

Isospin Breaking and Fine Tuning in Top-Color Assisted Technicolor

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Abstract

Recently, Hill has proposed a model in which new, potentially low-energy, top-color interactions produce a top-condensate (*a la* Nambu—Jona-Lasinio) and accommodate a heavy top quark, while technicolor is responsible for producing the W and Z masses. Here we argue that isospin breaking gauge interactions, which are necessary in order to split the top and bottom quark masses, are likely to couple to technifermions. In this case they produce a significant shift in the W and Z masses (i.e. contribute to $\Delta\rho_* = \alpha T$) if the scale of the new interactions is near 1 TeV. In order to satisfy experimental constraints on $\Delta\rho_*$, we find that either the effective top quark coupling or the top-color coupling must be adjusted to 1%. Independent of the couplings of the technifermions, we show that the isospin-splitting of the top and bottom quarks implies that the top-color gauge bosons must have masses larger than about 1.4 TeV. Our analysis can also be applied to strong extended technicolor (ETC) models that produce the top-bottom splitting via isospin breaking ETC interactions.

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1 Introduction

Recently, Hill has combined aspects of two different approaches to dynamical electroweak symmetry breaking into a model which he refers to as top-color assisted technicolor (TC²) [1]. In this model a top-condensate is driven by the combination of a strong isospin-symmetric top-color interaction and an additional (either weak or strong) isospin-breaking $U(1)$ interaction which couple only to the third generation quarks. He argues that the extreme fine-tuning that was required in pure top-condensate models can be done away with if the scales of the critical top-color and $U(1)$ interactions are brought down to a TeV. Given a top quark mass of around 175 GeV, such top-color interactions would produce masses for the W and Z that are far too small, hence there must be further strong interactions (technicolor) that are primarily responsible for breaking electroweak symmetry.

At low-energies, the top-color and hypercharge interactions of the third generation quarks may be approximated by four-fermion operators [1]

$$\mathcal{L}_{4f} = -\frac{4\pi\kappa_{tc}}{M^2} \left[\bar{\psi}\gamma_\mu \frac{\lambda^a}{2} \psi \right]^2 - \frac{4\pi\kappa_1}{M^2} \left[\frac{1}{3}\bar{\psi}_L\gamma_\mu\psi_L + \frac{4}{3}\bar{t}_R\gamma_\mu t_R - \frac{2}{3}\bar{b}_R\gamma_\mu b_R \right]^2, \quad (1.1)$$

where ψ represents the top-bottom doublet, κ_{tc} and κ_1 are related respectively to the top-color and $U(1)$ gauge-couplings squared, and where (for convenience) we have assumed that the top-color and $U(1)$ gauge-boson masses are comparable and of order M . The first term in equation (1.1) arises from the exchange of top-color gauge bosons, while the second term arises from the exchange of the new $U(1)$ hypercharge gauge boson which has couplings proportional to the ordinary hypercharge couplings. In order to produce a large top quark mass without giving rise to a correspondingly large bottom quark mass, the combination of the top-color and extra hypercharge interactions are assumed to be critical in the case of the top quark but not the bottom quark. The criticality condition¹ for top quark condensation in this model is then:

$$\kappa_{eff}^t = \kappa_{tc} + \frac{1}{3}\kappa_1 > \kappa_c = \frac{3\pi}{8} > \kappa_{eff}^b = \kappa_{tc} - \frac{1}{6}\kappa_1. \quad (1.2)$$

In order that the dynamical scale of the theory, M , be of order 1 TeV while the top quark mass is of order 175 GeV, some degree of fine-tuning is required. We can quantify the amount of fine-tuning involved to keep the top quark lighter than M in terms of

$$\frac{\Delta\kappa_{eff}}{\kappa_c} = \frac{\kappa_{eff}^t - \kappa_c}{\kappa_c}. \quad (1.3)$$

¹The criticality condition given in ref. [1] is inconsistent with the interaction Lagrangian in equation (1.1), i.e. equation (4) of ref. [1]. In the large- N_c limit, $\kappa_c = \pi/3$.

The gap-equation for the Nambu–Jona-Lasinio model implies that

$$\frac{\Delta\kappa_{eff}}{\kappa_c} = \frac{\frac{m_t^2}{M^2} \log \frac{M^2}{m_t^2}}{1 - \frac{m_t^2}{M^2} \log \frac{M^2}{m_t^2}}. \quad (1.4)$$

For $m_t=175$ GeV and $M=1$ TeV, we find that

$$\frac{\Delta\kappa_{eff}}{\kappa_c} \approx 12\%. \quad (1.5)$$

As emphasized by Hill, since M is of order 1 TeV, it is possible to accommodate a top quark mass of 175 GeV without excessively fine-tuning κ_{eff}^t .

In this paper we will argue that the extra $U(1)$ interaction in the TC² model is likely to also couple to technifermions in an isospin-breaking fashion. We will show that this produces a potentially large contribution to² $\Delta\rho_* \equiv \alpha T$, [2, 3] even if one assumes that there is *no* isospin breaking in the technifermion masses. Thus the isospin breaking $U(1)$ interaction in such models is likely to have to be weak. If this $U(1)$ interaction is weak, however, equation (1.2) implies fine-tuning of the top-color coupling κ_{tc} . We will show that either the tuning required to keep the top quark light compared to M (equation (1.4)), or that required for κ_{tc} because the $U(1)$ coupling is weak

$$\frac{\Delta\kappa_{tc}}{\kappa_c} = \left| \frac{\kappa_{tc} - \kappa_c}{\kappa_c} \right| \leq \frac{1}{3} \frac{\kappa_1}{\kappa_c}, \quad (1.6)$$

must be finer than 1%.

We further show that, independent of any assumption about the $U(1)$ couplings of the technifermions, the new top-color interactions for the top quark are themselves phenomenologically unacceptable at low scales. In particular, assuming that the top-bottom splitting is the *only* source of weak isospin breaking, if the top-color interactions are strong enough to produce a top-condensate, then the bound on $\Delta\rho_*$ requires that the mass of the top-color boson be larger than about 1.4 TeV. This bound is sufficient to rule out the possibility that these new gauge bosons produce a significant enhancement [4] of the top production cross section at FNAL.

In the next section we study the contributions to $\Delta\rho_*$ from technifermion loops. Next we calculate the contribution to $\Delta\rho_*$ from the new strong top interactions employed in the TC² model. Finally we compare these results with experimental data, and present our conclusions.

2 Technifermion Loops

Since the TC² model relies largely on technicolor to provide masses for the W and Z , the technifermions in the same weak doublets must be approximately degenerate.

²We are distinguishing here between isospin breaking that is present in the standard model, $\Delta\rho$, and isospin breaking from new physics, $\Delta\rho_*$.

It is difficult to achieve this degeneracy in any technicolor model, since the technifermions typically have some couplings to the top and bottom quarks, and thus the required isospin breaking dynamics can feed into technifermion mass splittings, and produce a contribution to $\Delta\rho_*$ [5]. Even though such a contribution is potentially dangerous, the effect is extremely difficult to calculate, and is quite model dependent, so we will not attempt to calculate it here.

However, even if the technifermions are degenerate, there will be additional significant (and positive) contributions to $\Delta\rho_*$ [6] if the additional $U(1)$ interaction couples to the right-handed technifermions in a manner that violates custodial symmetry. Note that some of the technifermions must couple to the additional $U(1)$: part of the top quark mass must come from ETC interactions in order to prevent the appearance of an extra set of light isotriplet Goldstone bosons. If the ETC interactions commute with $SU(2)_W$ and if the bottom quark also has a mass coming from the ETC interactions, then (since the top and bottom quarks must have differing $U(1)$ charges to allow for their different masses) the different right-handed technifermions to which top and bottom quarks couple must have different (and hence isospin-violating) charges.

In order to avoid these contributions to $\Delta\rho_*$, one must have custodially invariant $U(1)$ couplings to the technifermions. This will be difficult to accomplish while still evading the manifold anomaly cancellation constraints. (In order to do so, one will need to introduce exotic charges for the technifermions (cf. Bouchiat, Iliopoulos, and Meyer [7])). Furthermore, one must also allow for the third generation, which carries the new $U(1)$, to mix with the first and second generations, which carry custodial-symmetry violating hypercharge.

In this section we show that, if the additional $U(1)$ interactions violate custodial symmetry, the $U(1)$ coupling will have to be quite small to keep this contribution to $\Delta\rho_*$ small. We will illustrate this in the one-family technicolor [8] model, assuming that techniquarks and technileptons carry $U(1)$ -charges proportional to the hypercharge of the corresponding ordinary fermion³. We can rewrite the effective $U(1)$ interaction of the technifermions as

$$\mathcal{L}_{4T1} = -\frac{4\pi\kappa_1}{M^2} \left[\frac{1}{3} \bar{\Psi} \gamma_\mu \Psi + \bar{\Psi}_R \gamma_\mu \sigma^3 \Psi_R - \bar{L} \gamma_\mu L + \bar{L}_R \gamma_\mu \sigma^3 L_R \right]^2, \quad (2.1)$$

where Ψ and L are the techniquark and technilepton doublets respectively. Only the chiral piece of the four-fermion interactions will contribute to $\Delta\rho_*$.

The shift in the ratio of (zero-momentum) W and Z masses from physics beyond the standard model is given by [3]:

$$\Delta\rho_* \equiv \alpha T = \frac{e^2}{s^2 c^2 M_Z^2} \left(\Pi_{11}(q^2 = 0) - \Pi_{33}(q^2 = 0) \right), \quad (2.2)$$

³Note that this choice is anomaly-free.

where $\Pi_{11}(q^2)$ and $\Pi_{33}(q^2)$ are the contributions from new physics to the coefficients of $ig_{\mu\nu}$ in the W and Z vacuum polarizations, with the gauge couplings factored out. Following the standard notation, e is the electromagnetic gauge coupling, s and c are the sine and cosine of the weak angle, and M_Z is the mass of the Z gauge boson.

If the technifermions are degenerate, then the only contribution [6] to $\Delta\rho_*$ will come from the square of the right-handed interaction in equation (2.1). Thus the technifermion contribution to the vacuum polarization $\Pi_{33}(0)$ is just the product of two vacuum polarizations (see figure 1):

$$\begin{aligned}\Pi_{33}^T(0) &= -\frac{8\pi\kappa_1}{M^2} \left(\frac{1}{2} \text{Tr}(\sigma^3\sigma^3) \Pi_{LR}^T(0) \right)^2 \\ &= -\frac{2\pi\kappa_1}{M^2} (N_c + 1)^2 f^4 ,\end{aligned}\tag{2.3}$$

where the technipion decay constant, f , is related to the electroweak symmetry breaking scale, $v = 246$ GeV, by:

$$v^2 = (N_c + 1)f^2 .\tag{2.4}$$

Thus the contribution to $\Delta\rho_*$ from degenerate technifermions is:

$$\begin{aligned}\Delta\rho_*^T &= 8\pi\kappa_1 \frac{v^2}{M^2} \\ &= 152\% \kappa_1 \left(\frac{1 \text{ TeV}}{M} \right)^2 .\end{aligned}\tag{2.5}$$

We see immediately that, if M is of order 1 TeV, κ_1 must be extremely small. We will see precisely how small in section 4.

3 Top Quark Loops

Introducing new interactions for the top and bottom quarks will also generate additional contributions to $\Delta\rho_*$. In this section, we will focus on the contributions to $\Delta\rho_*$ coming from loops of top and bottom quarks.

The contribution to $\Delta\rho_*$ from one top-color gauge boson exchange (expressed in terms of vacuum polarizations for left-handed currents) is:

$$\Delta\rho_*^{\text{tc}} = \frac{e^2}{s^2 c^2 M_Z^2} \left(\frac{1}{2} \Pi_{LL}(q^2 = 0, m_t, m_b) - \frac{1}{4} \Pi_{LL}(q^2 = 0, m_t, m_t) \right) ,\tag{3.1}$$

where we have made explicit the dependence on the fermions (i.e. top and bottom) propagating in the two-loop graph⁴ (see figure 2). We are neglecting terms

⁴This effect is suppressed by f_t^2/M^2 and is not included in the leading order (in $1/M^2$) analysis of the gap equation.

suppressed by $(m_b/m_t)^2$. In the Nambu–Jona-Lasinio approximation we are considering, the two-loop vacuum polarization calculation reduces to the product of two one-loop vacuum polarizations. Keeping only (universal) logarithmically divergent terms in each of these one-loop pieces we find:

$$\Pi_{LL}(q^2 = 0, m_t, m_t) = -\frac{64\pi\kappa_{tc}}{9} \frac{f_t^4}{M^2}, \quad (3.2)$$

$$\Pi_{LL}(q^2 = 0, m_t, m_b) = -\frac{8\pi\kappa_{tc}}{9} \frac{f_t^4}{M^2}, \quad (3.3)$$

where

$$f_t^2 \equiv \frac{N_c}{8\pi^2} m_t^2 \log\left(\frac{M^2}{m_t^2}\right). \quad (3.4)$$

Note that, with this definition, in the TC² model the one-top-loop piece of the Z self energy contributes

$$\frac{e^2}{4s^2c^2} f_t^2 \quad (3.5)$$

to M_Z^2 .

Putting everything together we find the top-color exchange contribution to be:

$$\Delta\rho_*^{\text{tc}} = \frac{4\pi e^2\kappa_{tc}}{3s^2c^2} \frac{f_t^4}{M_Z^2 M^2}. \quad (3.6)$$

Since κ_1 must be small, equation (1.2) implies that κ_{tc} must be close to $3\pi/8$. Taking $\kappa_{tc} \approx 3\pi/8$, $m_t = 175$ GeV and $M = 1$ TeV, we have:

$$\Delta\rho_*^{\text{tc}} \approx 0.53\%, \quad (3.7)$$

which is about one half of the one-loop top quark contribution to $\Delta\rho$ in the standard model. Note that this contribution is nonzero, even though the top-color interactions themselves are *isospin-symmetric* – isospin violation is present in the form of the t - b mass splitting. We will see in the next section that experimental data constrain $\Delta\rho_*$ to be less than this value.

There is also a contribution to $\Delta\rho_*$ from the $U(1)$ interaction with one top quark loop and one technifermion loop. Assuming (again) that the techniquarks are degenerate, this contribution is:

$$\Delta\rho_*^{\text{T,t}} = 8\pi\kappa_1 \frac{f_t^2}{M^2}. \quad (3.8)$$

Thus for $m_t = 175$ GeV and $M = 1$ TeV:

$$\Delta\rho_*^{\text{T,t}} = 10.2\% \kappa_1. \quad (3.9)$$

The contribution from one $U(1)$ gauge boson exchange with two quark loops is even smaller.

4 Comparison with Data

In order to extract the S and T parameters, we have performed (using the procedure described in ref. [9]) a global fit to precision electroweak data. We have used the most precise measurement [10] of $\alpha_s(M_Z^2) = 0.115 \pm 0.002$. In figure 3 we show the ellipse in the $S - T$ plane which projects onto the 95% confidence interval on the T axis. This gives the range $-0.25 < T < 0.52$ ($-0.19\% < \Delta\rho_* < 0.40\%$) as the 95% confidence interval. Using a larger value for α_s results in an even tighter bound on T . For example, using $\alpha_s(M_Z^2) = 0.124$ (which is obtained from a global fit to precision electroweak data [11]) we find $-0.46 < T < 0.30$ ($-0.36\% < \Delta\rho_* < 0.23\%$).

If top-color exchange (with top and or bottom quarks in the loops, equation (3.6)) was the only contribution to $\Delta\rho_*$, the mass of the top-color gauge boson would have to be larger than about 1.4 TeV (using $\alpha_s(M_Z^2) = 0.124$ raises this bound to 2.2 TeV), in order for $\Delta\rho_*$ to remain acceptably small. While this is still of order 1 TeV, the bound is sufficient to rule out the possibility that these new gauge bosons produce a significant enhancement [4] of the top production cross section at FNAL⁵.

If we require that the total contribution to $\Delta\rho_*$ (the sum of equations (2.5), (3.6), and (3.8)) is less than 0.40%, we bound κ_1 from above and M from below. When κ_1 is small the top-color coupling must be adjusted carefully (see equation (1.6)). When M is large, more fine-tuning is required to keep the top quark light (equation (1.4)). For $M > 1.4$ TeV, we find that either $\Delta\kappa_{tc}/\kappa_c$ or $\Delta\kappa_{eff}/\kappa_c$ must be tuned to less than 1.0%. This trade-off in fine tunings is displayed in figure 4. For the “best” case where both tunings are 1.0%, $M = 4.5$ TeV. (If we require $\Delta\rho_*$ be less than 0.23%, we find that the couplings must be adjusted to approximately 0.7% and $M = 5.3$ TeV for the “best” case scenario.)

5 Conclusions

We have found that the TC² model can produce potentially dangerous shifts in the W and Z masses. If the new $U(1)$ interaction has custodial violating couplings to the technifermions, the experimental limit on isospin breaking in the W and Z masses forces the coupling of the $U(1)$ gauge boson to be small or the scale of the new interactions to be large. A small $U(1)$ coupling would imply a certain amount of fine-tuning. In particular, for the technifermions in the one-family model and $U(1)$ couplings proportional to hypercharge, we find that either the effective top quark coupling or the top-color coupling must be adjusted to an accuracy of at least 1%. For generic gauge boson masses (i.e. away from $M = 4.5$ TeV), the fine-tuning required is even more severe. Of course, with enough fine-tuning one

⁵Of course, the NJL calculations can only be taken as an approximation to the calculation in the full top-color theory. However, in order for top-color production to significantly enhance top quark production we need M to be of order 400 - 500 GeV [12].

could arbitrarily increase the top-color scale and dispose of the problem of isospin breaking, but the primary motivation of the model was to avoid fine-tuning.

These bounds can be mitigated if the $U(1)$ couplings do not violate custodial symmetry. We believe that constructing such a model will be difficult, given that one must cancel all gauge anomalies and allow for mixing between the third generation with the first two.

Independent of whether the $U(1)$ couplings to technifermions violate custodial symmetry, the contribution to $\Delta\rho_*$ from top-color exchange implies that the top-color gauge boson mass must be greater than approximately 1.4 TeV.

There are analogous effects in strong ETC models [13], for a recent example see ref. [14]. There, as well, the direct effect on $\Delta\rho_*$ from ETC gauge boson exchange can be suppressed at the expense of fine-tuning, while the lack of isospin symmetry in the technifermion spectrum remains problematic. The bound on the mass of the top-color gauge boson that we obtained from the contribution of top and bottom quark loops (discussed in section 3) can be directly taken over to the strong ETC case.

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Figure Captions

Figure 1. The $U(1)$ gauge boson contribution to the Z vacuum polarization.

Figure 2. The top-color gauge boson contribution to the W and Z vacuum polarizations, with top and bottom quarks in the loop.

Figure 3. The ellipse in the S - T plane which projects onto the 95% confidence range for T .

Figure 4. The amount of fine-tuning required in the TC^2 model. The dashed line is the amount of fine-tuning in $\Delta\kappa_{eff}$ required to keep m_t much lighter than M , see equation (1.4). The solid curve shows the amount of fine-tuning (see equation (1.6)) in $\Delta\kappa_{tc}$ required to satisfy the bound $\Delta\rho_* < 0.4\%$. The region excluded by the experimental constraint on $\Delta\rho_*$ is above the solid curve.