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Naturally Light Scalars

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Abstract

I argue that in certain chiral gauge theories composite scalars associated with chiral symmetry breaking can be light (i.e. lighter than naive scaling from QCD would suggest) without any fine-tuning. These scalars will be even lighter in chiral gauge theories that produce chiral symmetry breaking without confinement. I construct a model which demonstrates this last possibility.

1 Introduction

The fact that scalar mesons are much heavier than pseudoscalar and vector mesons was once considered to be something of a mystery. In fact, a mythical $\epsilon(730)$ once graced the pages of the Particle Data Book [1], in conformity with certain theoretical prejudices. Currently the lightest scalar composed primarily of a quark and anti-quark is thought to weigh in between 1300 and 1500 MeV [2]. (The $f_0(980)$ and $a_0(980)$, formerly known as the S^* and δ, are thought to be primarily K-$\overline{K}$ bound states [3, 4].) Recently Hill and Marinelli [4] pointed out that in QCD certain scalar mesons (those which are the parity partners of the pseudo-Goldstone bosons) receive a contribution to their masses from instantons, and thus, like the η', their masses can be larger than might be naively expected. In this paper I will explore the implications of Hill and Marinelli's observation for the spectra of certain chiral gauge theories. I will also consider chiral gauge theories that undergo chiral symmetry breaking without confinement. I will show that scalars can be quite light in certain chiral gauge theories. Such light scalars may be of use in building extensions of the standard model.

2 The Effect of Instantons

As noted by Hill and Marinelli [4], the effective interaction induced on quarks by instantons (the t'Hooft determinant interaction [5]) not only contributes to the η' mass, but also to the scalar meson masses. The key observation that they make is that when going from QCD to an effective theory of mesons, the t'Hooft determinant of quark bilinears is replaced by a determinant of a matrix of meson fields, $\mathcal{M}$. For n_f flavors of quarks, $\mathcal{M}$ is an $n_f \times n_f$ complex matrix, which can in general be written as a product of a Hermitian matrix H, a unitary matrix U, and a phase. The matrix H contains the scalar fields, U contains the pseudo-Goldstone boson fields, and the phase corresponds to the η' field (up to a normalization constant, f). Thus the t'Hooft interaction in the effective meson theory is proportional to:

$$\det\mathcal{M} + \text{h.c.} = \det\left(H\,U\right)\, e^{\frac{i\eta'}{f}} + \text{h.c.} = \det H\, e^{\frac{i\eta'}{f}} + \text{h.c.}\ . \tag{1}$$

As expected, the pseudo-Goldstone boson fields drop out. The scalars and the η' on the other hand receive contributions to their masses when chiral symmetry is spontaneously broken (i.e. when H gets a vacuum expectation value).

I will roughly estimate the size of this contribution to the η' mass by assuming that in the absence of instantons the η' is a pseudo-Goldstone boson. Neglecting the up and down quark masses, the mass (squared) matrix for the η and η' is:

$$M^2 = \frac{<\overline{\psi}\psi>}{3f^2} m_s \begin{pmatrix} 4 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{pmatrix} + \frac{1}{2}(I - \tau_3) M_I^2 \ , \tag{2}$$

where the first term is a standard chiral perturbation theory estimate [6], τ_3 is a Pauli matrix, and M_I^2 represents the instanton contribution.

Writing

$$M_I^2 = a \frac{<\overline{\psi}\psi>}{f^2} m_s \ , \tag{3}$$

I find

$$m_{\eta'}^2 + m_\eta^2 = (2 + a)\, m_s \frac{<\overline{\psi}\psi>}{f^2} \ , \tag{4}$$

$$m_{\eta'}^2 - m_\eta^2 = m_s \frac{<\overline{\psi}\psi>}{f^2} \sqrt{4 - \frac{4}{3}a + a^2} \ . \tag{5}$$

Using standard values for strange quark mass and the condensate ($m_s = 155\, MeV$, $<\overline{\psi}\psi> = (236\,\mathrm{MeV})^3$), equation (4) gives $M_I \approx 870$ MeV ($a = 3.16$); while this result verifies equation (5) to within 20%. If the contribution to the scalar mass (squared) is comparable to M_I^2, then in the absence of instantons, the scalar mass would be reduced to the range 970-1220 MeV.

3 Suppressing Instantons and the Effect of Confinement

How would things be different in chiral gauge theories? Recall that the effective interaction induced by instantons takes the form of a determinant, involving each of the (left and right-handed) fermions with non-Abelian gauge interactions. In producing a mass term in an effective Lagrangian for mesons, four of the fermions are replaced by two meson fields, and the remaining fermion lines must be contracted.

This is only possible if all of the remaining fermions get Dirac masses (i.e. the condensate $< \overline{\psi_L} \psi_R >$ is non-zero). But chiral gauge theories are characterized by the fact that not all the fermions can have gauge invariant mass terms. The generic situation for chiral symmetry breaking in chiral gauge theories is that not all the fermions get masses at the same scale. Thus, if the chiral gauge theory has more than four interacting fermions (counting left and right-handed fermions separately), and not all the fermions get dynamical masses then at least some of the scalars will not get a contribution to their masses from instantons. If the fermions that are massless at the chiral symmetry breaking scale under consideration (Λ_h the "heavy" scale) actually develop a mass at some lower scale (Λ_l the "light" scale), then the instanton contribution to the scalar mass squared will be suppressed[1] by a power of the light scale over the heavy scale. That is:

$$M_I^2 = \kappa \, \Lambda_h^2 \left(\frac{\Lambda_l}{\Lambda_h} \right)^{3n} \quad , \tag{6}$$

where n is just the number of light Dirac fermions.

As shown above, even in the absence of instanton contributions to their masses, the scalars can still be quite heavy, compared to the vector mesons. To proceed further, we must understand more about what makes scalars heavy. The heuristic explanation is that confinement makes them heavy. Although no one understands why non-relativistic quark models are so successful in describing the spectrum of hadrons containing light quarks, it will be helpful to consider what such models have to say about this issue. From the viewpoint of non-relativistic potential models the answer is quite clear. In a Coulombic potential the $1S$ (i.e. the analogues of the ρ and π) has a small splitting ($\mathcal{O}(\alpha^2)$) from the degenerate $2S$

[1] This suppression also applies to the mass squared of the analogue of the η'.

and $2P$ (i.e. the analogues of the ω and the scalar) levels. However, in a confining potential (e.g. a linear or even harmonic potential) the $1S$ can have a large splitting from the $2P$ (which is not degenerate with the $2S$).

Whether or not the results of non-relativistic quark models are trustworthy for the spectra of generic chiral gauge theories, one thing is certain: if the gauge symmetry left unbroken by the chiral symmetry breaking is not asymptotically free, then the long-distance contribution to the scalar mass can be calculable (since the coupling can be weak). In the case of a weak long-range coupling, a Coulombic potential should be a good approximation. In the next section I will construct a chiral gauge theory that, according to conventional wisdom, should demonstrate this behavior: it produces chiral symmetry breaking without confinement. Or in other words, it is asymptotically free above the chiral symmetry breaking scale, but not asymptotically free below this scale.

4 A Toy Model

In this section I will construct a model that dynamically breaks chiral symmetries, without confinement. Let $\mathbf{S_N}$ be the symmetric tensor representation of $SU(N)$, then the following reducible representations:

$$(\mathrm{N}+4) \times \mathbf{N} \oplus \overline{\mathbf{S_N}}\ , \tag{7}$$

and

$$\mathbf{N} \oplus \overline{\mathbf{N}}, \tag{8}$$

are each anomaly free. According to the most attractive channel (MAC) hypothesis [7], a condensate will form first in the MAC as determined by the strength of one gauge boson exchange[2], i.e. the sum of the quadratic Casimirs of the fermions minus the quadratic Casimir of the condensate, which I will call ΔC_2. The strength of the possible condensation channels for the fermions discussed above are:

$$\Delta C_2(\mathbf{N} \times \overline{\mathbf{S_N}} \to \overline{\mathbf{N}}) = \frac{(N+2)(N-1)}{N} , \tag{9}$$

$$\Delta C_2(\mathbf{N} \times \overline{\mathbf{N}} \to \mathbf{1}) = \frac{(N+1)(N-1)}{N} , \tag{10}$$

$$\Delta C_2(\overline{\mathbf{S_N}} \times \overline{\mathbf{S_N}} \to \overline{\mathbf{R_1}}) = \frac{2(N+2)}{N} , \tag{11}$$

$$\Delta C_2(\overline{\mathbf{N}} \times \overline{\mathbf{S_N}} \to \overline{\mathbf{R_2}}) = \frac{(N+3)}{N} , \tag{12}$$

$$\Delta C_2(\mathbf{N} \times \mathbf{N} \to \mathbf{A}) = \frac{N+1}{N} , \tag{13}$$

where $\mathbf{A}$ is the antisymmetric tensor representation, $\mathbf{R_1}$ is the representation with two symmetric and two antisymmetric indices, i.e. its Young tableaux is composed of two rows of two boxes, and $\mathbf{R_2}$ has a Young tableaux with two boxes in the first row, and one box in the second row. For $N > 3$, the MAC is

$$\mathbf{N} \times \overline{\mathbf{S_N}} \to \overline{\mathbf{N}} \tag{14}$$

which, for $N > 2$, breaks $SU(N)$ to $SU(N-1)$. Under this pattern of gauge symmetry breaking, $\overline{\mathbf{S_N}}$ decomposes into an $\overline{\mathbf{N-1}}$, an $\overline{\mathbf{S_{N-1}}}$, and a $\mathbf{1}$. Thus the condensation in (14) leaves (for each of the reducible, anomaly-free representations shown in (7)) the following massless fermions: $(N+5)$ $\mathbf{1}$'s, $(N+3)$ $\mathbf{N-1}$'s, and one $\overline{\mathbf{S_{N-1}}}$.

With n_χ copies of the chiral, reducible, anomaly-free representation shown in (7), and n_f of the vector representations (8), the coefficient of the one-loop β function for $SU(N)$ is proportional to

$$11N - n_\chi\left((N+4) + (N+2)\right) - 2n_f \tag{15}$$

[2]Of course, the MAC hypothesis could be wrong, see ref. [8] for a recent critique.

For the unbroken $SU(N-1)$, below the condensation scale, the one-loop β function coefficient is proportional to:

$$11(N-1) - n_\chi\left((N-1+4)+(N-1+2)\right) - 2n_f \tag{16}$$

For the $SU(N)$ gauge group to be asymptotically free while the $SU(N-1)$ subgroup is not (i.e. the $SU(N-1)$ subgroup is infrared free) the following inequality must be satisfied (taking $n_\chi = 1$ for simplicity)

$$\frac{9N-15}{2} < n_f < \frac{9N-6}{2} \ , \tag{17}$$

which has solutions for arbitrary N. Thus it is very simple to arrange for the properties I want at one loop. However, one must be more careful, since for certain values of N, n_χ, and n_f, there can be an infrared fixed point in the two-loop β function for small values of the gauge coupling $\alpha = g^2/(4\pi)$. For example, in the large N limit, with $n_\chi = 1$ and $n_f = 1 + (9N-15)/2$, the fixed point occurs at

$$\alpha_* \approx \frac{28\pi}{39N^2} \ . \tag{18}$$

Thus at two loops, this theory is asymptotically free for $\alpha < \alpha_*$, and not asymptotically free for $\alpha > \alpha_*$. For comparison, the standard crude estimate (calculated in the ladder "approximation") of the strength of the gauge coupling required for chiral symmetry breaking[3] is:

$$\alpha_c(N) = \frac{2\pi}{3\,\Delta C_2} = \frac{2\pi N}{3(N+2)(N-1)} \ . \tag{19}$$

Thus in the large N limit with $n_\chi = 1$ and $n_f = 1 + (9N-15)/2$, the coupling approaches its infrared fixed point from below, and this fixed point is at a value too

[3] That is the strength of the gauge coupling required to make the anomalous dimension of $\overline{\psi}\psi$ equal to one [9].

weak for chiral symmetry breaking to occur, so the low-energy effective theory is a (massless) conformal theory.

To produce a model that exhibits chiral symmetry breaking without confinement, one must choose N, n_χ, and n_f such that

$$\alpha_*(N, n_\chi, n_f) > \alpha_c(N) > \alpha_*(N-1, n_\chi, n_f). \tag{20}$$

If this condition is satisfied, then, starting from a weakly coupled theory at high energies, the coupling increases as the renormalization scale is lowered until it becomes strong enough for chiral symmetry breaking to occur. Below the chiral symmetry breaking scale, the coupling is above the fixed point of the unbroken gauge interactions, and as the renormalization scale is lowered further, the coupling decreases towards the new infrared fixed point.

As an example I will consider the model with $N = 4$, $n_\chi = 2$, and $n_f = 0$. For this model the estimate of the coupling at the chiral symmetry breaking scale, m, is

$$\alpha(m) \approx \alpha_c = \frac{2\pi}{3\, C_2(\mathbf{10})} = \frac{4\pi}{27} \approx 0.47\ , \tag{21}$$

while $\alpha_*(4,2,0) \approx 0.61$, and $\alpha_*(3,2,0) \approx 0.42$. (Another possible model is $N = 4$, $n_\chi = 1$, and $n_f = 8$, where $\alpha_*(4,1,8) \approx 0.58$, and $\alpha_*(3,1,8) \approx 0.22$.) Above the scale m there are sixteen $\mathbf{4}$'s and two $\overline{\mathbf{10}}$'s of $SU(4)$. At the chiral symmetry breaking scale $SU(4)$ breaks to $SU(3)$, and two Dirac fermions get masses, leaving eighteen $\mathbf{1}$'s, fourteen $\mathbf{3}$'s, and two $\overline{\mathbf{6}}$'s of $SU(3)$. Will any further condensation take place below this scale? According to the MAC hypothesis, the answer is no. The remaining condensation channels are less attractive (i.e. some of the gauge bosons have become massive) than the channel that has already condensed, and as the renormalization scale is moved towards the infrared, the coupling grows weaker, not stronger.

If the estimate for $\alpha(m)$, equation (21), is correct, then the unbroken gauge theory is not very strongly coupled. Recall that by itself the long-range gauge force would lead to an infinite number of bound states (labeled by n) of the massive fermions with binding energies given by:

$$E_n = -\frac{m\, C_2(\mathbf{3})^2 \alpha^2}{4n^2} = -\frac{4m\alpha^2}{9n^2} \ , \tag{22}$$

where $\alpha = \alpha(m\alpha)$, so the scalar (a $2P$ level) receives a long-distance contribution to its splitting from the analogue of the ρ (a $1S$ level) equal to $m\alpha^2/3$. The long-distance contribution to the splitting between the analogue of the ρ and the massless Goldstone bosons (some of which are eaten by gauge bosons) comes solely from hyperfine splitting, and is of order $m\alpha^4$. Thus the scalar is potentially as light as roughly $m/8$, i.e. roughly forty times lighter than a naive scaling from QCD would suggest.

What about short-distance contributions to the scalar mass from higher-dimension operators? The implicit assumption of the above discussion is that as the renormalization scale is lowered towards the chiral symmetry breaking scale (and the coupling constant grows), no irrelevant operators become relevant. Whether or not this happens can probably only be determined reliably in lattice simulations. The success of non-relativistic quark models can be taken as circumstantial evidence that short-distance contributions (aside from instanton contributions) to meson masses from higher-dimension operators are small in QCD. Of course, in the example I have considered, the short distance interactions are important for making the Goldstone bosons massless.

5 Conclusions

Using Hill and Marinelli's observation of the instanton contribution to scalar masses in QCD, I have pointed out that scalars associated with chiral symmetry breaking will be lighter in a class of chiral gauge theories than a naive scaling might suggest. I have also pointed out that in chiral gauge theories that exhibit chiral symmetry breaking but not confinement, the scalars will be even lighter. Finally a model was constructed that is asymptotically free above the chiral symmetry breaking scale, and (assuming the MAC hypothesis is correct) is non-asymptotically free (hence not confining) below this scale. If the simulation of chiral gauge theories on the lattice becomes feasible, then models like this may be useful in disentangling chiral symmetry breaking and confinement.

On a more speculative note, model builders who try to extend the standard model often end up with light scalars in their models. The scalars are usually kept light by some form of fine-tuning. The considerations above suggest that it may be possible to construct models with light-composite scalars without resorting to fine-tuning.

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