(q^2) = e^2. \quad (4.4)$$

In order to compare to experiments, we can relate our corrections of the neutral-current couplings to the generalized modifications of the Z couplings as defined by Burgess et al. [6],

$$\Delta \mathcal{L} = \frac{e}{s c} \sum_i \bar{f}_i \gamma^\mu (\delta \tilde{g}_L^{ff} \gamma_L + \delta \tilde{g}_R^{ff} \gamma_R) f_i Z_\mu, \quad (4.5)$$

where

$$\frac{1}{s c} = \frac{1}{s_0 c_0} \left[1 + \frac{(c_\phi^4 + c_\psi^4) v^2}{8M^2} \right]. \quad (4.6)$$

From (3.16) we obtain that¹

$$\delta\tilde{g}^{ff} = \frac{v^2}{4M^2} \left[q^f c_\psi^4 - t_3^f (c_\phi^4 + c_\psi^4) \right]. \quad (4.7)$$

For the individual couplings this implies

$$\begin{aligned} \delta\tilde{g}_L^{uu} &= \frac{-3c_\phi^4 + c_\psi^4}{24M^2} v^2 = \frac{1}{6}(-3c_1 + c_2), & \delta\tilde{g}_R^{uu} &= \frac{c_\psi^4}{6M^2} v^2 = \frac{2}{3}c_2, \\ \delta\tilde{g}_L^{dd} &= \frac{3c_\phi^4 + c_\psi^4}{24M^2} v^2 = \frac{1}{6}(3c_1 + c_2), & \delta\tilde{g}_R^{dd} &= -\frac{c_\psi^4}{12M^2} v^2 = -\frac{1}{3}c_2, \\ \delta\tilde{g}_L^{ee} &= \frac{c_\phi^4 - c_\psi^4}{8M^2} v^2 = \frac{1}{2}(c_1 - c_2), & \delta\tilde{g}_R^{ee} &= -\frac{c_\psi^4}{4M^2} v^2 = -c_2, \\ \delta\tilde{g}_L^{\nu\nu} &= -\frac{c_\phi^4 + c_\psi^4}{8M^2} v^2 = -\frac{1}{2}(c_1 + c_2), & & \end{aligned} \quad (4.8)$$

where $\delta\tilde{g}^{\mu\mu} = \delta\tilde{g}^{\tau\tau} = \delta\tilde{g}^{ee}$, and similarly $\delta\tilde{g}^{tt} = \delta\tilde{g}^{cc} = \delta\tilde{g}^{uu}$, $\delta\tilde{g}^{bb} = \delta\tilde{g}^{ss} = \delta\tilde{g}^{dd}$. We have introduced the notation

$$c_1 = \frac{c_\phi^4 v^2}{4M^2}, \quad c_2 = \frac{c_\psi^4 v^2}{4M^2}. \quad (4.9)$$

In the Appendix we calculate the shifts in the electroweak precision observables in terms of the parameters c_1 and c_2 defined above. We perform a two parameter global fit to the precision electroweak data given in Table 1 assuming a 115 GeV Higgs (as described in [13]), and find that c_1 and c_2 are tightly constrained as demonstrated in Fig. 1. The best fit in c_1 and c_2 has $\chi^2 \simeq 30.5$ with 23 observables. (We note in passing that shifting the Higgs to heavier masses worsens the best χ^2 fit to the data.) We will translate these constraints into bounds on \tilde{g} , \tilde{g}' and the relevant mass scales, after discussing the running of the gauge couplings.

5. Constraints on the High Energy Parameters

We now relate the constraints on c_1 and c_2 to the parameters of the $SU(3)_W \times SU(2) \times U(1)$ theory. At the matching scale $M_u = \text{Max}[M_{W_H}, M_{B_H}]$, the high energy gauge couplings are related to the electroweak couplings through

$$\frac{1}{g^2(M_u)} = \frac{1}{g_3^2} + \frac{1}{\tilde{g}^2} \quad (5.1)$$

$$\frac{1}{g'^2(M_u)} = \frac{3}{g_3^2} + \frac{1}{\tilde{g}'^2}. \quad (5.2)$$

¹Note that the correction to the Z coupling \tilde{g}^{ff} should not be confused with the high energy $SU(2) \times U(1)$ gauge couplings \tilde{g}, \tilde{g}' .

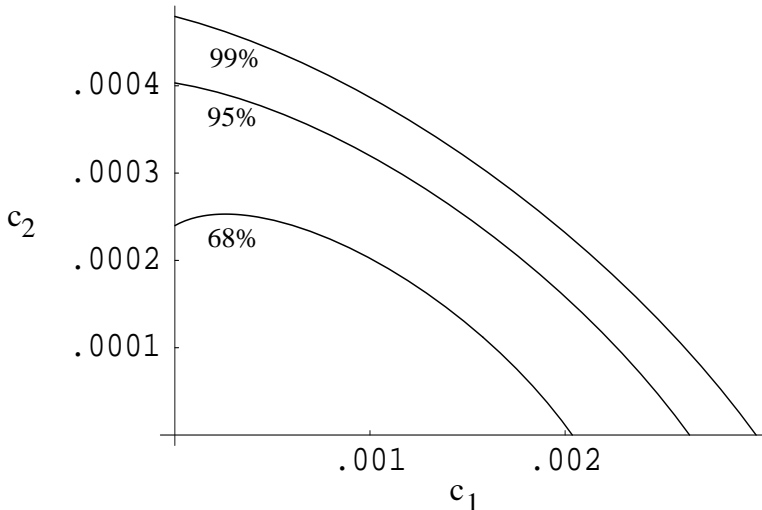


Figure 1: Confidence ellipses in c_1 - c_2 space.

As long as $g_3 \ll \tilde{g}, \tilde{g}'$ one expects the prediction $\sin^2 \theta_W = 1/4$ to approximately be satisfied. This is similar to the approximate SU(5) unification in the models in [14].

There are four unknowns (three gauge couplings and the matching scale) with two constraint equations. The electroweak couplings evaluated at the matching scale are related to the well-measured couplings at M_Z through the one-loop renormalization group equations

$$\frac{1}{g_a^2(M_u)} = \frac{1}{g_a^2(M_Z)} - \frac{1}{8\pi^2} \left[b_a \ln \frac{M_u}{M_Z} + \tilde{b}_a \ln \frac{M_u}{m_t} + (d_a + e_a) \ln \frac{M_u}{M_i} \right], \quad (5.3)$$

where $b_a = (53/9, -22/6)$, $\tilde{b}_a = (17/18, 1/2)$, $d_a = (-33, -11/3)$, and $e_a = (3/2, 1/2)$ for $a = [\text{U}(1)_Y, \text{SU}(2)_L]$. The last term represents the threshold correction from two contributions. The first (d_a) corresponds to the intermediate scale gauge bosons with mass $M_i = g_3 M / \sqrt{2}$ (that do not mix with the SM gauge bosons). The second (e_a) corresponds to all of the components of the scalar field Σ that are not eaten by W_H or B_H . This includes the four Goldstone bosons eaten by the intermediate scale gauge bosons and four uneaten scalars that we assume have mass M_i . We also note that these sharp threshold corrections at the masses of heavy fields are only approximate, but additional corrections do not significantly modify our results.

The high energy theory is therefore completely determined by \tilde{g} and \tilde{g}' . Any given point in this two-parameter space has a definite prediction for the matching scale M_u , the SU(3)_W coupling, and c_1 and c_2 . It is straightforward to determine the region of \tilde{g} - \tilde{g}' space that is excluded by large contributions to the electroweak precision observables, which we show in Fig. 2. Several comments are in order. Small couplings $\tilde{g}, \tilde{g}' \lesssim 0.5$ are forbidden since there is no solution to the matching conditions (5.1)-(5.2). Couplings larger than 3 are not shown since perturbation theory

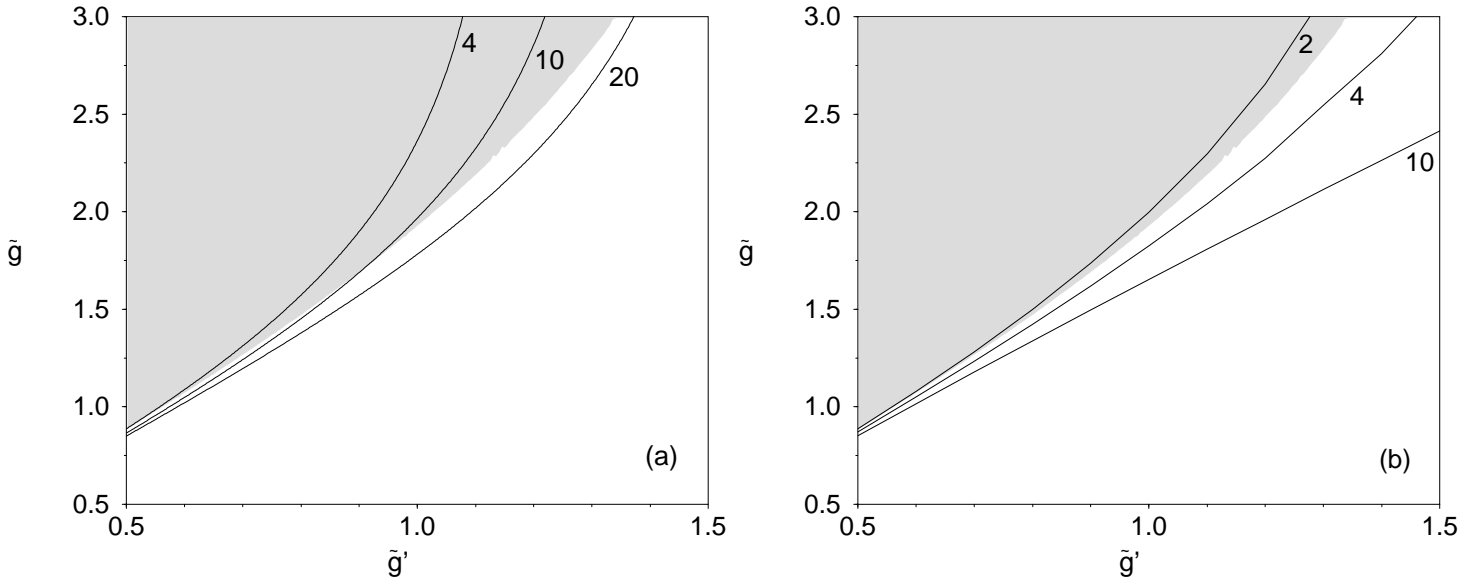


Figure 2: Region of \tilde{g} - \tilde{g}' space that is excluded (shaded region) to 95% C.L. by electroweak precision observables. The solid lines correspond to (a) contours of the matching scale M_u in TeV, and (b) contours of the intermediate gauge boson masses M_i in TeV.

begins to break down. We find that the matching (or unification) scale must be larger than 4 TeV throughout the physically acceptable region, and larger than 11 TeV for the region $\tilde{g}, \tilde{g}' > 1$.

It is clear from Fig. 2 that small changes in the high energy gauge couplings \tilde{g} , \tilde{g}' result in large changes to the matching scale. Consider for example what is needed to obtain a matching scale of order 20 TeV. The region of interest comprises $0.5 \lesssim \tilde{g}' \lesssim 1.4$, within which the *a priori* independent coupling \tilde{g} must be fine-tuned to approximately satisfy $\tilde{g} \simeq \sqrt{3}\tilde{g}'$. Along this line, the threshold corrections to $\sin^2 \theta_W(M_u)$ accidentally cancel out. However, the degree of fine-tuning needed for a fixed matching scale varies quite significantly with \tilde{g}' . We can quantify this by calculating the fractional shift in the high energy couplings that corresponds to a given fractional shift in the matching scale, shown in Fig. 3. For example, holding M_u fixed within 10% requires tuning \tilde{g} to be within (0.6%, 1.2%, 2.6%) for $\tilde{g}' = (0.75, 1, 1.25)$. This means that arranging that new physics be close to our bounds requires significant fine-tuning of the high energy gauge couplings. This fine tuning can be relaxed, but only by simultaneously going to moderate gauge couplings $\tilde{g}, \tilde{g}' > 1$ and matching scales well beyond our bounds, in the tens of TeV.

6. Gauge Boson Phenomenology

Directly resolving the physics of the matching scale is clearly well out-of-reach for future colliders (Tevatron, LHC, LC). For the region of physical interest ($g, g' > 1$) the heavy gauge bosons which

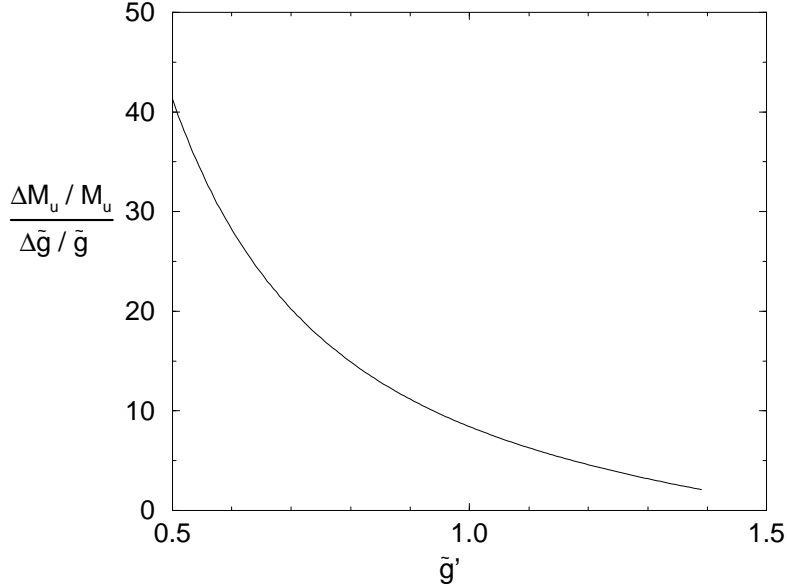


Figure 3: A measure of the fine-tuning of the high energy gauge couplings. The fractional change in the matching scale M_u normalized to the fractional change in the SU(2) coupling \tilde{g} is shown as a function of the U(1) coupling \tilde{g}' . For example, this means that holding M_u fixed within 10% requires tuning \tilde{g} to 10%/(y-axis value). We used $M_u = 20$ TeV, although the result shown is quite insensitive to the value of the matching scale.

couple directly to quarks and leptons have masses $M_{W_H} > 11$ TeV, $M_{B_H} > 6$ TeV. However, there are four gauge bosons, A^4 - A^7 that are generally significantly lighter than the heaviest gauge bosons W_H and B_H . This is shown in Fig. 2(b) where the contours correspond to the intermediate gauge boson mass M_i . A^4 - A^7 do not couple to the SM fermions. Hence, they do not contribute at tree-level to electroweak precision observables, which is why they are permitted to be much lighter than the heavy mixed states W_H and B_H .

The electroweak quantum numbers of A^4 - A^7 are $(2, 3/2)$ and $(2, -3/2)$, which means they have electric charges $\pm 2, \pm 1$. They can only be produced in pairs through couplings to the standard model gauge bosons. This is because they are $SU(2)_L$ doublets that do not couple to the standard model matter fields, so that every interaction vertex must contain at least two of these fields to be gauge invariant. Therefore in order to observe these particles one would have to pair produce them. For gauge couplings $\tilde{g}, \tilde{g}' > 1$, their mass is larger than 2.5 TeV, and so they are unlikely to be produced in sufficient quantities to be detected above backgrounds at the LHC. Similarly, in the minimal model without any new matter charged under $SU(3)_W$, these gauge bosons cannot decay. Again this is a consequence of the coupling by pairs to the electroweak gauge bosons that are triplets or singlets. This “doublet conservation” is not broken by electroweak symmetry breaking since the Higgs does not couple to these gauge bosons. The only other interaction is potentially with the physical $(3,0)$, $(1,0)$ components of the Σ scalar, but these also cannot be decay modes

for the same reason. Even adding non-renormalizable operators to the theory would not change this situation. The reason for this is conservation of the strange color s (the third color of the SU(3) group). This is not broken by the Σ vev, and gauge interactions leave the combination $s - \bar{s}$ invariant. This is a result of discrete symmetries of the model ($A^\mu \rightarrow -A^\mu$, $\Sigma \rightarrow -\Sigma$ and the SU(3) group element $\text{diag}(-1,-1,1)$), the product of which remains unbroken. For a decay mode of the $A^{4,\dots,7}$ gauge bosons $s - \bar{s} = \pm 1$ in the initial state, but is zero in any kinematically allowed final state, so there is no allowed decay mode.

Electrically charged, stable particles can lead to severe cosmological problems [15]. In fact, an order of magnitude estimate [16] shows that these $A^{4,\dots,7}$ gauge bosons would have a relic density much larger than the critical density, assuming their mass is of order $M_i = 2$ TeV, and the annihilation cross section is of order α^2/M_i^2 . This is obtained using the simple estimate [16] for the relic density

$$\Omega h^2 \sim \frac{7.7 \times 10^{-38} \text{ cm}^2}{\langle \sigma v \rangle}, \quad (6.1)$$

where Ω is the fractional energy density compared to the critical density, h is the Hubble constant in units of 100 km/s/Mpc, and $\langle \sigma v \rangle$ is the average annihilation cross section that we estimate to be of order α^2/M_i^2 . The result for $M_i = 2$ TeV is $\Omega \sim 15$. Thus these particles may overclose the Universe. But, there is an even stronger experimental constraint on the relic density. Stringent bounds on the relative abundances of new charged stable particles have been set by searches for heavy isotopes of ordinary nuclei. The typical relative abundance obtained for these charged gauge bosons is of the order $n_{A^{4,\dots,7}}/n_{\text{nucleons}} \sim 10^{-1}$. Searches for heavy isotopes [17], however, typically set a bound on this abundance of the order $n_{A^{4,\dots,7}}/n_{\text{nucleons}} < 10^{-15} - 10^{-20}$. This suggests that the presence of these stable charged gauge bosons $A^{4,\dots,7}$ is excluded.

One could try to avoid this constraint by adding light matter charged under just the $\text{SU}(3)_W$, to which the charged gauge bosons could decay. The difficulty with this approach is that the new matter automatically has electroweak quantum numbers and is therefore not immune to experimental constraints. One possibility is a vector-like pair of $\text{SU}(3)_W$ triplets unrelated to the SM matter. But this is both theoretically unappealing (why is the vector-like triplet mass scale near the electroweak scale?) as well as being constrained by electroweak measurements. In addition, a stable particle would still remain in the spectrum, since the conservation of the strange color introduced above requires that the lightest of the particles charged under this symmetry be stable. Generically, this particle will be charged under the unbroken $\text{U}(1)_{EM}$. In this case the mass of the stable particle could be much smaller than before (~ 100 GeV instead of TeV) and thus might not overclose the Universe. However the bound from isotope searches would be difficult to evade even in this case.

Another choice might be to assume some or all of the leptons are $\text{SU}(3)_W$ triplets. This causes $\text{SU}(2) \times \text{U}(1)$ anomalies, and so yet more matter must be added to the model just to cure this

problem. (Anomaly cancellation was also a generic problem in the “unified standard model” [7].) Furthermore, new operators would exist in the low energy effective theory affecting the ($SU(3)_W$ triplet) lepton couplings once A^4 - A^7 are integrated out. There is every reason to expect electroweak observables would then place as strong a constraint on the mass of these gauge bosons as we found on the mass of W_H and B_H . This means that the lower bound on the matching scale would be increased to of order 30 TeV.

Thus, we find that the “minimal” $SU(3)_W \times SU(2) \times U(1)$ idea has a potentially serious cosmological obstacle in the form of heavy, stable, charged gauge bosons. The solutions to this problem involve adding new light matter that is charged under the $SU(3)_W$ group, but this new sector is expected to be strongly constrained by electroweak precision measurements.

7. Conclusions

We have studied constraints on the recently proposed $SU(3)_W \times SU(2) \times U(1)$ electroweak model. By integrating out the heavy gauge bosons we calculated the low energy effective action in this model. We identified the corrections to the masses and couplings of standard model fields. This allowed us to calculate the corrections to the electroweak observables in this model, from which we performed a global fit to current experimental data. By fixing the value of the electric charge and $\sin^2 \theta_W$, the model was specified in terms of just two parameters. We found the excluded region in the space of $SU(2)$ and $U(1)$ couplings, and found that for the physically interesting region the unification scale is bounded to be larger than 11 TeV.

We also pointed out there are stable multi-TeV scale gauge bosons that are electrically charged, leading to cosmological difficulties. These gauge bosons would have been produced in the early universe, and we found that their present-day relic density would be larger than the critical density, and so would overclose the Universe. Furthermore, there are considerably stronger experimental constraints on the relic density of stable, charged particles coming from searches for heavy isotopes of nuclei. This suggests that modifications to the minimal model are needed to allow the gauge boson to decay. The simplest idea of allowing leptons or additional matter to transform under $SU(3)_W$ lead to further model-building difficulties (*e.g.* anomaly cancellation) and experimental constraints (new contributions to precision electroweak observables).

Acknowledgments

We thank J. Erler for providing us the SM predictions used in our fit. GDK thanks T. Plehn and D. Zeppenfeld for discussions. JE and JT thank A. de Gouvea for comments. The research of C.C.

is supported in part by the NSF, and in part by the DOE OJI grant DE-FG02-01ER41206. The research of J.E. and J.T. is supported by the US Department of Energy under contract W-7405-ENG-36. The research of G.D.K. is supported by the US Department of Energy under contract DE-FG02-95ER40896.

Appendix A: Predictions for Electroweak Observables

In this appendix we give the predictions for the shifts in the electroweak precision observables as a result of the new gauge dynamics $SU(3)_W \times SU(2) \times U(1)$. The electroweak observables depend on only two parameters, c_1 and c_2 . Using the results given in [6, 10] as well as the low-energy νe couplings:

$$g_{eV}(\nu e \rightarrow \nu e) = 2 \left(s^2 - \frac{1}{4} \right) , \quad g_{eA}(\nu e \rightarrow \nu e) = -\frac{1}{2} .$$

we find the following results:

$$\begin{aligned} \Gamma_Z &= (\Gamma_Z)_{SM} (1 - 0.89c_1 + 0.17c_2) \\ R_e &= (R_e)_{SM} (1 + 0.082c_1 + 0.91c_2) \\ R_\mu &= (R_\mu)_{SM} (1 + 0.082c_1 + 0.91c_2) \\ R_\tau &= (R_\tau)_{SM} (1 + 0.082c_1 + 0.91c_2) \\ \sigma_h &= (\sigma_h)_{SM} (1 - 0.0087c_1 - 0.096c_2) \\ R_b &= (R_b)_{SM} (1 - 0.018c_1 - 0.20c_2) \\ R_c &= (R_c)_{SM} (1 + 0.035c_1 + 0.39c_2) \\ A_{FB}^e &= (A_{FB}^e)_{SM} + 0.18c_1 + 2.0c_2 \\ A_{FB}^\mu &= (A_{FB}^\mu)_{SM} + 0.18c_1 + 2.0c_2 \\ A_{FB}^\tau &= (A_{FB}^\tau)_{SM} + 0.18c_1 + 2.0c_2 \\ A_\tau(P_\tau) &= (A_\tau(P_\tau))_{SM} + 0.78c_1 + 8.6c_2 \\ A_e(P_\tau) &= (A_e(P_\tau))_{SM} + 0.78c_1 + 8.6c_2 \\ A_{FB}^b &= (A_{FB}^b)_{SM} + 0.54c_1 + 6.0c_2 \\ A_{FB}^c &= (A_{FB}^c)_{SM} + 0.42c_1 + 4.7c_2 \end{aligned}$$

$$\begin{aligned}
A_{LR} &= (A_{LR})_{SM} + 0.78c_1 + 8.6c_2 \\
M_W &= (M_W)_{SM} (1 + 0.43c_1 + 1.4c_2) \\
M_W/M_Z &= (M_W/M_Z)_{SM} (1 + 0.43c_1 + 1.4c_2) \\
g_L^2(\nu N \rightarrow \nu X) &= (g_L^2(\nu N \rightarrow \nu X))_{SM} + 0.25(c_1 + c_2) \\
g_R^2(\nu N \rightarrow \nu X) &= (g_R^2(\nu N \rightarrow \nu X))_{SM} - 0.085(c_1 + c_2) \\
g_{eV}(\nu e \rightarrow \nu e) &= (g_{eV}(\nu e \rightarrow \nu e))_{SM} - 0.66(c_1 + c_2) \\
g_{eA}(\nu e \rightarrow \nu e) &= (g_{eA}(\nu e \rightarrow \nu e))_{SM} \\
Q_W(Cs) &= (Q_W(Cs))_{SM} + 73(c_1 + c_2)
\end{aligned}$$

We also give in Table 1 the experimental data [11, 18] and the SM predictions used for our fit.

| Quantity | Experiment | SM($m_h = 115$ GeV) |
|-----------------------------------|----------------------|----------------------|
| Γ_Z | 2.4952 ± 0.0023 | 2.4965 |
| R_e | 20.8040 ± 0.0500 | 20.7440 |
| R_μ | 20.7850 ± 0.0330 | 20.7440 |
| R_τ | 20.7640 ± 0.0450 | 20.7440 |
| σ_h | 41.5410 ± 0.0370 | 41.4800 |
| R_b | 0.2165 ± 0.00065 | 0.2157 |
| R_c | 0.1719 ± 0.0031 | 0.1723 |
| A_{FB}^e | 0.0145 ± 0.0025 | 0.0163 |
| A_{FB}^μ | 0.0169 ± 0.0013 | 0.0163 |
| A_{FB}^τ | 0.0188 ± 0.0017 | 0.0163 |
| $A_\tau(P_\tau)$ | 0.1439 ± 0.0041 | 0.1475 |
| $A_e(P_\tau)$ | 0.15138 ± 0.0022 | 0.1475 |
| A_{FB}^b | 0.0990 ± 0.0017 | 0.1034 |
| A_{FB}^c | 0.0685 ± 0.0034 | 0.0739 |
| A_{LR} | 0.1513 ± 0.0021 | 0.1475 |
| M_W | 80.450 ± 0.039 | 80.3890 |
| M_W/M_Z | 0.8822 ± 0.0006 | 0.8816 |
| $g_L^2(\nu N \rightarrow \nu X)$ | 0.3020 ± 0.0019 | 0.3039 |
| $g_R^2(\nu N \rightarrow \nu X)$ | 0.0315 ± 0.0016 | 0.0301 |
| $g_{eA}(\nu e \rightarrow \nu e)$ | -0.5070 ± 0.014 | -0.5065 |
| $g_{eV}(\nu e \rightarrow \nu e)$ | -0.040 ± 0.015 | -0.0397 |
| $Q_W(Cs)$ | -72.65 ± 0.44 | -73.11 |
| m_{top} | 174.3 ± 5.1 | 176.3 |

Table 1: The experimental results [11, 18] and the SM predictions for the various electroweak precision observables used for the fit. The SM predictions are for $m_h = 115$ GeV and $\alpha_s = 0.120$ and calculated [19] using GAPP [20].

References

- [1] S. Weinberg, Phys. Rev. D **5**, 1962 (1972).
- [2] E. J. Konopinski and H. M. Mahmoud, Phys. Rev. **92**, 1045 (1953).
- [3] S. Dimopoulos and D. E. Kaplan, hep-ph/0201148.
- [4] T. j. Li and L. Wei, hep-ph/0202090; L. J. Hall and Y. Nomura, hep-ph/0202107; S. Dimopoulos, D. E. Kaplan and N. Weiner, hep-ph/0202136; S. Dimopoulos and D. E. Kaplan, hep-ph/0203001.
- [5] M. Golden and L. Randall, Nucl. Phys. **B361** (1991) 3; B. Holdom and J. Terning, Phys. Lett. **B247** (1990) 88. M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); B. Holdom, Phys. Lett. B **259**, 329 (1991); H. Georgi, Nucl. Phys. B **363** (1991) 301.
- [6] C. P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, Phys. Rev. D **49**, 6115 (1994) [arXiv:hep-ph/9312291].
- [7] H. Georgi, E. E. Jenkins and E. H. Simmons, Phys. Rev. Lett. **62**, 2789 (1989) [Erratum-ibid. **63**, 1540 (1989)]; Nucl. Phys. B **331**, 541 (1990).
- [8] R. S. Chivukula, E. H. Simmons and J. Terning, Phys. Lett. B **346**, 284 (1995) [hep-ph/9412309].
- [9] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, 232 (2001) [hep-ph/0105239].
- [10] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).
- [11] J. Erler and P. Langacker, review in “The Review of Particle Properties,” D.E. Groom et. al. Eur. Phys. Jour. **C15** (2000) 95; updated version online: <http://www-pdg.lbl.gov/2001/stanmodelrpp.ps>.
- [12] D. C. Kennedy and B. W. Lynn, Nucl. Phys. B **322**, 1 (1989); D. C. Kennedy, B. W. Lynn, C. J. Im and R. G. Stuart, Nucl. Phys. B **321**, 83 (1989).
- [13] C. Csáki, J. Erlich and J. Terning, hep-ph/0203034.
- [14] N. Weiner, hep-ph/0106097; C. Csáki, G. D. Kribs and J. Terning, Phys. Rev. D **65**, 015004 (2002) [hep-ph/0107266]. H. C. Cheng, K. T. Matchev and J. Wang, Phys. Lett. B **521**, 308 (2001) [hep-ph/0107268].
- [15] A. De Rujula, S. L. Glashow and U. Sarid, Nucl. Phys. B **333**, 173 (1990); S. Dimopoulos, D. Eichler, R. Esmailzadeh and G. D. Starkman, Phys. Rev. D **41**, 2388 (1990).
- [16] B. W. Lee and S. Weinberg, Phys. Rev. Lett. **39**, 165 (1977); E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley, 1990), page 151.
- [17] T. K. Hemmick *et al.*, Phys. Rev. D **41**, 2074 (1990); P. Verkerk *et al.*, Phys. Rev. Lett. **68**, 1116 (1992); T. Yamagata, Y. Takamori and H. Utsunomiya, Phys. Rev. D **47** (1993) 1231.
- [18] LEP Electroweak Working Group, CERN-EP/2001-098, arXiv:hep-ex/0112021.
- [19] J. Erler, private communication.
- [20] J. Erler, hep-ph/0005084; <http://www.physics.upenn.edu/~erler/electroweak>.