

An Extended Technicolor Model

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Abstract

An extended technicolor model is constructed. Quark and lepton masses, spontaneous CP violation, and precision electroweak measurements are discussed. Dynamical symmetry breaking is analyzed using the concept of the big MAC.

1 Introduction

Recent work [1, 2, 3, 4, 5] on technicolor (TC) models indicates that it may be possible to describe the observed particle mass spectrum, while avoiding flavor changing neutral currents (FCNC's) and satisfying precision electroweak tests. That is, using a phenomenologically acceptable TC gauge group and technifermion count, and representing the extended technicolor (ETC) interactions by four-fermion interactions with arbitrary mass scales and arbitrary couplings for each of the ordinary fermions, one can produce the entire observed range of fermion masses, up to well over one hundred GeV for the t quark, without excessive fine tuning of parameters and without any phenomenological disasters. Though this exercise is interesting as

a sort of existence proof, it uses as many parameters as observables, so it is difficult to be sure if success is the result of having identified the correct physics.

To do better one must construct a model that explains the world rather than just describes it, i.e. a model with fewer parameters than the standard model. It is our purpose here to construct a plausible ETC model. After reviewing the constraints that must be satisfied, we will present the model. We then conclude with a discussion of quark and lepton masses, precision electroweak tests, and CP violation. One additional prediction for new phenomena will also be described.

2 Ingredients for Model Building

There are several ingredients that should be incorporated into a realistic ETC model. First of all, more than one ETC scale is expected. The absence of FCNC's (inferred from $K - \bar{K}$ mass splitting) requires the mass of the ETC bosons that connect to the s quark to be at least¹ about $\Lambda_{FCNC} = 1000 \text{ TeV} \cos\theta \sin\theta$, where θ is a model dependent mixing angle [7, 8]. For example, taking θ to be equal to the Cabibbo angle, we find $\Lambda_{FCNC} = 200 \text{ TeV}$. In order to have such a high ETC scale associated with the s , and still produce the correct mass, one may have to invoke walking [1]. Also, to obtain a t quark mass above 100 GeV without excessive fine tuning, it turns out that the ETC scale relevant to t mass generation should be at most about 10 TeV [2]. Such arguments, coupled with the observation of the hierarchy of family masses, suggest three different ETC scales, one for each family. In this paper we will take these scales to be roughly 10, 100, and 1000 TeV. With a reasonable running of gauge couplings, these scales can arise naturally via self-

¹Assuming a coupling of order 1, and the absence of a “TechniGIM” mechanism [6].

breaking gauge interactions, and may thus afford us with a natural explanation of the family mass hierarchy.

A realistic ETC model must also survive precision electroweak tests [9, 10]. It must produce a large t - b mass splitting, while keeping the radiative electroweak correction parameter $\Delta\rho_* \equiv \alpha T$ less than about 0.5%. The radiative electroweak correction parameter S can also be worrisome [10]. Experiments seem to be finding S to be very small or even negative, whereas QCD-like TC models give positive contributions to S (which grow with the number of technicolors, N_{TC}). Of course, QCD-like TC models may already be ruled out since they lead to large FCNC's, and furthermore it is difficult to reliably estimate S in TC models with non-QCD-like dynamics [11]. Nevertheless, the constraint on the S parameter seems to suggest that N_{TC} should be kept as small as possible.

An important constraint on ETC model building was originally elucidated by Eichten and Lane [7], who showed that the absence of a visible axion implies a limit on the number of spontaneously broken global U(1) symmetries², and hence a limit on the number of irreducible representations of the ETC gauge group. This points to some form of quark-lepton unification (such as Pati-Salam unification [13]), in ETC models.

Also, to avoid a plethora of massless, non-Abelian, Nambu-Goldstone bosons, a realistic ETC model should not have any exact, spontaneously broken, non-Abelian global symmetries. Thus there should not be repeated representations of the ETC gauge group.

An ETC model must also explain why neutrinos are special. The fact that

²Another possibility is that the axion is made very heavy by QCD, see ref. [12].

only extremely light left-handed neutrinos are seen in nature is one of the most puzzling features of the quark-lepton mass spectrum. It poses special problems for ETC model builders, since it is difficult to construct ETC models without right-handed neutrinos. With right-handed neutrinos present in the model, there are at least two simple explanations available for small neutrino masses: an implementation of the usual seesaw mechanism [14, 15], or the possibility that the technifermion masses do not feed down directly to the neutrinos. The latter possibility was suggested long ago by Sikivie, Susskind, Voloshin, and Zakharov (SSVZ) [16]. The model to be discussed in this paper will utilize this mechanism.

SSVZ considered an $SU(3)_{ETC}$ gauge group (which will appear in our model below 100 TeV), where a $\mathbf{3}$ of $SU(3)_{ETC}$ corresponds to two technifermions and one third-generation fermion. $SU(3)_{ETC}$ will be broken to $SU(2)_{TC}$ by another strong gauge interaction, referred to here as hypercolor (HC). The idea of SSVZ is to put leptons in unusual ETC representations. The left-handed leptons are placed in $\mathbf{3}$'s of $SU(3)_{ETC}$; the charge-conjugated, right-handed charged-leptons in $\bar{\mathbf{3}}$'s; and the charge-conjugated, right-handed neutrinos in $\mathbf{3}$'s. When $SU(3)_{ETC}$ breaks, all the technileptons are in equivalent $SU(2)_{TC}$ representations, but the ETC interactions of the ν_τ are different from those of the τ . In fact, it can be shown that to leading order in ETC exchange, the ν_τ does not receive a mass. ETC interactions must of course be extended beyond $SU(3)_{ETC}$. The ν_τ may receive a mass in higher orders in these interactions, but, as we will discuss later, in the model to be presented, at two loops, the $\nu_{\tau L}$ will remain massless.

Our model will also ensure that the muon neutrino (ν_μ) mass vanishes to a sufficiently high order in perturbation theory so as to satisfy the experimental

constraint on its mass. This will arise through a simple extension of the SSVZ mechanism to the second generation.

Whether or not the SSVZ mechanism is extended to the first generation, there would be other contributions to the ν_e mass that are much too large. If quarks and leptons are unified (as they must be in a realistic ETC model), then masses can feed down to ν_e from ordinary fermions as well as technifermions. For example, consider a Pati-Salam [13] unification scheme. The ν_e is placed in the same representation as the u quark, and there is a diagram that feeds the u quark mass down to the ν_e through the exchange of a heavy Pati-Salam gauge boson³. A standard calculation (for simplicity taking the Pati-Salam breaking scale, which provides the cutoff for the calculation, to be equal to the ETC scale of the first generation) then gives:

$$m_{\nu_e} \approx \frac{9\alpha_{PS}}{8\pi} m_u . \quad (1)$$

This gives a mass for the ν_e on the order of a fraction of an MeV, far above the experimental bound. In order to avoid such a disaster, the right-handed neutrino that is unified with the right-handed u quark must get either a large Majorana mass with itself, or a large Dirac mass with another $SU(2)_L$ singlet neutrino. The model presented here will employ the later possibility, and as a result there will be no right-handed neutrino in the first family.

Another problem that ETC models must face is intrafamily mass splittings. The most striking such splitting, and the most difficult to account for in models with a family of technifermions, is the t - τ splitting. One possible solution is that

³In a self-consistent calculation one should also include the ν_e self-energy coming from a single Pati-Salam gauge boson exchange.

this splitting comes from QCD effects [17, 18]. It is possible for small perturbations (like QCD) to have large effects in models where the (strong) ETC coupling is near critical [2]. One calculation [18] found that this effect could give a quark mass up to two orders of magnitude larger than that of the corresponding lepton. We will rely on the efficacy of this mechanism in our model.

3 A Realistic ETC Model

We now construct an ETC model, using the smallest possible TC group: $SU(2)_{TC}$. One family of technifermions will be included, since this allows for the smallest possible ETC gauge group. We require that:

- 1) there are no exact non-Abelian global symmetries,
- 2) quarks and leptons are unified so as to avoid a visible techniaxion,
- 3) fermions are only allowed to be singlets or triplets of $SU(3)_C$, i.e. we eschew quixes, queights, etc.,
- 4) all gauge anomalies vanish,
- 5) the standard model gauge groups are not embedded in the ETC group,
- 6) the ETC gauge group is asymptotically free,
- 7) the SSVZ mechanism is incorporated in order to keep the ν_τ light,
- 8) isospin and CP are not explicitly broken.

With these restrictions we can proceed straightforwardly. Starting with $SU(2)_{TC}$, the simplest way to gauge the family symmetries is to make use of $SU(5)_{ETC}$. In order to get a hierarchy of families, this gauge group should break

down in stages (i.e. $SU(5)_{ETC} \rightarrow SU(4)_{ETC} \rightarrow SU(3)_{ETC} \rightarrow SU(2)_{TC}$). In order to avoid a visible techniaxion, as discussed above, quarks and leptons are unified using the Pati-Salam [13] group $SU(4)_{PS}$. In order to break $SU(4)_{ETC}$ and $SU(3)_{ETC}$ down to $SU(2)_{TC}$, we will need an additional strong gauge group: $SU(2)_{HC}$. Thus, the gauge group for the model is taken to be $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$. The breaking scale for all these interactions will be on the order of 1000 TeV or lower.

To insure that the model contains only $\mathbf{3}$'s, $\bar{\mathbf{3}}$'s, and singlets of color, fermions are placed only in antisymmetric irreducible representations [19] of $SU(4)_{PS}$. As usual, the $U(1)_R$ is required in order to get the correct hypercharges for the right-handed fermions. Since, at the Pati-Salam breaking scale, Λ_{PS} , the $U(1)_R$ will mix with a generator of $SU(4)_{PS}$ (with $\alpha_{PS}(\Lambda_{PS}) \approx 0.07$), the $U(1)_R$ coupling must be very weak in order to get the right $U(1)_Y$ coupling in the low-energy effective theory. The $U(1)_R$ gauge group looks like a remnant of an $SU(2)_R$, but left-right symmetry has not been introduced since we expect that the requirement that the $SU(2)_L$ and $SU(2)_R$ gauge couplings be equal at the $SU(2)_R$ breaking scale would put this scale much higher than those being considered here. The reason for this is that the $U(1)_R$ coupling at Λ_{PS} is much weaker than the $SU(2)_L$ coupling at this scale.

The standard model fermions and one family of technifermions can be contained in the following representations ⁴:

$$(\mathbf{5}, \mathbf{1}, \mathbf{4}, \mathbf{2})_0 \quad (\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_{-1} \quad (\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_1 . \quad (2)$$

⁴Throughout we will make use of the convention of using the charge-conjugates of the right-handed fields instead of the right-handed fields themselves.

If these were the only fermions in the model, there would be no isospin splittings and no CKM mixing angles. Thus we must include additional fermions that can mix with some of the ordinary fermions, so that isospin breaking can arise spontaneously.

To motivate the choice of additional fermions, we next consider how to include CP violation in the model, without producing a strong-CP problem. This can be done if the Nelson-Barr solution to the strong-CP problem [20] can be implemented in our model. The Nelson-Barr mechanism allows complex phases to appear in the mass mixing between the standard model quarks and new exotic quarks. The determinant of the mass matrix, however, must remain real. To begin, this mechanism requires, in addition to the standard fermions already discussed, some exotic quarks that can mix with the ordinary quarks. These quarks should be $SU(2)_L$ singlets, so as not to contribute to S . The simplest way to do this (keeping in mind the restriction to antisymmetric representations) is to include particles that transform as $(\mathbf{6}, \mathbf{1})_0$ under $SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$. Such representations will decompose into particles with standard model quantum numbers $(\bar{\mathbf{3}}, \mathbf{1})_{2/3}$ and $(\mathbf{3}, \mathbf{1})_{-2/3}$. These correspond respectively to a charge-conjugate, right-handed, down-type quark, and a left-handed partner with which it can obtain a gauge invariant mass. One such “vector” quark and one hypercolored “vector” quark will be included, which we will refer to as the m and the G respectively. The G will be responsible for feeding down a mass to the m , and will also slow the running of the HC coupling above 10 TeV. We will return to a discussion of CP violation in section 4.

We also need extra particles to incorporate the SSVZ mechanism. Since they must have the quantum numbers of right-handed neutrinos, we can make use of the simplest possibility: that they are $SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$ singlets. It

can now be seen how isospin breaking can appear spontaneously in the model. The fermions of the standard model come from $\mathbf{4}$'s and $\bar{\mathbf{4}}$'s of $SU(4)_{PS}$. The additional Pati-Salam representations to be included are $\mathbf{1}$'s and $\mathbf{6}$'s, and these give only right handed neutrinos and “vector” down-type quarks. Thus there will be extra particles that can mix with neutrinos and down-type quarks, allowing for isospin breaking masses, and mixing angles.

We now explicitly write down the model. The gauge group is $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$, with the fermion content taken to be:

$$(\mathbf{5}, \mathbf{1}, \mathbf{4}, \mathbf{2})_0 \quad (\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_{-1} \quad (\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_1 \quad (3)$$

$$(\mathbf{1}, \mathbf{1}, \mathbf{6}, \mathbf{1})_0 \quad (\mathbf{1}, \mathbf{2}, \mathbf{6}, \mathbf{1})_0 \quad (4)$$

$$\begin{aligned} &(\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \quad (\mathbf{5}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \\ &(\bar{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \quad . \end{aligned} \quad (5)$$

The $(\mathbf{5}, \mathbf{1}, \mathbf{4}, \mathbf{2})_0$, the $(\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_{-1}$, and the $(\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_1$ in this list contain particles with quantum numbers corresponding to three families of ordinary fermions (plus charge-conjugated, right-handed neutrinos) and one family of technifermions, i.e. the $\mathbf{5}$ of $SU(5)_{ETC}$ corresponds to three families and two technicolors. The additional fermions are an economical set that will allow us to break ETC gauge symmetries, and isospin, as well as to incorporate the Nelson-Barr mechanism for CP violation. Note that the extra neutrino sector listed in (5) makes this a chiral gauge theory with respect to the gauge groups $SU(5)_{ETC}$ and $SU(2)_{HC}$. All the non-Abelian gauge interactions in the model are asymptotically free.

Next the pattern of symmetry breaking must be specified. An attractive and economical idea is that the breaking is completely dynamical, driven by the asymptotically free gauge theory itself at each stage (this phenomena is referred to

as “tumbling” [21]). Folklore then has it that the fermion condensates form in the most attractive channel (MAC) [21, 22]. The MAC is usually determined in one gauge boson exchange approximation, neglecting gauge boson masses that will be formed if the condensate breaks the gauge group. The one-gauge-boson exchange approximation may or may not be reliable⁵, and furthermore, the additional approximation of neglecting gauge-boson mass generation could be misleading. We will nevertheless adopt the MAC criterion here as a guideline.

We will argue that the breaking will in fact take place in the phenomenologically desired breaking channel at the lower ETC scales (approximately 100 TeV and below). For this purpose we will require that the $SU(5)_{ETC}$ and $SU(2)_{HC}$ couplings are relatively strong in order to drive the tumbling. (By contrast, the other gauge groups in the model, which produce the weakly coupled interactions of the standard model, will be too feeble to drive dynamical symmetry breaking.) At ETC scales of about 1000 TeV and above, the phenomenologically correct breaking channel will not be the MAC, and it will be necessary to assume that the breaking occurs in the desired channel. We take this as evidence that our model is complete below 1000 TeV, but perhaps not complete at higher scales.

Thus, to begin, we assume that the relatively strong $SU(5)_{ETC}$ gauge interactions and some additional new physics from higher scales trigger the formation of a condensate at the scale Λ_{PS} somewhat above 1000 TeV in the attractive channel $(\bar{\mathbf{5}}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_{-1} \times (\mathbf{5}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{1}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1})_{-1}$. This breaks the $U(1)_R$ and Pati-Salam symmetry, leading to the gauge group $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes$

⁵Some evidence for the reliability of the ladder approximation is discussed in ref. [23].

$U(1)_Y$ below Λ_{PS} . Hypercharge, Y , (normalized by $Q = T_{3L} + Y/2$) is given by

$$Y = Q_R + \sqrt{\frac{8}{3}} T_{15} , \quad (6)$$

where the $SU(4)_{PS}$ generator $T_{15} = \sqrt{\frac{3}{8}} \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1)$ is the $B - L$ generator, and Q_R is the $U(1)_R$ charge. Note that the $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{4}}, \mathbf{1})_{-1}$ condensate will give a large mass to the right-handed neutrinos that were unified with up-type quarks. This avoids the problem of quark masses feeding down to neutrino masses through Pati-Salam interactions, discussed in Section 2.

The MAC ⁶ for $SU(5)_{ETC}$ is $\mathbf{10} \times \overline{\mathbf{10}} \rightarrow \mathbf{1}$. The (massless) one-gauge-boson approximation gives a crude measure of the strength of the interaction, and we will use this as a guideline throughout the paper. In this approximation the interaction strength in this channel is proportional to the difference of Casimirs, $\Delta C_2 = C_2(\mathbf{10}) + C_2(\overline{\mathbf{10}}) - C_2(\mathbf{1}) = 36/5$. By contrast, the channel in which condensation is assumed here is the second most attractive channel (with respect to $SU(5)_{ETC}$) with $\Delta C_2 = 24/5$. As pointed out above, some additional new physics at Λ_{PS} and above may be necessary to produce the condensate in this channel.

The fermion content of the model below the Pati-Salam breaking scale, Λ_{PS} , (labeled by $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$) is:

$$\begin{array}{ll} (\mathbf{5}, \mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & (\mathbf{5}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} \\ (\overline{\mathbf{5}}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{-4/3} & \\ (\overline{\mathbf{5}}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{2/3} & (\overline{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_2 \end{array} \quad (7)$$

$$\begin{array}{ll} (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{2/3} \\ (\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{2}, \overline{\mathbf{3}}, \mathbf{1})_{2/3} \end{array} \quad (8)$$

⁶In our model this condensate would break the $SU(2)_{HC}$ group. Thus this channel may be disfavored given that $SU(2)_{HC}$ is relatively strong, since the broken HC gauge bosons will give a large positive contribution to the energy of the corresponding vacuum.

$$(\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \quad (\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 . \quad (9)$$

We have not listed the $(\overline{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ and the $(\mathbf{5}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ which have gotten a large Dirac mass from the dynamical symmetry breaking. We note that, except for $U(1)_Y$, all the remaining gauge groups are asymptotically free.

Next we assume that, at $\Lambda_5 \approx 1000$ TeV, a condensate forms in the attractive channel $(\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \times (\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\overline{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$. The $SU(5)_{ETC}$ MAC at this scale would again be $\mathbf{10} \times \overline{\mathbf{10}} \rightarrow \mathbf{1}$ with $\Delta C_2 = 36/5$. This condensate, however, would break $SU(2)_{HC}$, and might be disfavored as pointed out in footnote 6. The assumed breaking channel, $\mathbf{10} \times \mathbf{10} \rightarrow \overline{\mathbf{5}}$, is almost as strong with $\Delta C_2 = 24/5$, and it does not break $SU(2)_{HC}$. Note that the channel $(\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{5}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ is not a Lorentz scalar⁷, while $(\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{5}, \mathbf{3}, \mathbf{1}, \mathbf{1})_0$ is in a repulsive channel with respect to the $SU(2)_{HC}$ interactions, and will be prevented from forming if this gauge interaction is moderately strong. (Thus the $(\overline{\mathbf{10}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0$ should not develop a Majorana mass.) The condensate $(\overline{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ breaks the gauge symmetry to $SU(4)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, and the first family breaks off at this scale. The fermion content below 1000 TeV is (labeled according to $SU(4)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$):

$$\begin{array}{cccc}
(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & (\mathbf{4}, \mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} & (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} \\
(u, d)_L & & (\nu_e, e)_L & \\
\\
(\mathbf{1}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{-4/3} & (\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{-4/3} & & \\
u_R^c & & & (10) \\
\\
(\mathbf{1}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{2/3} & (\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})_{2/3} & (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})_2 & (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_2 \\
d_R^c & & e_R^c &
\end{array}$$

⁷It is assumed here that gauge theories do not spontaneously break Lorentz invariance.

$$\begin{array}{cc}
(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} \\
m_L & m_R^c
\end{array} \tag{11}$$

$$\begin{array}{cc}
(\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{2}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} \\
G_L & G_R^c
\end{array}$$

$$\begin{array}{cc}
(\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 & \\
(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 & (\mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 .
\end{array} \tag{12}$$

The names of standard model fermions have been written beneath the corresponding group representations (where $u_R^c = (u_R)^c$). We have also labeled the exotic, “vector” m quarks which should mix with the down-type quarks, and the hypercolored “vector” G quarks. Note that there is no ν_{eR}^c . We also note that all remaining non-Abelian gauge groups are again asymptotically free.

The next stage of breaking will be driven by the $SU(4)_{ETC}$ and $SU(2)_{HC}$ interactions. It will be argued to occur in the attractive channel $(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ at a scale taken to be around $\Lambda_4 \approx 100$ TeV. This breaks the gauge symmetry to $SU(3)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, and the second family splits off at this scale. This channel is not the MAC for $SU(4)_{ETC}$ alone: $\mathbf{6} \times \mathbf{6} \rightarrow \mathbf{1}$ ($\Delta C_2 = 5$) and $\mathbf{4} \times \bar{\mathbf{4}} \rightarrow \mathbf{1}$ ($\Delta C_2 = 15/4$) are more attractive. Nevertheless, both $\bar{\mathbf{4}} \times \mathbf{6} \rightarrow \mathbf{4}$ for $SU(4)_{ETC}$ ($\Delta C_2 = 5/2$), and $\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}$ for $SU(2)_{HC}$ ($\Delta C_2 = 3/2$) involve very attractive interactions (the latter is in fact the MAC for $SU(2)_{HC}$). We will next argue that the sum of these two interactions favors the chosen channel over all others.

It is not difficult to see that the most competitive other channel is the one involving the $SU(4)_{ETC}$ MAC: $(\mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1})_0$. To compare these two channels, we compute for each the sum of the gauge couplings evaluated

at Λ_4 , squared and weighted by the difference of Casimirs in the various channels. It is this combination that will appear in an effective potential, or gap equation analysis. For the channel involving the $SU(4)_{ETC}$ MAC, we have

$$\Delta C_2(\mathbf{6} \times \mathbf{6} \rightarrow \mathbf{1}) \alpha_4(\Lambda_4) + \Delta C_2(\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{3}) \alpha_2(\Lambda_4) = 5 \alpha_4(\Lambda_4) - \frac{1}{2} \alpha_2(\Lambda_4) , \quad (13)$$

while for the desired channel we obtain

$$\Delta C_2(\bar{\mathbf{4}} \times \mathbf{6} \rightarrow \mathbf{4}) \alpha_4(\Lambda_4) + \Delta C_2(\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}) \alpha_2(\Lambda_4) = \frac{5}{2} \alpha_4(\Lambda_4) + \frac{3}{2} \alpha_2(\Lambda_4) . \quad (14)$$

Thus if $\alpha_2(\Lambda_4) > \frac{5}{4} \alpha_4(\Lambda_4)$ then (14) will be larger than (13), and the desired channel will be preferred over the other. We assume that this is the case. Note that as long as $\alpha_2(\Lambda_4) < \frac{5}{3} \alpha_4(\Lambda_4)$, then it is still the $SU(4)_{ETC}$ interactions that make the dominant contribution to the dynamical symmetry breaking in the desired channel. A simple gap equation analysis (with constant couplings) indicates that dynamical symmetry breaking will proceed when $\Delta C_2(\bar{\mathbf{4}} \times \mathbf{6} \rightarrow \mathbf{4}) \alpha_4(\Lambda_4) + \Delta C_2(\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}) \alpha_2(\Lambda_4)$ reaches a critical value of $2\pi/3$. More sophisticated analyses that include the effects of running and gauge boson masses generally find that $2\pi/3$ is an underestimate of the critical value.

It is instructive to compare our analysis with a conventional MAC analysis, where one would compare the $SU(4)_{ETC}$ MAC with the $SU(2)_{HC}$ MAC, i.e. compare the first term in (13) with the second term in (14). Then one would find that as long as $\alpha_2(\Lambda_4) < \frac{10}{3} \alpha_4(\Lambda_4)$, the $SU(4)_{ETC}$ interaction in channel (13) would be dominant. The $\mathbf{6} \times \mathbf{6} \rightarrow \mathbf{1}$ channel would be preferred for condensation, which, for the range of couplings discussed above, would be the opposite conclusion to our more refined analysis. To summarize, we have suggested that when two (or more) relatively strong gauge interactions are at play, the favored breaking channel will be

determined by the sum of the interactions. As in the present example, the favored channel need not be the one involving the MAC of the strongest single interaction. We refer to the favored channel in this case as the big MAC. We assume that the coupling constants are in the correct range for the big MAC to be preferred.

The fermion content below Λ_4 (labeled according to $SU(3)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$) is:

$$\begin{array}{llll}
2(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & 2(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} & (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} \\
(u, d)_L, (c, s)_L & & (\nu_e, e)_L, (\nu_\mu, \mu)_L & \\
2(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-4/3} & (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-4/3} & & \\
u_R^c, c_R^c & & &
\end{array} \tag{15}$$

$$\begin{array}{llll}
2(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & 2(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})_2 & (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_2 \\
d_R^c, s_R^c & & e_R^c, \mu_R^c & \\
(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & & \\
m_L & m_R^c & & \\
(\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \mathbf{2}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & & \\
G_L & G_R^c & &
\end{array} \tag{16}$$

$$\begin{array}{ll}
(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 & (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0 \\
\nu_{\mu R}^c & \\
(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 & (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \\
X & F
\end{array} . \tag{17}$$

All the non-Abelian gauge groups at this stage are asymptotically free. We expect that the gauge couplings $\alpha_2(\Lambda_4)$ and $\alpha_3(\Lambda_4)$ are in the neighborhood of 0.5. For example, the values $\alpha_2(\Lambda_4) \approx 0.61$ and $\alpha_3(\Lambda_4) \approx 0.47$ are consistent with the big MAC analysis described above.

We note that the correct fermion content is now in place to employ the SSVZ

mechanism. Consider the technifermions and third generation fermions (which transform under $SU(3)_{ETC}$). The charge-conjugated, right-handed ν_τ and technineutrino are in a $\mathbf{3}$ (see line (17)) of $SU(3)_{ETC}$ as opposed to charge-conjugated, right-handed, quarks and charged leptons, which are in $\bar{\mathbf{3}}$'s. Note that the particle that will turn out to be the $\nu_{\mu R}^c$ has come out of the extra neutrino sector. We also note that the original fermion content (in lines (4) and (5)) above the Pati-Salam breaking scale did not suffer from Witten's anomaly [24] for the $SU(2)_{HC}$ gauge group. This ensures the presence of the particle we have labeled X , which did not appear in the original SSVZ toy model [16].

The final stage of ETC breaking occurs when the $SU(2)_{HC}$ and $SU(3)_{ETC}$ interactions get somewhat stronger, at a scale Λ_3 that will be roughly estimated to be around 10 TeV. The desired channel is the one in which the F condenses with itself: $(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$, breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$. This is the MAC for $SU(2)_{HC}$, and an attractive channel for $SU(3)_{ETC}$. The combination of the two interactions ensures that the F condenses with itself rather than with the X or the G , and provides another example of a big MAC. Again, as a guideline, we consider the sum of the gauge couplings, squared and weighted by the difference of Casimirs for this channel:

$$\Delta C_2(\bar{\mathbf{3}} \times \bar{\mathbf{3}} \rightarrow \mathbf{3}) \alpha_3(\Lambda_3) + \Delta C_2(\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}) \alpha_2(\Lambda_3) = \frac{4}{3} \alpha_3(\Lambda_3) + \frac{3}{2} \alpha_2(\Lambda_3). \quad (18)$$

Condensation should occur when expression (18) is about $2\pi/3$.

Note that since the coefficient of $\alpha_3(\Lambda_3)$ in (18) is less than the coefficient of $\alpha_4(\Lambda_4)$ in (14), $\alpha_3(\Lambda_3)$ must be larger than $\alpha_4(\Lambda_4)$ in order for dynamical symmetry breaking to occur at both Λ_3 and Λ_4 . This is consistent with the asymptotic freedom

of the $SU(3)_{ETC}$ and $SU(2)_{HC}$ gauge groups in our model. Some walking (recall that the “vector” G quarks help to reduce the one-loop HC β -function) of the HC gauge coupling will be required to make this condensation occur at a low enough scale (≈ 10 TeV).

For comparison, the MAC for $SU(3)_{ETC}$ is $\mathbf{3} \times \bar{\mathbf{3}} \rightarrow \mathbf{1}$. (Note that all the $\mathbf{3}$'s of $SU(3)_{ETC}$ in the model are $SU(2)_{HC}$ singlets, so there is no possibility of additional interactions to assist the condensation in this channel.) For this channel, the squared coupling weighted by the difference of Casimirs is:

$$\Delta C_2(\mathbf{3} \times \bar{\mathbf{3}} \rightarrow \mathbf{1}) \alpha_3(\Lambda_3) = \frac{8}{3} \alpha_3(\Lambda_3) . \quad (19)$$

Thus for $\alpha_2(\Lambda_3) > \frac{8}{9} \alpha_3(\Lambda_3)$, expression (18) is larger than (19), and the breaking proceeds as required: $(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \times (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0 \rightarrow (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$.

The condensate $(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_0$ breaks the ETC gauge symmetry down to TC: $SU(2)_{TC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The component of F that is neutral under TC does not get a mass from this condensate. This component does however condense with the X , at a slightly lower scale, Λ_{HC} . The G quarks will also condense at Λ_{HC} . Since the HC coupling is quite strong at $\Lambda_3 \approx 10$ TeV, with a standard running of this coupling Λ_{HC} will be very close to Λ_3 . Hypercolored particles are confined at Λ_{HC} , and the HC sector decouples from ordinary fermions and technifermions. We then have a one-family TC model, with an additional “vector” quark, m .

The fermion content below $\Lambda_3 \approx 10$ TeV (labeled according to $SU(2)_{TC} \otimes$

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is:

$$\begin{array}{llll}
\mathbf{3}(\mathbf{1}, \mathbf{3}, \mathbf{2})_{1/3} & (\mathbf{2}, \mathbf{3}, \mathbf{2})_{1/3} & \mathbf{3}(\mathbf{1}, \mathbf{1}, \mathbf{2})_{-1} & (\mathbf{2}, \mathbf{1}, \mathbf{2})_{-1} \\
(u, d)_L, (c, s)_L, (t, b)_L & (U, D)_L & (\nu_e, e)_L, (\nu_\mu, \mu)_L, (\nu_\tau, \tau)_L & (N, E)_L \\
\\
\mathbf{3}(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-4/3} & (\mathbf{2}, \bar{\mathbf{3}}, \mathbf{1})_{-4/3} & & \\
u_R^c, c_R^c, t_R^c & U_R^c & & \\
\\
\mathbf{3}(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & (\mathbf{2}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} & \mathbf{3}(\mathbf{1}, \mathbf{1}, \mathbf{1})_2 & (\mathbf{2}, \mathbf{1}, \mathbf{1})_2 \\
d_R^c, s_R^c, b_R^c & D_R^c & e_R^c, \mu_R^c, \tau_R^c & E_R^c
\end{array} \tag{20}$$

$$\begin{array}{ll}
(\mathbf{1}, \mathbf{3}, \mathbf{1})_{-2/3} & (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{2/3} \\
m_L & m_R^c
\end{array} \tag{21}$$

$$\begin{array}{ll}
\mathbf{2}(\mathbf{1}, \mathbf{1}, \mathbf{1})_0 & (\mathbf{2}, \mathbf{1}, \mathbf{1})_0 \\
\nu_{\mu R}^c, \nu_{\tau R}^c & N_R^c
\end{array} . \tag{22}$$

Note that $\mathbf{2}$'s and $\bar{\mathbf{2}}$'s of $SU(2)_{TC}$ are equivalent. All interactions except $U(1)_Y$ are asymptotically free. The model at this stage consists of the usual three families (left and right-handed except that there is no right-handed ν_e), one conventional family of technifermions, and the ‘‘vector’’ quark m .

At the technicolor scale Λ_{TC} , the $SU(2)_{TC}$ coupling becomes strong enough that $\frac{3}{2\pi} \Delta C_2(\mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}) \alpha_{TC}(\Lambda_{TC}) = \mathcal{O}(1)$. Technifermions then get dynamical masses, $SU(2)_L \times U(1)_Y$ breaks to $U(1)_{em}$, and the masses of the quarks and leptons are generated by the ETC interactions linking the various particles in (20) and (22). We turn next to a description of these masses and other features of the model.

4 Features and Problems of the Model

In this section we will discuss the mass spectrum of ordinary fermions, some of the phenomenology of the TC sector, and CP violation. To begin we note that the

three, well-separated, ETC scales in the model provide a natural starting point for an explanation of the pattern of family masses. Furthermore, as we will discuss in more detail below, it is possible that QCD interactions will adequately split quark masses from lepton masses. The combination of the above mentioned effects with the SSVZ mechanism (which suppresses neutrino masses) will then generate the overall gross features of the quark and lepton spectrum.

We first discuss the masses of the third family (t, b, ν_τ, τ) . We note that in a moderately walking TC theory, $\Lambda_3 \approx 10$ TeV is a natural scale to generate the mass of the τ . In order to explain the $t - \tau$ hierarchy of nearly two orders of magnitude it will be assumed that the ETC interactions linking the t to the U are near-critical [2] at Λ_3 , i.e. $\alpha_3(\Lambda_3)$ is very close to, but below, a critical value $\alpha_c(3)$, given by a crude Schwinger-Dyson equation (in the ladder approximation⁸) analysis to be

$$\alpha_c(3) = \frac{2\pi}{3 \Delta C_2(\mathbf{3} \times \mathbf{\bar{3}} \rightarrow \mathbf{1})} = \frac{\pi}{4}. \quad (23)$$

Then as pointed out in Section 2, the additional effect of the QCD interaction in the gap equation for the t and U can dramatically enhance m_t relative to m_τ [18]. In particular, if $\alpha_3(\Lambda_3)$ is within 1-10% of $\alpha_c(3)$, then it is possible to produce an m_t in the 150 GeV range with $m_\tau = 1.8$ GeV. We note that if $\alpha_3(\Lambda_3)$ is near-critical, then the mass of the techniquarks (which sets the scale for the W and Z masses) can be substantially larger than the intrinsic TC scale, Λ_{TC} . The scale Λ_{TC} could be as low as 100 GeV [5]. Since $\alpha_{TC}(\Lambda_3) = \alpha_3(\Lambda_3)$, the TC coupling must be moderately walking from Λ_3 down to Λ_{TC} in order for Λ_{TC} to be much smaller than Λ_3 . For the TC coupling in this range, the perturbative expansion for the β function may be unreliable. The same is true for some of the ETC and HC gauge couplings relevant

⁸See footnote 5.

at higher scales. In this paper, we will not attempt to compute these β functions. Instead, we will simply point out the qualitative behavior that is necessary in each energy range.

Consider the ETC and HC couplings in the range from $\Lambda_4 \approx 100$ TeV to $\Lambda_3 \approx 10$ TeV. Suppose, as discussed in section 3, that the $SU(2)_{HC}$ coupling $\alpha_2(\Lambda_4) = 0.61$, and $\alpha_4(\Lambda_4) = 0.47$. This makes expression (14) equal to $2\pi/3$, and $\alpha_2(\Lambda_4) > \frac{5}{4} \alpha_4(\Lambda_4)$. For the model to work, the coupling α_3 must walk from Λ_4 to Λ_3 be near $\alpha_c(3) = \pi/4 \approx 0.8$ at Λ_3 . Also α_2 must be walking in order for Λ_3 to be an order of magnitude smaller than Λ_4 .

In order to estimate masses of the quarks and leptons, we need estimates for the condensates of the technifermions. However our model is far from QCD-like, so we cannot simply scale-up the QCD condensate. Instead we use the t and τ masses as inputs to determine the relevant condensates, and use these estimates to calculate the masses of particles in the second and first families. We expect that the τ mass is given roughly by the standard one-ETC-gauge-boson-exchange graph:

$$m_\tau \approx 3\pi\alpha_3(\Lambda_3) \frac{\langle \bar{E}_R E_L \rangle}{\Lambda_3^2} . \quad (24)$$

The coefficient $3\pi\alpha_3(\Lambda_3)$ can be understood as follows. The one-ETC-gauge-boson-exchange graph is given by $3\alpha_3(\Lambda_3)C/(4\pi)$ times an integral of the technielectron self-energy. This integral is $4\pi^2$ times the technielectron condensate⁹ $\langle \bar{E}_R E_L \rangle$. The constant C comes from the squares of ETC generators, and for the representations in our model turns out to be $N/2$, where N is the number of heavy ETC gauge bosons which contribute to the graph. For $SU(3)_{ETC} \rightarrow SU(2)_{TC}$, $N = 2$;

⁹Our convention for the condensate is the negative of the more usual convention.

for $SU(4)_{ETC} \rightarrow SU(3)_{ETC}$, $N = 3$, and so on. Thus, rewriting equation (24), we take the charged-technilepton condensate to be:

$$\begin{aligned} \langle \bar{E}_R E_L \rangle &\approx \frac{4}{3\pi^2} m_\tau \Lambda_3^2 \\ &\approx 0.024 \text{ TeV}^3 . \end{aligned} \tag{25}$$

Since the mass of the t is comparable to the techniquark mass, the corresponding Schwinger-Dyson equations are near-critical and non-linear, and we do not expect a simple formula like (24) to apply for m_t . We expect that just below Λ_3 , the dynamical mass of the U techniquark, Σ_U , is roughly constant (for a larger range than the technilepton mass), and is close to m_t . Thus, we will simply use the estimate:

$$\begin{aligned} \langle \bar{U}_R U_L \rangle &\approx \frac{1}{4\pi^2} \int_0^{\Lambda_3^2} dk^2 \frac{k^2 \Sigma_U(k)}{k^2 + \Sigma_U^2(k)} \\ &\approx \frac{1}{4\pi^2} \int_0^{\Lambda_3^2} dk^2 m_t \\ &\approx \frac{m_t \Lambda_3^2}{4\pi^2} \\ &\approx 0.38 \text{ TeV}^3 , \end{aligned} \tag{26}$$

where we have made the approximation that the integral is dominated at momenta near Λ_3 , and taken $m_t = 150 \text{ GeV}$.

The mass of the ν_τ is suppressed as in the SSVZ mechanism described in Section 2. While the E_L and E_R^c transform as a $\mathbf{3}$ and a $\bar{\mathbf{3}}$ under $SU(3)_{ETC}$, the N_L and N_R^c both transform as $\mathbf{3}$'s. Thus, a Dirac mass will not feed down to the ν_τ unless there is some mixing of ETC gauge bosons, since the one ETC gauge boson exchange graph is identically zero. The ν_τ does not receive a mass even at two loops. In fact one can show that a mass cannot feed down to the ν_τ from the

technineutrino mass alone, to all orders in perturbation theory. The reason for this is that the technineutrino mass transforms as part of a $\mathbf{3}$ of $SU(3)_{ETC}$, while the ν_τ mass transforms as part of a $\mathbf{6}$; the appropriate component of the $\mathbf{6}$ can only be made from an even number of $\mathbf{3}$'s, but there must be an odd number of mass insertions in order to have a helicity flip. We expect, however, that at three loops particles other than neutrinos can feed down a mass to the ν_τ . As we will see however, there are more important effects that will couple the $\nu_{\tau R}^c$ to the ν_e . We will return to this when we discuss the first generation.

The remaining member of the third family is the b quark. The mechanism for generating its mass is quite different from that for the t quark. The t gets its mass only through the standard one ETC gauge boson exchange, while the b mass can be suppressed by mixing with the m quark. Since the Schwinger-Dyson equations for the mass of the b and the mass of the D techniquark are coupled, the reduced b mass feeds back into the mass (renormalized near the ETC scale, $\Lambda_3 \approx 10$ TeV) of the D techniquark, which lowers its mass, and further lowers the b mass. Thus this model may not have a problem accommodating a large t - b mass splitting. The calculation of the b quark mass will require further information about the mixing with the m quark, which depends on physics at and above 1000 TeV.

With the interactions discussed so far, the m quark remains massless. In order for it to gain a mass, and to mix with the down-type quarks, there must be additional physics, which will take the form of higher dimension operators in the low-energy effective theory below $\Lambda_5 \approx 1000$ TeV. An example of an operator that

would give the m a mass is:

$$\mathcal{L}_{4f} = \frac{g^2}{\Lambda_5^2} \overline{G}_R G_L \overline{m}_R m_L + \text{hermitian conjugate} , \quad (27)$$

where we expect $g^2/4\pi$ to be $\mathcal{O}(1)$. Then when the G gets a mass at Λ_{HC} , this mass will feed down to the m through the four-fermion operator (27). In order to estimate this mass, we will need the value of the condensate $\langle \overline{G}_R G_L \rangle$ cut off at the scale Λ_5 . We recall that the anomalous dimension of the mass operator (in ladder approximation¹⁰) in an $SU(N)$ gauge theory is:

$$\gamma_N(\alpha) = 1 - \sqrt{1 - \frac{\alpha}{\alpha_c(N)}} , \quad (28)$$

where $\alpha_c(N)$ is the generalization of equation (23) to the appropriate gauge group, and we are assuming $\alpha < \alpha_c(N)$. We also recall that for an extremely slowly running coupling between the symmetry breaking scale μ and a larger scale Λ , the condensate $\langle \overline{\psi}\psi \rangle$ cut off at Λ is roughly given by

$$\langle \overline{\psi}\psi \rangle_\Lambda \approx \langle \overline{\psi}\psi \rangle_\mu \left(\frac{\Lambda}{\mu} \right)^{\gamma_N(\alpha)} . \quad (29)$$

Of course the coupling does run; for the purposes of a crude calculation we will use an average coupling $\overline{\alpha}$. In order to make an estimate, we split the range of momenta into two, from $\Lambda_{HC} \approx \Lambda_3 \approx 10$ TeV to $\Lambda_4 \approx 100$ TeV, and from Λ_4 to $\Lambda_5 \approx 1000$ TeV. We expect α_2 to run from $\alpha_2(\Lambda_{HC}) = \alpha_c(2) = 4\pi/9 \approx 1.4$, to $\alpha_2(\Lambda_4) \approx 0.6$ (as discussed above) over the lower range. We will take $\alpha_2(\Lambda_5) = 0.4$. Thus we have $\overline{\alpha}_2 = 1$ over the lower range, and $\overline{\alpha}_2 = 0.5$ over the upper range. We assume that the $\langle \overline{G}_R G_L \rangle$ condensate is at least as big as a scaled-up QCD

¹⁰See footnote 5.

condensate (i.e. $4\pi f^3$). The mass of the m is then:

$$\begin{aligned}
m_m &\approx \frac{g^2}{\Lambda_5^2} \langle \bar{G}_R G_L \rangle \left(\frac{\Lambda_4}{\Lambda_3} \right)^{\gamma_2(\bar{\alpha}_2=1.0)} \left(\frac{\Lambda_5}{\Lambda_4} \right)^{\gamma_2(\bar{\alpha}_2=0.5)} \\
&\approx \frac{g^2}{(1000 \text{ TeV})^2} 4\pi (10 \text{ TeV})^3 \left(\frac{100 \text{ TeV}}{10 \text{ TeV}} \right)^{0.5} \left(\frac{1000 \text{ TeV}}{100 \text{ TeV}} \right)^{0.2} \\
&\approx g^2 60 \text{ GeV} .
\end{aligned} \tag{30}$$

With $g^2/4\pi > 0.2$, our estimate for m_m is above the current experimental lower bound ($\approx 110 \text{ GeV}$) for such a particle. Of course m_m does not correspond to the physical mass of the m , since it must mix with the down-type quarks, and this could change the value of the physical mass.

It is also important to comment on the masses of the single family of technifermions in this model. With the ETC coupling at Λ_3 close enough to criticality so that the t is much heavier than the τ , the techniquarks will be much heavier than technileptons, [18]. Also, since the technielectron has attractive ETC interactions in the scalar channel while the technineutrino has repulsive ETC interactions, the technielectron will be heavier than the technineutrino. Thus this model can provide a realization of the technifermion mass pattern suggested in ref. [5]. It was shown there that with this breaking of $SU(2)_R$ the electroweak radiative correction parameter S will be smaller than is estimated in QCD-like TC models, and may even be negative. We also expect that the lightness of the technineutrinos will lead to a very light techni- ρ (composed of technineutrinos and antitechnineutrinos), that may be light enough to be seen at LEP II [5]. This model will also generate a significant (m_t dependent) correction [25] to the $Z \rightarrow b\bar{b}$ vertex, which should be accurately measured soon. The spectrum of pseudo-Nambu-Goldstone bosons should also be

similar¹¹ to that sketched out in ref. [5].

Later in this section we will need an estimate of the technineutrino condensate, in order to estimate ordinary neutrino masses. Since the technineutrinos have repulsive ETC interactions, the integral representing the technineutrino condensate should converge rapidly above 100 GeV (which we take as an order of magnitude estimate of the technineutrino mass [5]). So we take

$$\begin{aligned}
\langle \bar{N}_R N_L \rangle &\approx \frac{1}{4\pi^2} \int_0^{\Lambda_3^2} dk^2 \frac{k^2 \Sigma_N(k)}{k^2 + \Sigma_N^2(k)} \\
&\approx \frac{(100\text{GeV})^3}{8\pi^2} \\
&\approx 1.3 \times 10^{-5} \text{TeV}^3 .
\end{aligned} \tag{31}$$

We also note that the vacuum alignment problem [26] of $SU(2)_{TC}$ theories, with one family of degenerate technifermions, should not be present in this model. Recall that, in the absence of ETC interactions, the contribution of the (unbroken) electroweak gauge bosons to the vacuum energy causes the N_L to condense with the E_L rather than N_R^c . This technilepton condensate breaks $U(1)_{em}$ rather than $SU(2)_L$, which obviously does not correspond to the observed vacuum¹². In our model, however, the strong ETC interactions will lower the energy of the vacuum where E_L condenses with E_R^c (cf. ref. [4]). Moreover since the techniquarks condense at a higher energy scale, at the scale where the technileptons condense, the W^\pm and Z have already gotten the bulk of their masses from the techniquark condensate, and hence their (destabilizing) contribution to the vacuum energy will be

¹¹However there will be two more pseudo-Nambu-Goldstone bosons in the model discussed here, since there is no distinction between N_L and N_R^c . We leave a detailed examination of the pseudos for future work.

¹²By contrast, the techniquark condensate breaks electroweak gauge symmetry in the correct fashion due to the presence of QCD interactions [26].

suppressed.

We turn next to a discussion of the second family (c, s, ν_μ, μ) . With a moderate enhancement from walking, the mass of the μ can be obtained naturally with an ETC scale of $\Lambda_4 \approx 100$ TeV. We know that α_3 must run from $\alpha_3(\Lambda_3) \approx \alpha_c(3) \approx 0.79$ to $\alpha_3(\Lambda_4) = \alpha_4(\Lambda_4) \approx 0.47$ (as discussed earlier), so we take $\bar{\alpha}_3 = 0.7$. We then have:

$$\begin{aligned}
m_\mu &\approx \frac{9\pi\alpha_4(\Lambda_4)}{2} \frac{\langle \bar{E}_R E_L \rangle}{\Lambda_4^2} \left(\frac{\Lambda_4}{\Lambda_3} \right)^{\gamma_3(\bar{\alpha}_3)} \\
&\approx 6.7 \frac{0.024 \text{ TeV}^3}{(100 \text{ TeV})^2} \left(\frac{100 \text{ TeV}}{10 \text{ TeV}} \right)^{0.67} \\
&\approx 100 \text{ MeV} .
\end{aligned} \tag{32}$$

The same physics that gives a large t mass will also enhance the c mass relative to the μ . Assuming that the correct t mass is generated, as discussed above, we can roughly estimate the c mass as:

$$\begin{aligned}
m_c &\approx \frac{9\pi\alpha_4(\Lambda_4)}{2} \frac{\langle \bar{U}_R U_L \rangle}{\Lambda_4^2} \left(\frac{\Lambda_4}{\Lambda_3} \right)^{\gamma_3(\bar{\alpha}_3)} \\
&\approx 6.7 \frac{0.38 \text{ TeV}^3}{(100 \text{ TeV})^2} \left(\frac{100 \text{ TeV}}{10 \text{ TeV}} \right)^{0.67} \\
&\approx 1 \text{ GeV} .
\end{aligned} \tag{33}$$

The results for m_μ and m_c are quite good for such crude estimates. One could hope to do better with a more refined analysis of the Schwinger-Dyson equations. We further expect that mixing with the m quark will reduce the mass of the s quark, just as in the case of the b quark.

The ν_μ is the heaviest neutrino in our model. It does not receive a mass at one loop, but it does at two loops. The extra loop is necessary to mix two different 100 TeV ETC gauge bosons. The mixing breaks $SU(3)_{ETC}$ and $SU(4)_{ETC}$, and so

should be of order $10 \text{ TeV} \times 100 \text{ TeV}$. Thus we expect the ν_μ neutrino mass to be given roughly by:

$$\begin{aligned}
m_{\nu_\mu} &\approx 18\pi^2 \alpha_4^2(\Lambda_4) \frac{\langle \bar{N}_R N_L \rangle}{\Lambda_4^4} \frac{\Lambda_3 \Lambda_4}{16\pi^2} \\
&\approx 40 \frac{1.3 \times 10^{-5} \text{ TeV}^3}{(100 \text{ TeV})^3} \frac{10 \text{ TeV}}{16\pi^2} \\
&\approx 30 \text{ eV} .
\end{aligned} \tag{34}$$

Note that the coefficient in equation (34) is $g_4^2(\Lambda_4) = 4\pi\alpha_4(\Lambda_4)$ times that in equations (32) and (33), since there is an extra ETC gauge boson exchange. The $1/16\pi^2$ is the standard estimate of the suppression due to an extra loop. It is interesting that 30 eV is the right mass for a stable Dirac neutrino to close the universe, but considerations of structure formation indicate that a lighter neutrino mass is preferred. However, the neutrino mass estimates in our model are more unreliable than those of other fermions, since the neutrino masses only arise at two loops, and there is, as yet, no experimental input to determine the technineutrino condensate in equation (31).

Finally we briefly discuss the first family (u, d, ν_e, e) . An ETC scale of roughly $\Lambda_5 \approx 1000 \text{ TeV}$ will be sufficient to give naturally the correct mass for the e . To see this, we again split the range of momenta into two parts, from Λ_3 to Λ_4 , and from Λ_4 to Λ_5 . As discussed above, $\alpha_4(\Lambda_4) = 0.47$, and we take $\alpha_4(\Lambda_5) = \alpha_5(\Lambda_5) = 0.1$. Thus we have $\bar{\alpha}_4 = 0.35$. A crude calculation then gives

$$\begin{aligned}
m_e &\approx 6\pi\alpha_5(\Lambda_5) \frac{\langle \bar{E}_R E_L \rangle}{\Lambda_5^2} \left(\frac{\Lambda_4}{\Lambda_3}\right)^{\gamma_3(\bar{\alpha}_3)} \left(\frac{\Lambda_5}{\Lambda_4}\right)^{\gamma_4(\bar{\alpha}_4)} \\
&\approx 1.9 \frac{0.024 \text{ TeV}^3}{(1000 \text{ TeV})^2} \left(\frac{100 \text{ TeV}}{10 \text{ TeV}}\right)^{0.67} \left(\frac{1000 \text{ TeV}}{100 \text{ TeV}}\right)^{0.32} \\
&\approx 1 \text{ MeV} .
\end{aligned} \tag{35}$$

Note that most of the walking enhancement comes from the momentum range 10 TeV to 100 TeV.

The QCD enhancement of the t and c quark masses, discussed above, will put the u and d masses in the right range of 5 – 10 MeV:

$$\begin{aligned}
m_u &\approx 6\pi\alpha_5(\Lambda_5) \frac{\langle \bar{U}_R U_L \rangle}{\Lambda_4^2} \left(\frac{\Lambda_4}{\Lambda_3}\right)^{\gamma_3(\bar{\alpha}_3)} \left(\frac{\Lambda_5}{\Lambda_4}\right)^{\gamma_4(\bar{\alpha}_4)} \\
&\approx 1.9 \frac{0.38 \text{ TeV}^3}{(1000 \text{ TeV})^2} \left(\frac{100 \text{ TeV}}{10 \text{ TeV}}\right)^{0.67} \left(\frac{1000 \text{ TeV}}{100 \text{ TeV}}\right)^{0.32} \\
&\approx 10 \text{ MeV} .
\end{aligned} \tag{36}$$

The estimates for m_e and m_u are encouraging, and again suggest that a more refined analysis of the Schwinger-Dyson equations is merited. The size and sign of the $u - d$ mass splitting remains unexplained so far. It must arise from mixing with the m quark driven by additional, high energy interactions.

At two loops, ν_{eL} gets a mass with the $\nu_{\tau R}^c$. As with the ν_μ we must mix two different ETC gauge bosons, but in this case only one is associated with $SU(5)_{ETC}$ breaking (and thus has a mass around Λ_5), while the other is associated with $SU(3)_{ETC}$ breaking (and thus has a mass around Λ_3). The mixing term requires two $SU(3)_{ETC}$ breaking dynamical masses, one from the $X - F$ mass, and one from the F mass. Thus (taking $\alpha_3(\Lambda_3) = 0.79$, and $\alpha_5(\Lambda_5) = 0.1$) we have

$$\begin{aligned}
m_{\nu_e} &\approx 12\pi^2\alpha_3(\Lambda_3)\alpha_5(\Lambda_5) \frac{\langle \bar{N}_R N_L \rangle}{\Lambda_5^2 \Lambda_3^2} \frac{\Lambda_3^2}{16\pi^2} \\
&\approx 9.4 \frac{1.3 \times 10^{-5} \text{ TeV}^3}{(1000 \text{ TeV})^2} \frac{1}{16\pi^2} \\
&\approx 1 \text{ eV} .
\end{aligned} \tag{37}$$

Thus the $\nu_{\tau R}^c$ becomes part of a Dirac neutrino, the ν_e .

To recap the neutrino sector, at two-loop order we have two Dirac neutrinos (ν_μ and ν_e), while the ν_τ is purely left-handed. At higher orders, $\nu_{\tau L}$ may get a mass with the $\nu_{\tau R}^c$, but this will only serve to mix the $\nu_{\tau L}$ with the ν_e . We note that this model does not generate the MSW solution [27] to the solar neutrino problem. We also note that the extra right-handed neutrinos in this model will pose no problems for Big Bang nucleosynthesis [28].

The CKM mixing angles among the quarks, and the mixing angles between down-type quarks and the m quark must arise from new physics at the 1000 TeV scale and above. This physics may be related to the interactions that were invoked to break $SU(4)_{PS}$ and $SU(5)_{ETC}$. Because of this we cannot yet obtain reliable estimates of mixing angles, CP violating parameters, and masses of down-type quarks.

Next we turn to the mechanism for CP violation. It was pointed out earlier that our model contains the additional¹³ “vector” quark, m , necessary to implement the Nelson-Barr mechanism. This mechanism can function if the theory is CP conserving (i.e. $\theta_{ETC} = \theta_{PS} = 0$), and if CP is spontaneously broken by the appearance of complex phases in the masses which connect the ordinary down-type quarks with the m quark. More specifically the ETC breaking dynamics must give rise to a (d, s, b, m) mass matrix of the form:

$$\begin{pmatrix} d_R^c \\ s_R^c \\ b_R^c \\ m_R^c \end{pmatrix} \begin{pmatrix} d_L & s_L & b_L & m_L \\ & \text{real} & & M_1 \\ & & \text{real} & M_2 \\ & & & M_3 \\ 0 & 0 & 0 & \text{real} \end{pmatrix}, \quad (38)$$

where at least one of M_1 , M_2 , and M_3 is complex. Under these conditions, CP violating phases will appear in the CKM matrix of the ordinary fermions, but the

¹³It may be more realistic to consider models where there is more than one “vector” quark.

determinant of the mass matrix is real, so the effective strong CP violating parameter $\bar{\theta}$ is identically zero at tree level in the low-energy effective theory. Furthermore, since the breaking is soft, higher-order corrections will be finite and small. In the work of Nelson and Barr [20], the form (38) was arranged by a particular choice of elementary Higgs fields and couplings. Whether this form will appear in our dynamical model at the appropriate breaking scale is not clear. This will depend on the details of dynamical breaking at 1000 TeV and above¹⁴.

It is worth noting that there is not necessarily a problem with CP domain walls [29], if inflation occurs and the reheating temperature is below the scale where CP is spontaneously broken [30]. Since baryogenesis must take place below the inflationary reheating scale, this scenario is consistent if baryogenesis occurs at the electroweak scale¹⁵. We also note that a TC theory with a family of technifermions (as ours is) will provide a first-order electroweak phase transition [32] (as opposed to one-doublet TC models, which have second-order, or extremely weak first-order, phase transitions [32, 33]), and thus allows for the possibility of electroweak baryogenesis.

5 Conclusions

We have constructed a potentially realistic (not obviously wrong) ETC model, that can incorporate many of the right ingredients: $m_t \gg m_b$, $m_\tau \gg m_{\nu_\tau}$, a family hierarchy, no bad FCNC's, no visible techniaxion, and CP violation with no strong CP problem. We have made estimates for some of the quark and lepton masses in

¹⁴For a discussion of how CP may be broken dynamically see refs. [29, 34].

¹⁵For a review of electroweak baryogenesis see ref. [31].

this model. New physics is expected in the form of a light (less than a few hundred GeV) techni- ρ composed of light technineutrinos. We stress again that our model contains an attractive tumbling scheme below 1000 TeV. The phenomenologically desired channel can be the most attractive when both of the two strong gauge interactions are taken into account. While there is thus an understanding of how dynamical symmetry breaking is achieved through tumbling at lower scales, our understanding of the breaking at high scales is incomplete. This could be a result of our ignorance of strongly coupled, chiral gauge theories, or it may mean that the model is not complete at the highest scales. It remains to be seen whether the model can survive a more detailed scrutiny, and in particular whether extensions of the model can provide quantitative estimates of down-type quark masses and mixing angles.

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