A LIGHT DILATON IN GAUGE THEORIES?

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We study gauge theories having a slowly varying running coupling and chiral symmetry breaking. It has been suggested that such theories contain a light dilaton, a pseudo-Goldstone boson associated with approximate scale invariance. We calculate its mass by studying the scaling properties of the effective action describing chiral symmetry breaking. We also consider the effect of a physical high energy cutoff as motivated in a technicolor context. Our results indicate that a light dilaton is unlikely

We wish to study asymptotically free gauge theories containing no explicit dimensionful parameters and having a small β-function over the momentum range relevant for chiral symmetry breaking. Below we shall exhibit theories of this type for which we can trust the lowest order perturbative β-function throughout this momentum range. But if the β-function is small then the theory possesses an approximate scale invariance, and chiral symmetry breaking would imply a spontaneous breaking of this scale symmetry. This makes necessary a pseudo-Goldstone boson associated with broken approximate scale invariance, a pseudo-dilaton (PD) [1,2]. The nonzero β-function is the only explicit source of scale symmetry breaking, and the PD mass is related to it in some way. We might guess that the mass squared is proportional to the value of the β-function at the scale of chiral symmetry breaking [3]. Others [4] suggest that the mass is given by the confinement scale Λ, the inverse size of glueball states. The theories we study will have Λ less than the chiral symmetry breaking scale Λ_ψ.

The PD will appear in the effective theory immediately below Λ_ψ. In this paper we will focus on short distance contributions to the PD mass, from physics at energy scales above Λ_ψ. But there will also be contributions from lower scales. For example, at scales $<\Lambda_\psi$ but $>\Lambda$ there are still gluons in the effective theory and there should be an induced PD coupling to $(F^a{}_{\mu\nu})^2$. At scales $<\Lambda$, after integrating out the gluons, we will then find further contributions to the PD mass. We do not attempt to calculate these contributions, but they should be characterized by Λ. The short distance contributions we do calculate may well be larger than Λ in which case our calculation would give a useful lower bound on the PD mass.

A gauge theory with a slowly varying running coupling is of interest [1–9] as a technicolor [10] interaction responsible for breaking the weak interactions. This was first discussed [5] in the context of a nontrivial ultraviolet fixed point where anomalous scaling was shown to break the naive connection between fermion masses and flavor changing neutral currents (FCNCs) in extended technicolor theories. FCNCs are suppressed and technipion masses are increased. Then in ref. [7] we analyzed the Schwinger–Dyson equation in ladder approximation in which we used a running coupling satisfying $p\partial_p\alpha(p) = -b\alpha^2(p)$. We numerically solved for the self-energy $\Sigma(p)$ for various b. We found that the smaller b was the less rapidly $\Sigma(p)$ fell with p and that this implied a suppression of FCNCs. This point was made [2,3,8] again for $b=0$ where $\Sigma(p)$ could be studied analytically. A detailed discussion of the behavior of $\Sigma(p)$ for small b and the connection with FCNCs may be found in ref. [9].

An extended technicolor scale supplies a physical high energy cutoff at which the technicolor interactions should no longer be considered in isolation. Thus with the technicolor application in mind we will consider gauge theories in which we insert a high momentum cutoff $M \gg \Lambda_\psi$ by hand. This introduces another source of explicit scale breaking into the theory and produces a further high energy contribution to the PD mass which we will treat. (In the case that one studies a Schwinger–Dyson equation with constant coupling ($b=0$) one finds [11] that an ultraviolet cutoff is actually necessary for a chiral symmetry breaking solution to occur.)

The pseudo-Goldstone boson under discussion arises due to a chiral symmetry breaking condensate. We will therefore study the effective action which describes the formation of this condensate. In the standard formalism [12] the effective action is a functional of a nonlocal order parameter, the fermion propagator (parameterized by $[\not{p}+\Sigma(p)]^{-1}$ in Landau gauge). We make the standard two-loop approximation [13]. Renormalization effects are accounted for by inserting the running coupling at the gauge vertices. This introduction of the running coupling will be the origin of explicit breaking of scale invariance. In euclidean space our effective action for n flavors of fermions in representation r having dimension $d(r)$ and quadratic Casimir $C_2(r)$ reads

$$\Gamma = \frac{nd(r)}{8\pi^2} \int_0^\infty \mathrm{d}p\, p^3 \left[\frac{4\Sigma^2(p)}{p^2+\Sigma^2(p)} - 2\ln\left(\frac{p^2+\Sigma^2(p)}{p^2}\right)\right]$$

$$- \frac{3nd(r)C_2(r)}{8\pi^3} \int_0^\infty \mathrm{d}p \frac{p^2\Sigma(p)}{p^2+\Sigma^2(p)} \int_0^\infty \mathrm{d}k \frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)} \min\{k/p, p/k\}\alpha(\max\{p,k\}) . \tag{1}$$

Minimizing Γ with respect to the fermion self-energy, $\Sigma(p)$, gives rise to the Schwinger–Dyson equation in the ladder approximation,

$$p\Sigma(p) = \frac{3C_2(r)}{2\pi} \int_0^\infty \mathrm{d}k \frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)} \min\{p/k, k/p\}\alpha(\max\{p,k\}) . \tag{2}$$

We will use this equation in its nonlinear form to numerically solve for $\Sigma(p)$. We can then study the behavior of Γ under scaling transformations by inserting this solution into Γ and transforming $\Sigma(p)\to e^{\rho}\Sigma(e^{-\rho}p)$. This transformation follows from the scaling transformation of the fermion field $\psi(x)\to e^{\rho}\psi(e^{\rho}x)$. We label the magnitude of the transformed effective action by $\Gamma(\rho)$.

In the effective theory below Λ_ψ the potential for the PD field, $\sigma(x)$, should exhibit the same explicit scale breaking found in Γ; the potential and Γ should transform in a similar way under a scale transformation. This can be accomplished by taking $\Gamma(\rho)$, promoting the number ρ in this function to a field $\sigma(x)/F_\sigma$, and identifying the result with the potential $V(\sigma(x))$. The scale transformation of a dilaton field is $\sigma(x)\to\sigma(e^{\rho}x)+\rho F_\sigma$ and thus the sense in which $V(\sigma(x))$ transforms the same way as Γ is given by

$$V(\sigma(e^{\rho}x)+\rho F_\sigma)\,|_{\sigma(x)=0} = \Gamma(\rho) . \tag{3}$$

From $\Gamma(\rho)$ we can extract the PD mass,

$$m_\sigma^2 = \partial^2_{\sigma(x)} V(\sigma(x))\,|_{\sigma(x)=0} = F_\sigma^{-2}\partial^2_\rho \Gamma(\rho)\,|_{\rho=0} . \tag{4}$$

We will now give a situation in which we are justified to use just the first term in the following perturbative evolution equation for the running coupling,

$$p\partial_p\alpha(p) = -b\alpha^2(p) + c\alpha^3(p) + d\alpha^4(p) + \dots . \tag{5}$$

We note that chiral symmetry breaking takes place when $\alpha(p)$ somewhat exceeds the critical value [11,13] $\alpha_C \equiv \pi/3C_2(r)$ (by an amount which is small when b is small) [7]. It is then convenient to define $\tilde{\alpha}(p)\equiv \alpha(p)C_2(r)$ and write the evolution equation as

$$p\partial_p\tilde{\alpha}(p) = -\tilde{b}\tilde{\alpha}^2(p) + \tilde{c}\tilde{\alpha}^3(p) + \tilde{d}\tilde{\alpha}^4(p) + \dots , \tag{5'}$$

where $\tilde{b}\equiv b/C_2(r)$, $\tilde{c}\equiv c/C_2(r)^2$, $\tilde{d}\equiv d/C_2(r)^3$, etc. Note that $\tilde{\alpha}(\Lambda_\psi)\approx\alpha_C C_2(r)=\pi/3\approx 1$.

We wish to make a formal extrapolation to a noninteger number of flavors n; specifically $0<n<1$. Then for n small enough and for some number of colors N we have $b\approx N$, $c\approx N^2$, $d\approx N^3$, etc. Thus $\tilde{b}$ is small and all higher order terms can be neglected if we choose the fermion representation such that $C_2(r)\gg N$. Below Λ_ψ the fermions which have condensed out no longer contribute to the β-function and this in general will modify the value of b below Λ_ψ. But we may also neglect this effect in this case of small n.

Thus in the case of small n and large $C_2(r)$ we are justified to use the lowest order β-function over the whole momentum range of interest. By varying $\tilde{b}$ we will develop a sense for how departures from scale invariance as represented by $\tilde{b}$ translates into a PD mass. This should also give us some feeling for more general situations.

We also wish to find the PD mass dependence on a high energy cutoff M. This scale breaking effect will be introduced via the running coupling by simply setting $\alpha(p)=0$ for $p>M$. This will approximate an embedding of our gauge group at the scale M into a strongly asymptotically free gauge group whose coupling above M rapidly becomes small.

For $\tilde{b}>0$, $\alpha(k)$ at small enough k eventually blows up. This behavior for $\alpha(k)$ is obviously unphysical, and we therefore need to introduce a scale μ below which we take $\alpha(k)=\alpha(\mu)$. But we wish to study results which are μ independent. This is accomplished as long as we can choose μ sufficiently below $\Sigma(0)$, since the integrals in (1) and (2) are damped by powers of $k/\Sigma(0)$ for $k\ll\Sigma(0)$. We will monitor the μ dependence of our results and this will provide an upper bound on the value of $\tilde{b}$ we study.

We thus use the coupling

$$\begin{aligned}\alpha(p)&=[b\ln(\mu/\Lambda)]^{-1}, &&\text{for}\quad p<\mu,\\ &=[b\ln(p/\Lambda)]^{-1}, &&\text{for}\quad \mu<p<M,\\ &=0, &&\text{for}\quad p>M.\end{aligned} \tag{6}$$

Λ is less than μ and is approximately the confinement scale of the theory.

We now must face the question of what are we comparing the PD mass to? We need to identify a physical mass scale characterizing the chiral symmetry breaking which we can hold fixed while varying M and $\tilde{b}$. Candidates for Λ_ψ are $\Sigma(0)$, the scale k' such that $\alpha(k')=\alpha_C$, or the scale k'' such that $\Sigma(k'')=k''$. They all vary in a nontrivial manner relative to each other as $\tilde{b}$ is varied. We will instead choose F_π. In the technicolor context it is F_π, the technipion decay constant, which determines the mass of the W and Z bosons and thus sets the physical scale. An explicit expression for F_π has been derived [14] in terms of $\Sigma(k)$:

$$F_\pi^2=\frac{d(r)}{(2\pi)^2}\int_0^\infty \mathrm{d}k\,\frac{2k^3[\Sigma^2(k)-\frac{1}{4}k\Sigma(k)\Sigma'(k)]}{[k^2+\Sigma^2(k)]^2}\,. \tag{7}$$

Thus for a given M and $\tilde{b}$ we hold F_π fixed and simultaneously find a solution to the Schwinger–Dyson equation by varying the Λ appearing in (6).

To find the PD mass from (4) we need $\partial_\rho^2\Gamma(\rho)|_{\rho=0}$. We derive an expression for this directly from (1) by inserting $\Sigma(p)\to e^\rho\Sigma(e^{-\rho}p)$ and using $\partial_\rho\Gamma(\rho)|_{\rho=0}=0$. We then find

$$m_{\mathrm{PD}}^2=3nd(r)(A+B)/(F_\sigma^2 4\pi^3), \tag{8}$$

$$\begin{aligned}A=&-\frac{M^6\Sigma^2(M)\tilde{\alpha}(M)}{[M^2+\Sigma^2(M)]^2}+\left(\frac{4M^2\Sigma(M)\tilde{\alpha}(M)-M^3\Sigma'(M)\tilde{\alpha}(M)-M^2\Sigma(M)\tilde{b}\tilde{\alpha}^2(M)}{M^2+\Sigma^2(M)}\right.\\ &\left.+\frac{2M^2\Sigma^2(M)[\Sigma'(M)M-\Sigma(M)]\tilde{\alpha}(M)}{[M^2+\Sigma^2(M)]^2}\right)\int_0^M \mathrm{d}k\,\frac{k^3\Sigma(k)}{k^2+\Sigma^2(k)}\end{aligned} \tag{9}$$

(where $\Sigma'(M)\equiv\partial_p\Sigma(p)|_{p=M}$),

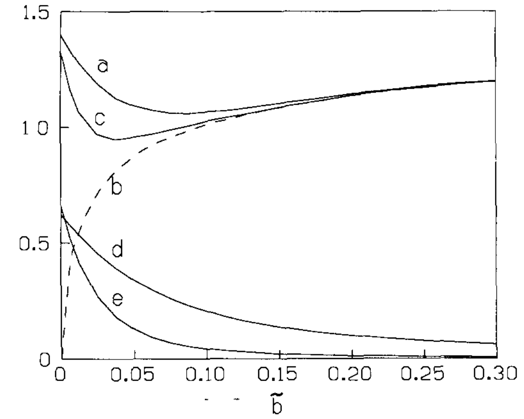

Fig. 1. (a) $f(\tilde{b},R)/10$ for $R=4\times10^3$ ($f(\tilde{b},R)$ is related to the PD mass in eq (11)), (b) contribution to curve (a) neglecting scale breaking effect of cutoff M, (c) $f(\tilde{b},R)/10$ for $R=10^5$, (d) $m_{\rm TP}/F_\pi$ for $R=4\times10^3$ ($m_{\rm TP}$ is technipion mass), (e) $m_{\rm TP}/F_\pi$ for $R=10^5$

$$B=4\tilde{b}\int_0^\mu dk\frac{k^3\Sigma(k)}{k^2+\Sigma^2(k)}\int_\mu^M dp\frac{p\Sigma(p)}{p^2+\Sigma^2(p)}[\tilde{\alpha}^2(p)-\tilde{b}\tilde{\alpha}^3(p)/2]$$

$$+4\tilde{b}\int_\mu^M dk\frac{k^3\Sigma(k)}{k^2+\Sigma^2(k)}\int_k^M dp\frac{p\Sigma(p)}{p^2+\Sigma^2(p)}[\tilde{\alpha}^2(p)-\tilde{b}\tilde{\alpha}^3(p)/2]+\tilde{b}\frac{\mu^2\Sigma(\mu)\tilde{\alpha}^2(\mu)}{\mu^2+\Sigma^2(\mu)}\int_0^\mu dk\frac{k^3\Sigma(k)}{k^2+\Sigma^2(k)}\,. \quad (10)$$

A and B are the contributions due to scale breaking introduced by the cutoff M and a nonzero $\tilde{b}$, respectively.

We will give our results for m_σ in terms of a function $f(\tilde{b},R)$ where $R\equiv d(r)^{1/2}M/F_\pi$:

$$m_\sigma F_\sigma/F_\pi^2=[n/d(r)]^{1/2}f(\tilde{b},R)\,. \quad (11)$$

$f(\tilde{b},R)$ is obtained from (7) and (8) after pulling out the factors of n and $d(r)$. We will not attempt to calculate F_σ, but it is expected to be of order F_π.

In the following we use $\tilde{\alpha}(\mu)=20$. The dependence of our results on μ increases as $\tilde{b}$ increases. The maximum value of $\tilde{b}$ we use, $\tilde{b}=0.3$, corresponds to $\partial(\log m_\sigma)/\partial(\log\mu)\lesssim0.03$. Also, for comparison with the following, we note that $\Sigma(0)/F_\pi$ varies from ≈5 at $\tilde{b}=0$ to ≈10 at $\tilde{b}=0.3$.

In fig. 1 we display $f(\tilde{b},R)/10$ for $R=4\times10^3$ (curve a). The contribution to $f(\tilde{b},R)/10$ arising from the running coupling, the B term in (8), is also indicated (curve b). As expected, this vanishes as $\tilde{b}\to0$. But one surprise is the large size of this contribution for rather small $\tilde{b}$. Thus even a slow logarithmic behavior of the running coupling translates into a rather large PD mass. The other surprise is that the remaining contribution due to the scale breaking effect of the cutoff M is so large. Thus in precisely the limit in which one naively expects a real dilaton to emerge, $\tilde{b}\to0$, we find that the dilaton can gain significant mass from very high energy effects.

$f(\tilde{b},R)/10$ for $R=10^5$ is given by curve c. We see that a light PD would require a very large R, and thus a very large physical cutoff M. But we also note the $d(r)^{1/2}$ factor in R. This along with the $d(r)^{-1/2}$ factor in (11) means that a light PD mass may emerge if the dimension of the representation r is large enough.

In a technicolor context we have effective four fermion operators generated at the scale M. For example there must be terms of the form $M^{-2}\bar{\psi}\psi\bar{\rm f}{\rm f}$ in order to generate a mass for the quark or lepton f. We are therefore interested in the quantity

$$\langle\bar{\psi}\psi\rangle=d(r)\int^M\frac{d^4k}{(2\pi)^4}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}\,. \quad (12)$$

This quantity also allows us to compare our PD mass to a typical neutral, color singlet technipion mass $m_{\rm TP}$. $m_{\rm TP}$ may arise due to the presence of a term of the form $M^{-2}\bar{\psi}\psi\bar{\psi}\psi$ which implies $m_{\rm TP}/F_\pi\approx\langle\bar{\psi}\psi\rangle/MF_\pi^2$. This ratio, independent of $d(r)$, is displayed for $R=4\times10^3$ (curve d) and $R=10^5$ (curve e). We remind the reader that the enhancement of $\langle\bar{\psi}\psi\rangle$ for small $\tilde{b}$ implies a larger M for a given quark or lepton mass and thus a suppression of FCNCs.

We may use the expression for $\langle\bar{\psi}\psi\rangle$ to determine the PD Yukawa couplings to quarks and leptons. These couplings originate in the $M^{-2}\bar{\psi}\psi\bar{f}f$ terms. We find them in the same way we found $V(\sigma(x))$; we make the substitution $\Sigma(k)\rightarrow e^{\rho}\Sigma(e^{-\rho}k)$ in (12) and write the result as some function $h(\rho)$ times $\langle\bar{\psi}\psi\rangle$, with $h(0)=1$. We promote ρ to a field as before and then find that the Yukawa coupling is $h(\sigma(x)/F_\sigma)m_f\bar{f}f$. For example in the case that $\tilde{b}$ is small enough [9], $\Sigma(p)\propto 1/p$ for large p, and we find $h(\rho)=e^{2\rho}$. Then the Yukawa coupling is $\exp[2\sigma(x)/F_\sigma]m_f\bar{f}f$. The factor of 2 agrees with ref. [3].

We have found that the PD mass seems to bear no relation to the confinement scale Λ, which in our case is of order $\Lambda_\psi\exp(-1/\tilde{b})$ and possibly very small. But we note that there may be a dilaton-like particle in the pure glue sector, a glueball with a mass of order Λ. Such a particle should not be associated with an approximate scale invariance of the full underlying theory. Also, this glueball would not have the Yukawa couplings expected of the PD.

An uncertainty in our analysis follows from the neglect of corrections to the effective action (1) not accounted for by the use of the running coupling. But it would be surprising to see a large decrease in the PD mass in a more complete treatment. Our results should also be indicative in cases where we cannot justify the lowest order β-function. For example if a strongly interacting theory happened to be approximately scale invariant over some range of energies, perhaps due to a nearby fixed point, then this energy range would have to be very large for a light dilaton to emerge.

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