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Cosmology of One Extra Dimension with Localized Gravity

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Abstract

We examine the cosmology of the two recently proposed scenarios for a five dimensional universe with localized gravity. We find that the scenario with a non-compact fifth dimension is potentially viable, while the scenario which might solve the hierarchy problem predicts a contracting universe, leading to a variety of cosmological problems.

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The main theme of the first 20 years of the hierarchy problem has been to modify particle physics around the TeV scale. In the past two years it has become evident that another successful route to solving the hierarchy problem is to modify the nature of gravitational interactions at distances shorter than a millimeter [1, 2, 3]. This modification can be most simply achieved by introducing compact extra dimensions. One recent proposal which attracted enormous attention is to lower the fundamental Planck scale M_* all the way to the TeV scale, by introducing large extra dimensions [2]. The observed Planck scale is then just an effective scale valid for energies below the mass scale of the Kaluza-Klein (KK) excitations. The consequence of this proposal is that the necessary size of the extra dimensions is determined by the KK reduction formula

$$R = \left(\frac{M_{Pl}}{M_*}\right)^{\frac{2}{n}} \frac{1}{M_*}, \tag{1}$$

where M_* is the fundamental Planck scale of the order of 1 TeV, $M_{Pl} = 10^{18}$ GeV, and n is the number of extra dimensions. Applying this formula for one extra dimension one obtains $R \sim 10^{13}$ cm, which would immediately suggest that this possibility is excluded because gravity would be modified at the scale of our solar system. For $n \geq 2$, R is sufficiently small so that this scenario is not excluded by short-distance gravitational measurements. However, Randall and Sundrum recently realized [4, 5] that the case of one extra dimension is very special, and the naïve KK reduction formula (1) may not be applicable in this case. The basic reason behind this is that in this scenario the standard model fields have to be confined to a three-dimensional wall ("three-brane"), and such branes act like sources for gravity in the extra dimensions. The behavior of Green's functions in one dimension is dramatically different from the case of two or more dimensions. Indeed, in one dimension Green's functions grow linearly, while in the case of more than two dimensions there is an inverse power law $1/r^{d-2}$ (and logarithmic growth for $d = 2$). Thus branes act like small perturbations on the system for the case of two or more extra dimensions, and one expects the KK reduction formula (1) to be applicable. For just one extra dimension, however, the presence of branes can significantly alter the bulk gravitational fields, which may invalidate arguments based on the naïve KK reduction formula.

Indeed, Randall and Sundrum (RS) have presented a new static classical solution to Einstein's equations with one extra dimension (taken to be S^1/Z_2), and branes with non-vanishing tensions placed at the orbifold fixed points [4]. In this solution, for large brane separations, the effective four-dimensional Planck scale is independent of the size of the extra dimension, in agreement with the expectation that (1) is invalid for the case of one extra dimension. For their solution to work RS found that the brane tensions must be of opposite sign, and that there must be a negative bulk cosmological constant which stabilizes the system.

Two possible applications of this solution have been proposed [4, 5]. In one case (which we refer to as RS1), our universe ("the visible brane") is the brane with negative tension, and the exponential "warp-factor" appearing in the RS solution will yield a natural new solution to the hierarchy problem [4]. In the second (RS2) proposal [5], the visible brane is the one with positive tension. In this case the hierarchy problem is not solved, however, the second brane can be moved to infinity, thus providing an exciting example of a non-compact extra direction [6], which nevertheless correctly reproduces Newton's law on the visible brane.

Binétruy, Deffayet and Langlois (BDL), however, have pointed out recently [7] that five dimensional theories with branes tend to have non-conventional cosmological solutions, once matter on the walls is included.* In this letter, we apply the results of BDL to the models presented by RS. We find that the equation for the scale factor on the visible brane (for small matter densities) coincides with the conventional Friedmann equation, up to the overall sign of the source terms. This sign depends on the sign of the cosmological constant (tension) on the visible brane. In the case of negative brane tension the source terms of the Friedmann-like equation have the opposite sign from standard cosmology. The change in sign implies that the universe would collapse on a timescale on the order of the Hubble time at the start of the expansion, for any matter with energy density ρ, pressure p, on our wall with an equation of state $p = w\rho$ and $w < 1/3$. So whereas in the radiation-dominated phase the universe expands as in the conventional cosmology, after the transition from the radiation-dominated to matter-dominated (or quintessence-dominated) epoch when the universe was a few thousand years old, the universe would not expand as in conventional cosmology, but would rather collapse within a few thousand years. As we argue, this conclusion relies on knowing the expansion rate during the era of matter-radiation equality, which is provided by the success of the standard big-bang nucleosynthesis (BBN), and the current baryon and radiation densities. So in order to avoid this conclusion this scenario requires either a non-standard BBN or a modification of the RS1 solution, for example through the introduction of additional fields. We emphasize that crucial to these conclusions is the fact that we live on a negative-tension brane, and that there is only one extra dimension. Both of these facts, however, are also crucial ingredients to the solution to the hierarchy problem presented by RS. For the case of the positive brane tension, however, the conventional expanding solution is reproduced in this model.

We begin by summarizing the work of BDL [7]. The scenario considered here is a five-dimensional spacetime compactified on the line segment S^1/Z_2. The bulk coordinate is labeled by y, which is taken to be in the interval $-1/2 \leq y \leq 1/2$, where the points $-1/2$ and $1/2$ are identified. In this notation the coordinate y is dimensionless. The Z_2 symmetry identifies the points y and $-y$, so we can restrict ourselves to $0 \leq y \leq 1/2$. Two three-branes are placed at the fixed points of the discrete symmetry, $y = 0$ and $y = 1/2$. In our notation, the visible brane is located at $y = 0$, and a hidden brane is located at $y = 1/2$. The compactness of the extra dimension requires the existence of two branes, since each brane must absorb the gravitational flux lines from the other.

The most general metric for a five-dimensional spacetime which preserves three-dimensional rotational and translational invariance is given by

$$ds^2 = n^2(\tau, y)d\tau^2 - a^2(\tau, y)d\vec{x}^2 - b^2(\tau, y)dy^2. \tag{2}$$

Note that here we use the metric signature $(+, -, -, -, -)$. The induced metric on the visible

*For other recent results on the cosmological aspects of theories with large extra dimensions see [8-16].

brane is obtained by evaluating the metric tensor at $y = 0$. The metric is determined by solving Einstein's equations in the presence of some energy density in the bulk and on the three-branes. A solution which describes a static four-dimensional Lorentz invariant universe is given by $a(\tau, y) = n(\tau, y) = f(y)$. This still allows for a non-trivial dependence of the metric on the bulk coordinate. This is the key point in the solution of [4] to the hierarchy problem, for localizing gravity in the bulk [4, 5], and for obtaining a non-compact fifth dimension with a conventional Newton's force law [5]. For an expanding four-dimensional universe, however, we must have $a \neq n$. The metric is determined by solving Einstein's equations

$$G_{AB} \equiv R_{AB} - \frac{1}{2} g_{AB} R = \kappa^2 T_{AB}. \tag{3}$$

Here A, $B = 0, 1, 2, 3, 5$, and κ^2 is the five-dimensional Newton's constant, which is related to the five-dimensional Planck scale by $M_*^3 = \kappa^{-2}$. The components of the Einstein tensor G_{AB} with the above ansatz are given in (8)–(11) of [7].

The five-dimensional stress-energy tensor T_{AB} is the sum of contributions from the bulk and from the two branes. The width of the branes is neglected since it is $O(1/M_*)$, which is much smaller than the distance scales of cosmological interest. Thus the stress-energy tensor is approximated as

$$T^{AB}(x, y) = \tilde{T}^{AB}(x, y) + \frac{S^{AB}_{vis}(x)}{b_0(\tau)} \delta(y) + \frac{S^{AB}_{hid}(x)}{b_{1/2}(\tau)} \delta(y - 1/2), \tag{4}$$

where $b_0(\tau) \equiv b(\tau, 0)$ and $b_{1/2}(\tau) \equiv b(\tau, 1/2)$. Here $\tilde{T}^{AB}$ is the stress-energy tensor in the bulk, and S^{AB}_{vis} (S^{AB}_{hid}) is the 4-dimensional stress-energy tensor of the visible (hidden) brane: $S^{A}_{B\,vis} = (\rho, -p, -p, -p, 0)$ and $S^{A}_{B\,hid} = (\rho_*, -p_*, -p_*, -p_*, 0)$. At this point the composition of the energy density on the walls is completely general.

Using the above energy-momentum tensor BDL derived an equation for the scale factor on the visible brane which is independent of the details of the global solution to Einstein's equations†:

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = -\frac{\kappa^2}{3b_0^2} \tilde{T}_{55} - \frac{\kappa^4}{36} \rho(\rho + 3p). \tag{5}$$

The time derivatives which appear are with respect to t, where $dt \equiv n(\tau, 0) d\tau$, and $a_0(t) = a(\tau, 0)$ is the scale factor on the visible brane. There are several important features of this equation. First, the energy density and pressure of the second brane do not appear. This follows from the local nature of Einstein's equations [17]. Second, as noted in [7], the Hubble parameter $H \equiv \dot{a}_0/a_0 \propto \rho$ rather than $\sqrt{\rho}$ as in the conventional cosmology. Finally, this equation depends only on a_0. In the case that $\tilde{T}^5_5$ is time-independent (as will be the case later), this equation completely determines $a_0(t)$. Therefore it is not necessary to determine the solutions to Einstein's equations for the whole bulk.

†This equation can be derived [7] by first calculating the discontinuities in the derivatives of the functions a and n at the position of the branes by matching the δ–functions in Einstein's equations. This information about the discontinuity of the derivatives together with the discontinuity and the average value of the 5,5 component of Einstein's equation results in (5).

We assume that no energy is flowing from the bulk onto the brane so that $T^{05} = 0$. Then the discontinuity in the $G_{05} = 0$ equation [7] gives the usual conservation of energy condition:

$$\dot{\rho} + 3\frac{\dot{a}_0}{a_0}(\rho + p) = 0. \tag{6}$$

Thus for $p = w\rho$, $\rho \propto a^{-3(w+1)}$ as in standard cosmology. The conservation of energy equation (6) and the Friedmann-like equation (5) are the central equations which are used in what follows.

Now we briefly review the solution presented by Randall and Sundrum [4, 5]. In their scenario the two branes have some tension, V_{vis}, V_{hid}, and the bulk contains a cosmological constant Λ. In terms of our earlier notation, $\rho = -p = V_{vis}$, $\rho_* = -p_* = V_{hid}$, and $\tilde{T}^A_B = \Lambda(1,1,1,1,1)$. A static Lorentz invariant solution is obtained with $a(\tau, y) = n(\tau, y) = e^{\sigma(y)}$, and $b(\tau, y) = b(y) = b_0 = \text{const.}$, the latter having been obtained by a coordinate transformation on y. Then the 5,5 component of Einstein's equation gives

$$\sigma'^2 = -\frac{\kappa^2 b_0^2}{6}\Lambda. \tag{7}$$

Thus $\Lambda < 0$ is required. It is then convenient to introduce

$$m^2 \equiv -\frac{\kappa^2}{6}\Lambda. \tag{8}$$

Therefore $\sigma = \pm m b_0 |y|$. (The absolute value is required so that a'' is singular at $y = 0$, matching the δ–function sources in Einstein's equations.) The discontinuity equation for either a or n then requires that

$$V_{vis} = \mp\frac{6}{\kappa^2}m = -V_{hid}. \tag{9}$$

We note that the case in which $V_{vis} \neq -V_{hid}$ leads to the brane inflating solutions found in [8]. Since we are interested in non-inflationary solutions, we assume that the tensions of the two branes are adjusted to satisfy (9). Since $sgn(V_{vis})$ is crucial in determining the expansion rate of our wall discussed below, we emphasize that the correlation between the sign of the tension of the visible brane and the growth of the scale factor away from the visible brane is:

$$\text{RS1} : n(y) = a(y) = e^{+mb_0|y|} \Longleftrightarrow V_{vis} < 0, \tag{10}$$

$$\text{RS2} : n(y) = a(y) = e^{-mb_0|y|} \Longleftrightarrow V_{vis} > 0. \tag{11}$$

For RS2, gravity is localized about our brane [5]. This allows for a non-compact fifth dimension that is consistent with the short-distance force experiments. By contrast, case RS1 with $V_{vis} < 0$ provides a potential solution to the hierarchy problem [4]. The reason for this is the following. The metric on the distant brane contains a conformal factor e^{mb_0} in units where the conformal factor on our wall is one. This implies that mass scales on our wall and the distant wall are then related by $m_{vis} = e^{-mb_0/2} m_{hid}$, so that a large hierarchy of mass scales is possible if $m_{hid} \sim M_{Pl} \sim M_*$ and $mb_0 \sim 100$.

This non-trivial scale factor significantly modifies the naïve relation $M_{Pl}^2 \sim M_*^3 b_0$ between the fundamental and derived Planck scales. The correct relation for $mb_0 \gg 1$ is in fact [4]

$$8\pi G_N = \frac{1}{M_{Pl}^2} = \kappa^2 m = \frac{\kappa^4}{6}|V_{vis}|. \tag{12}$$

It is remarkable that this is independent of the size of the extra dimension. For RS1 the coupling of the KK excitations of the graviton to matter are given by $\sim e^{mb_0/2}/m$ [5]. Since these excitations would appear as resonances in collider experiments, this coupling must be $O(\text{TeV}^{-1})$ or smaller, thus implying $m \sim M_{Pl}$ (since $e^{mb_0/2} \sim 10^{15}$). Therefore a satisfactory resolution to the hierarchy problem requires $m \sim \kappa^{-2/3} \sim M_{Pl}$.

However, both of these solutions to Einstein's equations are static and do not describe a universe with time-dependent scale factor a. While some work has been done on inflating solutions [8], we will attempt to uncover cosmological solutions which reproduce the successes of the usual Friedmann equation for a flat universe. This seems difficult at first thought, given the earlier statement that in five-dimensional brane models $H \propto \rho$. However, we will see that the presence of large background cosmological constants changes this conclusion, and the usual Friedmann equation is reproduced, up to the sign of the source term.

We begin by perturbing the RS solution by placing an additional energy density on the two branes without a compensating change to the bulk cosmological constant. That is, we consider $\rho = V_{vis} + \rho_{vis}$ and $p = -V_{vis} + p_{vis}$ where V_{vis} is given by (9) and ρ_{vis}, p_{vis} are the energy density and pressure measured by an observer living on the visible brane, with equation of state $p_{vis} = w\rho_{vis}$. Key to our results will be that we work in the limit $\rho_{vis} \ll |V_{vis}|$. Given that $|V_{vis}| \sim M_{Pl}^4$ in these models, this limit is the correct one for describing our (post-inflationary) universe. Substituting these expressions for ρ and p into (5) gives, for either RS1 or RS2,

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = -\frac{\kappa^4}{36}V_{vis}(3p_{vis} - \rho_{vis}) - \frac{\kappa^4}{36}\rho_{vis}(\rho_{vis} + 3p_{vis}). \tag{13}$$

The $O(\Lambda)$ and $O(\kappa^4 V_{vis}^2)$ terms cancel using (8) and (9). Note that the presence of the background energy density allows for $H^2 \propto \rho_{vis}$ as in conventional cosmology. This differs from the observations of BDL because here the brane matter is a perturbation to the background RS solution. It is also clear that the presence of the prefactor V_{vis} in (13) implies that for one of RS1 or RS2 solutions, (13) will have a negative sign relative to the conventional Friedmann equation. In fact, we find that the "wrong-signed" Friedmann-like equation corresponds to RS1, the solution with $V_{vis} < 0$. To see this, substitute the formula for the Planck mass, (12), into (13):

$$\text{RS1}: \ \frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{4\pi G_N}{3}(3p_{vis} - \rho_{vis}) - \frac{\kappa^4}{36}\rho_{vis}(\rho_{vis} + 3p_{vis}). \tag{14}$$

For small densities ($\rho_{vis} \ll M_{Pl}^4$) we can neglect the second term on the RHS, since $\kappa^4 \sim 1/M_{Pl}^6$. The first term on the RHS contains a negative sign relative to the Friedmann equation in the conventional cosmology. Obviously this sign is flipped in the RS2 scenario and we get the correct Friedmann equations up to small corrections.

It is interesting to note that for a radiation-dominated (RD) universe this sign problem has no effect: since $p_{vis} = \rho_{vis}/3$ the first term on the RHS of (14) identically vanishes, and one obtains the same equation for both RS1 and RS2. It is then important to see whether the conventional RD solution is obtained in this case. The approximate solution for the scale factor can be easily found. We take

$$\rho_{vis}(t) = \rho_{vis}(t_i)\left(\frac{a_0(t)}{a_0(t_i)}\right)^{-4} , \tag{15}$$

then

$$a_0(t) \sim t^{\frac{1}{2}}\left(1 - \frac{\kappa^4}{36}\frac{\rho^2_{vis}(t_i)a_0(t_i)^8}{t^2} + \dots\right) . \tag{16}$$

Thus at late times this reduces to the evolution of standard RD cosmology. Note, that this solution is different from the $a_0(t) \sim t^{1/4}$ solution presented in [7] (even though (5) reduces to exactly the same equation in the case of a RD universe). The reason is simply that a non-linear second order differential equation can have more than one solution, and the initial condition will determine which is the relevant one. Since (16) reproduces the standard RD cosmology, it is plausible that the potential problems with BBN found in [7] are in fact solved by the existence of the extra solution (16). From the solution (16) we can see that both RS1 and RS2 reproduce the conventional RD solution. Since for RS2 (13) gives the conventional Friedmann equation for any type of matter (up to small corrections), we conclude that the RS2 solution is viable. Therefore in the following we will only concentrate on the RS1 solution.

What is the effect of the wrong sign in the Friedmann equation for the RS1 solution? Assume that after some time t_{eq} the energy density of the universe is dominated by a component with an equation of state having $w < 1/3$. In what follows, the subscript "eq" will denote quantities measured at time t_{eq} of matter-radiation equality. Then, e.g., $\rho_{vis}(t) = \rho_{eq}(a_0(t)/a_0(t_{eq}))^{-3(w+1)}$. Next, for $w \neq 1/3$ and energy densities $\rho_{vis} \ll M^4_{Pl}$, the second term on the RHS of (14) is subdominant to the first, so it is neglected. It is then convenient to introduce new variables $\tilde{t} \equiv H_{eq}t$ and $x(\tilde{t}) \equiv a_0(t)/a_0(t_{eq})$, where $H(t)$ is the Hubble parameter. The initial conditions at $t = t_{eq}$ are then $x(t_{eq}) = 1$ and $\dot{x}(t_{eq}) = 1$, where the overdot denotes a derivative with respect to $\tilde{t}$. Then in these units (14) is

$$\frac{\ddot{x}}{x} + \left(\frac{\dot{x}}{x}\right)^2 = -\frac{\lambda}{2}\frac{1}{x^{3(w+1)}}, \tag{17}$$

where the dimensionless constant $\lambda \equiv 8\pi G_N \rho_{eq}(1-3w)/(3H^2_{eq})$ is introduced. Note that for $w < 1/3$ one has $\lambda > 0$. (One obtains conventional cosmology by the replacement $\lambda \to -\lambda$.) Also note that if $H^2 = 8\pi G_N\rho/3$ were the correct flatness constraint equation then $\lambda = 1 - 3w$. Since it is expected that $H^2 \sim G_N\rho$, then $\lambda \sim O(1)$. Multiplying the above equation by $y \equiv x^2$ then gives

$$\ddot{y} = -\lambda y^{-(3w+1)/2}. \tag{18}$$

That is, the expansion of our universe is described by the one-dimensional motion of a particle in the classical potential

$$V(y) = \frac{2}{1-3w}\lambda y^{(1-3w)/2}. \tag{19}$$

For $w < 1/3$ this describes an *attractive* potential. From this we understand that the universe reaches a maximum size, and then collapses. This is one of the main points of this letter: this result is in conflict with our current understanding of cosmology from several viewpoints. At the very least, observationally the universe seems to be accelerating [18] rather than decelerating. (We will examine a sharper conflict with BBN and the age of the universe below.) By contrast, in conventional cosmology there is an extra negative sign on the RHS of (18), so for $w < 1/3$ the potential is inverted and the universe expands forever (since curvature terms have not been included). In the RS1 scenario, the period of this oscillation is $O(1)$ in these dimensionless units if $\lambda \sim O(1)$. In fact, for $w = 0$ (non-relativistic matter), the solution is

$$\tilde{t} = \int \frac{x dx}{\sqrt{1+\lambda-\lambda x}} = \frac{2}{\lambda^2}\left(\frac{1+\lambda-\lambda x}{3} - 1 - \lambda\right)\sqrt{1+\lambda-\lambda x}. \tag{20}$$

In RS1, the period t_U of oscillation (i.e., the age of the universe) for $\lambda = 1$ is then $t_U = 20/3$. So in the original units $t_U \sim H_{eq}^{-1}$ as expected. By inspection the maximum size of the universe is $x_{max} = 1 + 1/\lambda$ for $w = 0$. Finally, note that for the conventional MD cosmology ($\lambda = -1$) the above formula correctly reproduces $\tilde{t} \sim x^{3/2}$.

In the RS1 scenario, the classical potential (19) determines a first-order equation for x, in other words, it determines $H^2 = H_{eq}^2 \dot{x}^2/x^2$. In fact,

$$H^2 = \frac{EH_{eq}^2}{2x^4} - \frac{8\pi G_N}{3}\rho_{vis}. \tag{21}$$

This contains an arbitrary integration constant $E = 2 + 2\lambda/(1-3w)$. Note that the above equation (21) together with the conservation of energy (6) implies the Friedmann equation (14) for arbitrary values of the integration constant E. In conventional cosmology $\lambda = -1 + 3w$ so $E = 0$, and there is an obvious extra minus sign in the second term, reducing (21) to the usual flatness equation. In RS1 (21) is the analog of the flatness equation. In what follows, we assume that $\rho_{vis} > 0$, which implies

$$E > \frac{16\pi G_N \rho_{eq}}{3H_{eq}^2} > 0\ . \tag{22}$$

The preceding arguments indicate that in the RS1 scenario, after the time of matter-radiation equality, the universe collapses on a timescale given by H_{eq}^{-1}. The expansion rate H_{eq} is obtained as follows. Assuming standard BBN, we know the radiation temperature $T_{BBN} \sim$ MeV, and expansion rate H_{BBN} during BBN. We use the present-day values of the radiation and baryon energy densities, together with conservation of energy and (16) describing the RD cosmology, to determine H_{eq}. We use $a \sim t^{1/2}$ from (16) rather than the solution $a \sim t^{1/4}$, which is known to have difficulties with the He^4 abundance [7]. Then

$$\frac{T_{eq}}{T_{BBN}} = \frac{a_{BBN}}{a_{eq}} = \left(\frac{t_{BBN}}{t_{eq}}\right)^{1/2} = \left(\frac{H_{eq}}{H_{BBN}}\right)^{1/2}. \tag{23}$$

The first equality follows from energy conservation, and the last two equalities from (16). Next, conservation of energy is used, together with the present-day value $\rho_\gamma/\rho_{crit} \sim 10^{-5}$, and $T_{now} \sim 2.7$ K, to give $T_{eq} \sim 5$ eV. Inserting this result, together with $T_{BBN} \sim$ MeV, and $H_{BBN} \sim T_{BBN}^2/M_{Pl} \sim 10^{-21}$ MeV, into (23) gives $H_{eq} \sim 10^{-32}$ MeV (the standard BBN result). Therefore, a standard BBN cosmology implies in the RS1 scenario that an MD universe collapses on a time scale of $t_U \sim H_{eq}^{-1} \sim$ few $\times 10^3$ years. It is clear from these arguments that the age of the universe has not been used as an input, so this last result may be viewed as the RS1 standard BBN cosmology prediction for the age of the universe.

In order to evade the previous arguments RS1 requires some non-standard version of nucleosynthesis. In particular, in order for the universe to exist for billions of years, the Hubble parameter at the start of MD must be orders of magnitude larger than in standard BBN: $H_{eq}^2 \sim 10^3 \times (\frac{8\pi}{3} G_N \rho_{vis})$.

To conclude, we have considered the cosmology of five-dimensional theories with localized gravity on a three-brane introduced in [4, 5]. We have found that the solution with a non-compact extra dimension is potentially viable, since it reproduces the conventional cosmological solutions. However, the solution which may solve the hierarchy problem predicts a contracting universe with a lifetime of a few thousand years (assuming standard BBN). One way to avoid this difficulty might be a non-conventional BBN. But it seems more likely that these problems could be avoided by introducing additional matter fields in the bulk (which are anyway required to stabilize the radius of the extra dimension). The difficulty with this possibility will be maintaining the features which led to the solution of the hierarchy problem in the first place.

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