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Neutrino Cosmology and Limits on Extended Technicolor

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Abstract

Using Big Bang Nucleosynthesis limits on the number of light neutrinos we derive a cosmological lower bound of ≈ 2 TeV on the scale of extended technicolor interactions (or any other new interactions) for the third family in models where heavy gauge bosons couple to both left and right-handed neutrinos.

1 Introduction

Recent work [1-4] on technicolor (TC) and extended technicolor (ETC) theories suggests that it may be possible to describe the observed charged-particle mass spectrum while keeping flavor changing neutral currents supressed, the ρ parameter close to 1, and S small [5]. Of course, if ETC purports to be a theory of mass, it must also accomodate the fact that neutrinos are light, or massless. Until recently, perhaps because of the formidable problems associated with explaining the observed quark and charged lepton masses, this latter challenge has received little attention.

The chief difficulty in such an effort is that it is hard to construct ETC

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models without right-handed neutrinos. While this may present a challenge to reproducing a realistic neutrino mass spectrum, it also suggests that neutrino physics can place new cosmological constraints on ETC models. If new right-handed states are introduced into the theory, one must ensure that these states are not fully populated at the time of Big Bang Nucleosynthesis (BBN), or else the predicted helium abundance may exceed observational limits. This in turn places upper limits on the reaction rates of right-handed neutrinos. Since it is massive ETC gauge boson exchange that is responsible for the interactions of the right-handed neutrinos, a new upper limit on ETC interaction strengths places a new lower bound on the scale where the ETC gauge group is broken.

2 Why Are Neutrinos Light?

The fact that only extremely light left-handed neutrinos are seen in nature is one of the most puzzling features of the quark-lepton mass spectrum. As we have noted, this poses special problems for ETC theories. With right-handed neutrinos present in the theory, there is at least one simple explanation available for the fact that they have not yet been seen: an implementation of the usual seesaw mechanism [6, 7]. The idea of the seesaw mechanism is that right-handed neutrinos get large Majorana masses, so that the left-handed neutrinos end up with masses given by a Dirac mass squared divided by the Majorana mass. It is natural to assume that the neutrino Dirac masses are of the same order as their charged leptonic partners' masses. It is also natural to take the Majorana masses to be of the same order as some ETC scale (or scales) if condensates of bilinears of right-handed neutrinos (Majorana condensates) are involved in the dynamical breaking of the ETC gauge symmetry. If there is a hierarchy of ETC scales to arrange for a hierarchy of charged lepton masses (m_e, m_μ, m_τ), each of which is expected to be of order $(1\text{TeV})^3$ divided by the square of the corresponding ETC scale, then the masses of the associated neutrinos are fixed at the same time. One finds that in this case $m_{\nu_\mu} \approx O(10-50)\text{eV}$, and $m_{\nu_\tau} \approx O(10-50)\text{keV}$. This mass spectrum is of course incompatable with the MSW solution of the solar neutrino problem. More importantly, m_{ν_τ} is unacceptably large, requiring exotic decay mechanisms in order to satisfy the constraints of cosmology [8]. As a result, we will not consider this mechanism further in this work.

A second possibility is that TC produces Dirac masses for the technineutrinos, but that these masses do not feed down to the ordinary neutrinos. One example is a suggestion made long ago by Sikivie, Susskind, Voloshin, and Zakharov (SSVZ) [9]. SSVZ considered an $SU(3)_{ETC}$ model that describes the interactions of one family of technifermions and the third family of ordinary fermions. They concentrated only on the lepton sector, which contains the following left-handed and charge-conjugated right-handed fermions (where $E_R^c = (E_R)^c$) labeled with their $SU(3)_{ETC} \otimes SU(2)_L \otimes U(1)_Y$ charges:

$$\begin{pmatrix} N_{L1} & N_{L2} & \nu_{\tau L} \\ E_{L1} & E_{L2} & \tau_L \end{pmatrix} \sim (\mathbf{3},\mathbf{2})_{-1} \quad ; \quad N_{R1}^c, N_{R2}^c, \nu_{\tau R}^c \sim (\mathbf{3},\mathbf{1})_0$$
$$E_{R1}^c, E_{R1}^c, \tau_R^c \sim (\overline{\mathbf{3}},\mathbf{1})_2 \qquad (1)$$

down It is straightforward to devise mechanisms to break $SU(3)_{ETC}$ to $SU(2)_{TC}$, with the addition of particles that also cancel $SU(3)_{ETC}$ anomalies [1]. Then when the TC interactions get strong, E_L condenses with E_R^c, and N_L condenses with N_R^c.

[1] The SSVZ model suffers from an $SU(2)$ Witten anomaly [10], which can be avoided by the addition of one multiplet.

Note that N_L and N^c_R. technifermions both transform as **3**'s under $SU(3)_{ETC}$, as opposed to the E_L and the E^c_R which transform as a **3** and a $\overline{\mathbf{3}}$ respectively. As a result, while the standard one-ETC-gauge-boson-exchange graph feeds down a mass to the τ, the corresponding graph for the ν_τ vanishes for group theoretical reasons. Thus the ν_τ remains massless. Note that the $\nu^c_{\tau R}$ in this model is sterile, aside from ETC interactions.

This provides one example of the general class of models we will constrain here, ie. those involving small (or vanishing) Dirac masses for the existing neutrinos, with sterile right handed components save for new interactions mediated by heavy gauge bosons such as ETC gauge bosons.

3 ETC, QCD, and BBN

One might imagine that having light right-handed partners, Dirac or Majorana, for standard model neutrinos would destroy the agreement between BBN calculations and observations of helium [11], which now suggest that the number of equivalent light neutrinos, N_ν, is less than[2] about 3.5 [12]. However, extra neutrinos are not necessarily in thermal equilibrium with electrons and photons at temperatures as low as 1 MeV. If the extra neutrinos have Dirac masses with the standard left-handed ν's, then the combination of weak interactions and helicity flips can contribute to thermal population of right-handed states; one then finds that the Dirac mass must be less than about 250 keV [8, 13]. If the neutrino masses are smaller than this,

[2] In fact, if the primordial helium abundance is constrained to be less than 24%, as is sometimes argued, the limit is closer to 3.3 [12]. We will conservatively assume here, however, that the actual abundance can be somewhat larger, as the existing error in this estimate is dominated by large systematic effects. Note that at present the BBN prediction is only consistent if the helium abundance is greater than $\approx 23.6\%$.

population of the right-handed states can only occur by new interactions. Since this proceeds by ETC gauge boson exchange, requiring that the right-handed states do not become populated places a lower bound on the mass of the ETC gauge boson divided by the ETC gauge coupling.

If the decoupling temperature of a particular neutrino is above the QCD phase transition temperature ($T_{QCD} \approx 150$ MeV), then the subsequent reheating of particles coupled to the radiation gas will cause such a decoupled neutrino to contribute less than about 0.1 of a fully populated neutrino state to the radiation energy density below (T_{QCD}). This is currently compatible with BBN limits. We therefore investigate what ETC scale a decoupling temperature of T_{QCD} corresponds to. One can derive independent bounds in this manner for each ETC scale in the generic case where separate scales exist for each family. However, in such models it is generally the third family that is associated with the lowest ETC breaking scale. We thus focus on the tau neutrino here.

There are two ETC processes which can populate right-handed tau neutrino states directly: $\nu_L\overline{\nu}_L \to \nu_R\overline{\nu}_R$ and $\tau\overline{\tau} \to \nu_R\overline{\nu}_R$. Of these two processes only the former is significant at temperatures of order T_{QCD}. This is because the rate for these processes is proportional to the square of the density of incoming particles and at $T \approx 150$ MeV, the number density of τ leptons is suppressed by $\approx e^{-10}$ compared to neutrinos. Thus, even though the cross section for τ annihilation at these temperatures is $\approx 10^2$ times larger than for neutrino annihilation (due to the larger center of mass energies involved), their number density is sufficiently small to make this reaction insignificant. The cross section for right-handed τ neutrino production by ETC interactions for center of mass energy $2E$ can then be straightforwardly derived to be:

$$\sigma = \left(\frac{g}{M}\right)^4 \frac{E^2}{6\pi} A^2 \ , \tag{2}$$

where g is the gauge coupling, M is the mass of the ETC gauge boson, and A^2 represents a group theory factor, which, for example, in the case of $SU(3)_{ETC}$ is 1/9.

To determine the rate of population of right-handed neutrinos at temperature T, one must solve a Boltzmann equation for their number density:

$$\frac{d\eta_R}{dt} = -3\frac{\dot{R}}{R}\eta + \langle\sigma v\rangle\eta_L^2 - \langle\sigma v\rangle\eta_R^2 \ , \tag{3}$$

where η_L and η_R are the number densities of left and right-handed ν_τ's, R is the cosmological scale factor, and the angular brackets imply a thermal average at temperature T. The incident neutrinos are relativistic, so $v = 1$, and it is straightforward to evaluate the thermal average of (2). One finds:

$$\langle\sigma v\rangle = \frac{12.94\, T^2}{6\pi}\left(\frac{g}{M}\right)^4 A^2 \ . \tag{4}$$

Next, following standard methods [14] we can use Einstein's equations for the Friedmann-Robertson-Walker expansion to rewrite equation (3) in a form which is more easily solved. Writing a dimensionless quantity $\tilde{\eta} = \eta/T^3$ one finds

$$\frac{d\tilde{\eta}_R}{dT} = -\left(\frac{3}{8\pi G g*}\right)^{1/2} \langle\sigma v\rangle(\tilde{\eta}_L^2 - \tilde{\eta}_R^2) \ , \tag{5}$$

where

$$g_* = \frac{\pi^2}{30}\left(g_b + \frac{7}{8}g_f\right) \ , \tag{6}$$

gives the contribution of helicity states in the radiation gas to the total energy density, and g_b and g_f are the number of boson and fermion helicity states in the radiation gas at temperature T.

Now, define the thermal equilibrium value,

$$\tilde{\eta}_{thermal} = \frac{3}{4}\frac{\zeta(3)}{\pi^2} = k, \tag{7}$$

for a single neutrino helicity state. After the QCD phase transition at $T_{QCD} \approx 150$ MeV, there is rapid reheating of all light or massless states in thermal equilibrium at that time. Before any possible reheating of right-handed neutrinos (which we are assuming are essentially decoupled) therefore, it is reasonable to assume an initial condition $\tilde{\eta}_R = \epsilon_i k$, where

$$\epsilon_i = \frac{g_*(T < T_{QCD})}{g_*(T > T_{QCD})} \approx 0.28 \; . \tag{8}$$

This takes into account the temperature suppression of states not in thermal equilibrium during the period of reheating following QCD the phase transition, which we assume is adiabatic. By assumption the right-handed neutrino states are not included in the helicity sums. To arrive at the value quoted, we include up, down, and strange quarks in our initial ($T > T_{QCD}$) radiation gas.

Given this initial condition, we wish to see if the final abundance, $\tilde{\eta}_R$, rises to a value larger than its initial value after the QCD phase transition. In particular, we are interested in whether this quantity approaches ≈ 0.5 of a neutrino in thermal equilibrium, that is $\tilde{\eta}_R(T \approx 1\text{MeV})/k \approx 0.5$ (i.e. $N_\nu < 3.5$). Using equation 4, we can then integrate (5) analytically. Using our initial condition for ϵ_i, we find

$$\epsilon_f = \tilde{\eta}_R(T_f)/k = \left[\frac{\left(\frac{1+\epsilon_i}{1-\epsilon_i}\right)\exp\left(\frac{2}{3}P[T_{QCD}^3 - T_f^3]\right) - 1}{\left(\frac{1+\epsilon_i}{1-\epsilon_i}\right)\exp\left(\frac{2}{3}P[T_{QCD}^3 - T_f^3]\right) + 1}\right] \; , \tag{9}$$

where

$$P = \frac{12.94k}{6\pi}\left(\frac{3}{8\pi G g*}\right)^{1/2}\left(\frac{g}{M}\right)^4 A^2 \; . \tag{10}$$

One can also derive a useful approximate result, in which the dependence of ϵ_f on the fundamental parameters is clearer. This is obtained by ignoring the second term in (5). In this case one finds:

$$\epsilon_f \approx \epsilon_i + \frac{P}{3}\left(T_{QCD}^3 - T_f^3\right) \ . \tag{11}$$

This result will be good as long as both ϵ_i and ϵ_f remain small. We find that for $\epsilon_f < 1/2$, the bounds obtained using this result are in good agreement with those obtained from (10). Note that a larger T_{QCD} would result in a more stringent bound on the ETC scale, so 150 MeV represents a conservative choice.

In order to use these results to bound M/g we must take two further factors into account. First, below the pion and muon annihilation thresholds (when $T \approx m_\pi, m_\mu$) right-handed neutrinos can be further diluted. We can account for this by approximating the reheating due to pion and muon annihilations as instantaneous and then integrating in stages. We first integrate to find ϵ_f just above the pion threshold, and then calculate the dilution of this quantity (as in eq. (8)) which would result from reheating of the thermally coupled radiation gas due to pion annihilations, assuming again that the right-handed neutrinos are decoupled at this point. We then input this value as ϵ_i in a new integration (using either (9) or (11)) down to $T = m_\mu$, using the new appropriate g_* value in this range, and then repeat the above procedure, and then integrate down to $T \approx 1$ MeV.

Finally, we must remember that the quantity of interest is not ϵ itself, but rather $\epsilon^{4/3}$. This is due to the fact that the quantity which is bounded by BBN is the neutrino energy density and not its number density. The former is proportional to the 4/3 power of the latter.

We display in figure 1 our final result for the quantity $\epsilon_f^{4/3}$, based on succesive iterations of (9), across the pion/muon thresholds as described above. If there is a single extra right-handed neutrino, in order for it to contribute less than ≈ 0.5 extra neutrino species to the expansion rate during nucleosynthesis, then $M/g > 2.2$ - 4 TeV, depending on the value of A^2. Note that where the curve approaches unity the extra neutrino is beginning to be in thermal equilibrium near T_{QCD}.

Thus, cosmology places a severe constraint on ETC models of this type. If we use the more restrictive BBN bound of 3.3 effective neutrino species, then the lower bound on M/g would rise only slightly to 3-5 TeV. These results are significant because ETC scales this small are likely to be required to get a t quark mass above 100 GeV. Note that if a model had 6 or more extra neutrino helicity states, then at least some of the extra neutrinos would have to decouple above the τ and c quark thresholds (or have rapid decays) in order that $\epsilon_i < 0.1$. Using equation (11), we find that the lower bound would then be pushed up to $M/g > O(20)$ TeV.

While our analysis was carried out in the context of one class of models, the arguments are generic, and must be taken into account in any ETC model which purports to explain neutrino masses, or for that matter in any model that has heavy gauge bosons which couple to right handed neutrinos. For example, the recent model of Randall [4] has 3 extra right-handed neutrinos which form 3 Dirac particles with 3 more sterile neutrinos. The right-handed neutrinos have ETC interactions, but only with right-handed, up-type quarks. Thus below T_{QCD} these neutrinos will be produced by $\pi^+\pi^- \to \nu_R\overline{\nu}_R$. Below $T = m_\pi$, they are produced by $\gamma\gamma \to \nu_R\overline{\nu}_R$, and by bremsstrahlung off protons and neutrons (both reactions proceed through virtual π^0's). Since the ETC scale in this model is only 1.5 TeV, arguments of the type presented here could provide severe constraints on it, unless some mechanism allows the extra neutrinos to decay before the BBN era.

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Figure 1: Contribution of an extra neutrino to the energy density at the time of BBN expressed as a fraction of that due a standard model neutrino, as a function of the ETC scale. A^2 is a group theoretic factor depending upon the ETC gauge group, and is 1/9 for $SU(3)_{ETC}$. For comparison, the result is also displayed for a new U(1) interaction,

where $A^2 = 1$