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MONOPOLE NON-ANNIHILATION AT THE ELECTROWEAK SCALE—NOT!

Evalyn Gates, Lawrence M. Krauss*, and John Terning†
Center for Theoretical Physics
Sloane Physics Laboratory
Yale University
New Haven, CT 06511

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Abstract

We examine the issue of monopole annihilation at the electroweak scale induced by flux tube confinement, concentrating first on the simplest possibility—one which requires no new physics beyond the standard model. Monopoles existing at the time of the electroweak phase transition may trigger W condensation which can confine magnetic flux into flux tubes. However we show on very general grounds, using several independent estimates, that such a mechanism is impotent. We then present several general dynamical arguments limiting the possibility of monopole annihilation through any confining phase near the electroweak scale.

*Also Department of Astronomy. Research supported in part by the NSF, DOE, and the Texas National Research Laboratory Commission. Bitnet: Krauss@Yalehep.
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The “monopole problem” has been with us since the advent of Grand Unified Theories (GUTs), which allow the formation of these non-singular stable topological defects when a semi-simple gauge group is broken to a lower symmetry group that includes an explicit $U(1)$ factor. These objects typically have a mass $m_M \simeq m_X/\alpha$, where m_X is the mass of the gauge bosons in the spontaneously broken GUT theory, and α is the fine structure constant associated with the gauge coupling of the theory.

Shortly after it was recognized that monopoles could result as stable particles in spontaneously broken GUT models [1], and also that they would be produced in profusion during the phase transition associated with the GUT symmetry breaking in the early universe [2], it was also recognized that they posed a potential problem for cosmology. Comparing annihilation rates with the expansion rate of the universe after a GUT transition, it was shown [3, 4] that the monopole to photon ratio would “freeze out” at a level of roughly 10^{-10}. Not only would such an initial level result in a cosmic mass density today which is orders of magnitude larger than the present upper limit, but direct observational limits on the monopole abundance in our neighborhood are even more stringent [5].

This cosmological problem was one of the main motivations for the original inflationary scenario[6]. However one of the chief challenges to the original inflationary solution of the monopole problem was the necessity of having a reheating temperature which is high enough to allow baryogenesis, but low enough to suppress monopole production. In addition, recent work on large scale structure, including observed galaxy clustering at large scales, large scale velocity flows, and the absence of any observable anisotropy in the microwave background, has put strong constraints on such models.

With the recent recognition that even something as exotic as baryogenesis may be possible within the context of the standard electroweak theory (supplemented by minor additions), it is worth examining the issue of whether the monopole problem may be resolved purely through low energy physics. A canonical method by which one might hope to achieve complete annihilation is by confining monopole-antimonopole pairs in flux tubes, such as might occur if $U(1)_{em}$ were broken during some period. Proposals along this line, based on introducing new physics have been made in the past, eg. [7, 10]. The possibility that plasma effects might also play a role in producing an effective confining potential at early times has also been proposed[8, 9]. Most recently, the possibility that such a phase might briefly occur near the electroweak breaking scale, for multi-Higgs models, has also been raised [12]. By far the simplest possibility, however, is that flux tube confinement of monopoles might occur in the standard model at relatively low energies, unsupplemented by any new physics. We explore this issue in detail here, and then go on to examine the general dynamical obstacles facing any model involving monopole confinement at the electroweak scale.

1. Monopole Confinement in the Standard Model:

It has been known for some time that the electroweak vacuum in the broken phase is unstable in the presence of large ($\geq m_W^2/e$) magnetic fields[13]. The instability is due to the coupling between the magnetic field H and the magnetic moment of the massive W gauge bosons. Due to this coupling the effective mass of the W at tree level is

$$m_{W_{\text{eff}}}^2 = m_W^2 - eH \tag{1}$$

where $e = g \sin\theta_W$ (all expressions are given in Heaviside-Lorentz units for electromagnetism). This effective mass squared becomes negative for $H_c^{(1)} \geq m_W^2/e$. The general resolution [13] of the instability is the formation of a condensate of W and Z bosons, which sets up currents that antiscreen the magnetic field. The vacuum then acts as an anti-type II superconductor, and the energy is minimized by the formation of a periodic network of magnetic flux tubes. As we shall describe in some length later, Ambjørn and Olesen have also shown, at least for the special case $m_H = m_Z$, that if the magnetic field increases above $H_c^{(2)} = \frac{m_W^2}{e\cos^2\theta_W}$, the full $SU(2)_L \otimes U(1)$ symmetry is restored [15]. (Top quark loops, for a sufficiently heavy top quark, might affect the field at which symmetry is restored[14], but this issue is not yet resolved, and in any case will not affect the arguments presented here.) Thus for an external magnetic field $H_c^{(1)} < H < H_c^{(2)}$, the electroweak vacuum passes through a transition region where a W condensate exists and the magnetic field is confined in a periodic network of flux tubes.

It is possible to imagine how such a phase might arise naturally in a way which might lead to monopole-antimonopole annihilation at the electroweak scale in the early universe (This idea has also been suggested elsewhere in the literature [11]). First of all, the magnetic field necessary to produce such a phase could come from the monopoles themselves, provided the electroweak transition is second order, or sufficiently weakly first order. In the approximation of a second order transition, the mass of the W boson generically has a temperature dependence of the form

$$m_W^2(T) \approx m_W^2(0)[1 - T^2/T_c^2] \tag{2}$$

where T_c (≈ 300 GeV) is the critical temperature associated with electroweak breaking. Thus, just below the transition temperature T_c, relatively small magnetic fields could trigger W condensation. (This argument carries through if the transition is sufficiently weakly first order so the VEV of the scalar field is small in the broken phase which nucleates.) A remnant density of GUT-scale monopoles could provide such a magnetic field. Once the condensate forms, monopoles would become confined to the network of flux tubes, whose width is related to the W mass, as we shall describe. Once the width of the flux tubes is of the same order as the distance between monopoles, the monopoles would experience a linear potential and begin to move towards each other. If the flux tubes exist for a sufficiently long time, the monopoles could annihilate, and their density would correspondingly decrease.

This picture is very attractive in principle. However, we now demonstrate, using a series of arguments which probe this scenario in successively greater detail, that the parameters associated with such a transition at the electroweak scale generally preclude it from being operational. Moreover, we present dynamical arguments relevant for any scenario involving monopole annihilation via flux tube confinement at the electroweak scale.

2. Kinemetic Arguments: Non-annihilation via Magnetic Instabilities:

(a) A Global Argument: In figure 1, we display a phase diagram describing the W condensation picture discussed above, as a function of both temperature T and background magnetic field H. At $T = 0$, for the case examined by Ambjørn and Oleson[15, 16, 17], in the region $1 < He/m_W^2 < 1/\cos^2\theta_W$ a magnetic flux tube network extremizes the energy and both the ϕ (Higgs) and $|W|^2$ fields develop non-zero expectation values. For finite temperature the phase boundaries evolve as shown, in response to the reduction in the W mass with temperature, up to $T = T_c$, where they meet. Thus, the phase in which flux tubes and a W condensate are energetically preferred falls in between these two curves.

While the actual magnetic field due to the presence of a density of monopoles and anti-monopoles will be complicated and inhomogenous, we first approximate it by a homogenous mean field H_m, whose precise value is not important for this discussion. (As we will later show, given the remnant density of monopoles predicted to result from a GUT transition, the value of this field will be well below the zero temperature critical field $m_W^2(0)/e$ at the time of the electroweak phase transition.) As the universe cools from above T_c, this background magnetic field will eventually cross the upper critical curve for the existence of a flux tube phase.

We now imagine that immediately after this happens, flux tubes form, and monopole annihilation instantaneously begins. We shall later show that this is far from the actual case. Nevertheless, this assumption allows us to examine constraints on monopole annihilation even in the most optimistic case. As monopole-antimonopole annihilation proceeds, the mean background magnetic field falls quickly. At a certain point this mean field will fall *below* the lower critical curve, and if it is this background field which governs the energetics of W condensation, the W condensate will then become unstable, the magnetic field lines will once again spread out, and monopole-antimonopole annihilation will cease. As can be seen in the figure, the net reduction in the magnetic field expected from this period of annihilation will be minimal. Quantitatively the final field (neglecting dilution due to expansion during this period) will be a factor of $\cos^2\theta_W$ smaller than the initial field. This is hardly sufficient to reduce the initial abundance of monopoles by the many orders of magnitude required to be consistent with current observations.

(b) A Local Argument: The above argument points out the central problem for a monopole annihilation scenario based on magnetic field instabilities at the electroweak scale. In order to arrange for flux tubes to form, and confinement of monopoles to occur, the field must be tuned to lie in a relatively narrow region of parameter space. Nevertheless, a potential problem with the above argument, even if it were less sketchy, is that flux tube formation, and monopole annihilation, may more likely be related to local and not global field strengths. For example, even if the globally averaged magnetic field is reduced by annihilation, the local field between a monopole-antimonopole pair connected by a flux tube may remain above the critical field, so that the flux tube will presumably persist, and annihilation can proceed. We now demonstrate that even under the most optimistic assumptions about the magnitude of local fields, for almost all of electroweak parameter space, local flux tube formation at a level capable of producing a confining potential between monopole- antimonopole pairs will not occur. We first consider the case for which solutions (involving a periodic flux tube network) were explicitly obtained by Ambjørn and Oleson[16].

The area A of flux tubes forming due to the W condensate can be obtained by minimizing the classical field energy averaged over each cell in the periodic network in the presence of a background H field [16]:

$$\overline{\mathcal{E}}_{min} A = \frac{m_{\mathrm{W}}^2}{e}\int_{\mathrm{cell}} \mathrm{f}_{12} d^2x - \frac{m_{\mathrm{W}}^4}{2e^2}A + \left(\lambda - \frac{g^2}{8\cos^2\theta_W}\right)\int \left(\phi^2 - \phi_0^2\right)^2 d^2x, \qquad (3)$$

where f_{12} is the magnetic field, and λ is the ϕ^4-coupling in the Lagrangian, and ϕ_0 is the Higgs VEV. Utilizing the topological restrictions on the flux contained in the flux tubes (containing minimal flux $2\pi/e$),

$$\int_{cell} f_{12} d^2x = \oint \vec{A}\cdot d\vec{\ell} = 2\pi/\mathrm{e}, \qquad (4)$$

this yields an expression for A, determined by the energy density $\overline{\mathcal{E}}_{min}$, which is in turn a function of the external magnetic field:

$$A = \frac{2\pi m_{\mathrm{W}}^2}{e^2\left[\overline{\mathcal{E}}_{\mathrm{min}} + m_{\mathrm{W}}^4/2e^2 - \left(\lambda - \frac{g^2}{8\cos^2\theta_W}\right)\int \left(\phi^2 - \phi_0^2\right)^2 d^2x\right]}. \qquad (5)$$

Taking the Bogomol'nyi limit[18] $\lambda = \frac{g^2}{8\cos^2\theta_W}$, corresponding to $m_H = m_Z$, the classical field equations simplify, and the properties of the flux tubes can be derived. In particular, one can show [15] that the area of the flux tubes is restricted to lie in the range

$$2\pi\cos^2\theta_W < A m_W^2 < 2\pi. \qquad (6)$$

From our point of view, it is important to realize that this result is equivalent to the statement that a W condensate can only exist between the two critical values of the magnetic field

$$\frac{m_W^2}{e\cos^2\theta_W} > H > \frac{\mathrm{m}_\mathrm{W}^2}{e}. \tag{7}$$

Moreover, it gives a one to one correspondence between the area of the flux tube and the background magnetic field value in this range. We shall use this correspondence, both in the Bogomol'nyi limit and beyond, to examine the confinement properties of such a flux tube network connecting monopole-antimonopole pairs.

Magnetic monopoles are formed at the GUT transition with a density of about one monopole per horizon volume. This corresponds to a value of $\frac{n_M}{s} = 10.4{g_*}^{1/2}(T_{GUT}/M_{Pl})^3 \sim 10^2(T_{GUT}/M_{Pl})^3$, where n_M is the number density of monopoles, g_* is approximately the number of helicity states in the radiation at the time t_{GUT}, M_{Pl} is the Planck mass, and s is the entropy of the universe at this time. Since T_{GUT} could easily exceed 10^{15} GeV for SUSY GUTs, it is quite possible that the initial monopole abundance left over from a GUT transition is $\frac{n_M}{s} > 10^{-10}$. Preskill has shown that in this case monopoles will annihilate shortly after the GUT transition until $\frac{n_M}{s} \sim 10^{-10}$[3], and this value remains constant down to the electroweak scale. Since $s = (2\pi^2/45)g_*T^3$, the monopole number density at the electroweak transition ($T_c \sim 300GeV$) of $\approx 0.13\ GeV^3$ (assuming $g_*(T_c) \approx 100$) corresponds to a mean intermonopole spacing of $L \approx 2\ GeV^{-1}$. [1] From this, we can calculate the mean magnetic field produced by the monopoles with Dirac charge $h = 2\pi/e$. In general, because the monopole background is best described as a "plasma" involving both monopoles and antimonopoles, the mean magnetic field will be screened at distances large compared to the intermonopole spacing. However, because we will demonstrate that even under the most optimistic assumptions, monopole-antimonopole annihilation will not in general occur, we ignore this mean field long-range screening, and consider the local field in the region between a monopole-antimonopole pair to be predominantly that of nearest neighbors, i.e. a magnetic dipole. While the field is not uniform in the region between the monopole and antimonopole, we will be

[1] Note that monopoles actually begin to dominate the mass density of the universe somewhat before this temperature if they start with an initial value of $\frac{n_M}{s} > 10^{-10}$. As we later describe, this can have dramatic effects upon the expansion rate at this temperature.

interested in the minimum value of the field here. We shall make the (optimistic) assumption that if this field everywhere exceeds the critical value m_W^2/e on the line joining the two monopoles, that an instability of the type described above, involving a condensate of W fields and an associated magnetic flux tube, can occur along this line.

For a monopole-antimonopole pair separated by a distance L, the minimum field will be halfway between them, and will have a magnitude $H = 2h/\pi L^2 = 4/eL^2$. For this field to exceed the minimum Ambjørn-Oleson field m_W^2/e then implies the relation: $L < 2/m_W$. For a value $m_W = 81 GeV$ this relation is manifestly not satisfied for the value of L determined above. However, assuming a second order transition, as we have described, the W mass increases continuously from zero as the temperature decreases below the critical temperature, implying some finite temperature range over which the (fixed) background field due to monopoles will lie in the critical range for flux tube formation. In this case, the magnetic field would enter this range from *above*. In order that the magnetic field lie in the range given by inequality (7), we find

$$2/m_W > L > 2\cos\theta_W/m_W. \tag{8}$$

Nevertheless, even if a flux tube forms connecting the monopole-antimonopole pair, this will not result in a confining linear potential until the width of the flux tube $2r < L$. A bound on this width can be obtained from the lower bound on the area of the flux tube (equation (6)):

$$2r > 2\sqrt{2}\cos\theta_W/m_W \ , \tag{9}$$

when the magnetic field is at its upper critical value of $m_W^2/e\cos^2\theta_W$. This implies the constraint

$$L > 2\sqrt{2}\cos\theta_W/m_W \ . \tag{10}$$

As can be seen, inequalities (8) and (10) are mutually inconsistent. Hence, there appears to be no region in which both a Ambjørn-Oleson type superconducting phase results, and at the same time monopole-antimonopole pairs experience a confining potential. We expect the situation will be similar to the quark-hadron phase transition when the transition is second order. In that case, it is impossible to distinguish between a dense plasma of confined quarks and a gas of free quarks, because the mean interquark spacing is small compared to the confinement scale. Here there will be no physical impact of a short superconducting phase, because the confinement scale is larger than the distance between monopoles required to trigger the phase transition. We expect no significant monopole annihilation during the short time in which this phase is dynamically favored as the W mass increases.

This result has been derived in the Bogomol'nyi limit, when $m_H = m_Z$. What about going beyond this limit? First, note that the energy density of the external magnetic field, $\mathcal{E} = H^2/2$, provides an upper bound on $\overline{\mathcal{E}}_{min}$. Then from equation (5) one can show that as long as $\lambda > g^2/8\cos^2\theta_W$ $(m_H > m_Z)$, the flux tube area, for a fixed value of the field, is *larger* than it is in the Bogomol'nyi limit. While we have no analytic estimate of the upper critical field, and hence no lower bound on the flux tube area, the scaling between area and magnetic field will still be such that for a given monopole-antimonopole spacing, and hence a given magnetic field strength, the area of the corresponding flux tube will be larger than in the Bogomol'nyi limit. Hence the inconsistency derived above will be exacerbated. Only in the narrow range $m_Z/2 \lesssim m_H < m_Z$ (still allowed by experiment) is there a remote possibility that even in principle, flux tube areas may be reduced sufficently so that confining potentials may be experienced by monopoles triggering a W condensate. However, in this range, the energy (4) can be reduced by increasing ϕ, so we expect that instabilities arise in this range which are likely to make a W condensate unstable in any case.

3. Dynamical Arguments Against Annihilation:

Even if a confining potential may be achieved through flux tube formation, there are dynamical reasons to expect monopole annihilation will not be complete. These arguments apply to any scenario involving a confining phase for monopoles, and suggest that estimates based on the efficacy of monopole annihilation may be overly optimistic. In the first place, we can estimate the energy of a monopole-antimonopole pair separated by a string of length L. For a long flux tube of radius

r, considerations of the electromagnetic field energy trapped in the tube imply a net energy stored in the flux tube of

$$E = \frac{L}{2\alpha r^2}. \tag{11}$$

Considering the case when $L \approx 2r$, when confinement would first begin, we find the energy associated with the string tension is $E = \frac{2}{\alpha L} \approx 130\ GeV$. This is significantly smaller than the mean thermal energy associated with a transition temperature $T_c \approx 300\ GeV$. Hence, if the string tension does not vary significantly over the period during which the magnetic field exceeds the critical field, the string tension exerts a minor perturbation on the mean thermal motion of monopoles, and will not dramatically affect their dynamics. The only way this would not be the case would be if the monopole-antimonopole pair moved towards one another at a rate which could keep the magnetic field between them sufficiently large so as to track the increase in the minimum critical field as $m_W(T)$ increased to its asymptotic value. However, this cannot in general occur, because thermal velocities are sufficiently large so as to swamp the motion of the monopole-antimonopole towards each other. Using the mean thermal relative velocity of monopoles at $T = T_c$, we can calculate how much time, δt, it would take to traverse a distance equal to the initial mean distance between monopoles. This time becomes an ever smaller fraction of the horizon time, t, as the universe expands. Since the thermal velocity is $\ll 1$, non-relativistic arguments are sufficient. We find $\delta t/t \approx 6 \times 10^{-4}$, for $m_M \approx 10^{17} GeV$, and $T_c \approx 300\ GeV$. During such a small time interval, $m_W(T)$ remains roughly constant, and hence so does the string tension. We find that during the time δt the flux tube induced velocity of the monopole-antimonopole pair remains a small fraction of the mean thermal velocity, for $m_M > 10^{15} GeV$. Thus, monopoles and antimonopoles will not in general move towards one another as m_W increases. Since $r(T)$ will not change significantly between $H_c^{(1)}$ and $H_c^{(2)}$ as m_W increases, if the mean inter-monopole spacing remains roughly constant, monopole annihilation will, on average, not proceed before the field drops below its critical value.

What about the more general case of a brief superconducting phase which might result if $U(1)_{em}$ is broken for a small temperature range around the electroweak scale [12]? In this case, the flux tube area is not driven by the strength of the background magnetic field, and hence is not tied to monopole-antimonopole spacing. Nevertheless, dynamical arguments suggest that annihilation, even in this case, may be problematic. We describe three obstacles here: (a) as above, the field energy contained in the string may not be enough to significantly alter the dynamics of a thermal distribution of monopoles; (b) even in the event that this energy is sufficiently large, the time required to dissipate this energy will in general exceed the lifetime of the universe at the time of the $U(1)_{em}$ breaking transition; (c) the time required for monopoles to annihilate even once they have dissipated most of the string energy and are confined within a "bag" may itself be comparable to the lifetime of the universe at the time of the transition.

(a) Consider the energy (11) stored in the flux tube. The radius, r, will depend upon the magnitude of the VEV of the field responsible for breaking $U(1)_{em}$. Until this field achieves a certain minimum value, flux tubes will not be sufficiently thin to produce a confining potential for monopoles. Even if this VEV quickly achieves its maximum value, one must investigate whether or not this field energy is large compared to the thermal energy at that time, in order to determine whether the monopoles will be dynamically driven towards each other. As long as $r^{-1} \approx e\phi_0 \approx eT_c$, where in this case ϕ_0 represents the VEV of the field associated with $U(1)_{em}$ breaking and T_c represents the transition temperature, then $E \gg T$, so that the condition of a confining potential is in general satisfied. Nevertheless, one must also verify that this inequality is such that the Boltzmann tail of the monopole distribution with velocities large enough to be comparable to this binding energy is sufficiently small (i.e. that sufficiently few monopoles have thermal motion which is not significantly affected by the confining potential). If we assume that such monopoles do not annihilate, then to avoid the stringent limits on the monopole density today probably requires $E > O(25-30)T$. (This includes the fact that if monopoles annihilate, since they dominate the energy density of the universe at this time, they will increase the entropy by a factor of up to 10^5, for $m_M \approx 10^{17} GeV$.) Determining L by scaling from the initial density, we find that if $\phi_0 = \rho T_c$, then the ratio of the binding energy to the transition temperature, $E/T_c \approx 3800\rho^2$, independent of T_c. This implies a rather mild constraint on the VEV of the field associated with $U(1)_{em}$ breaking: $\rho > 0.09$.

(b) Monopole's must dissipate the large energy associated with the string field energy if they are to annihilate. There are two possible ways in which this energy can be dissipated: thermal scattering, and the emission of radiation [3, 19]. Utilizing the estimates of energy loss by radiation given by Vilenkin [19] we find that this process requires $\approx 10^{15}$ times longer to dissipate the string energy than the lifetime of the universe at the time of the transition.[2] Hence, we concentrate on the possibility of dissipating the energy by thermal scattering. We shall assume here that $\rho \approx 1$, so that the initial average monopole- antimonopole pair energy is $\approx 3800T$. The energy loss by collisions with thermal particles in the bath is [19] (Note that Callen-Rubakov scattering might also play a role, although its efficiency as an energy loss mechanism for monopoles is questionable. In any case its cross section is also of this order, and thus the estimates given here are appropriate even if this is included.) $dE/dt \approx -bT^2v^2$, where $b = 3\zeta(3)/(4\pi^2)\sum(q_i/2)^2$, and the sum is over all helicity states of charged particles in the heat bath. At $T \approx 100$ GeV, $b \approx 0.7$. Recall that at this time, monopoles will likely dominate the energy density of the universe. Utilizing the appropriate relationship between temperature and time resulting from the faster expansion rate in this case and setting $\eta = \frac{n_M}{S}$, we find

$$E_\infty = E_i \exp\left[\frac{-0.04M_{Pl}}{m_M}\left(\frac{T_i}{\eta m_M}\right)^{1/2}\right] . \tag{12}$$

For $m_M \approx 10^{17}GeV, E_\infty \approx 0.96E_i$, so that monopole-antimonopole pairs will apparently never dissipate their string energy in a matter dominated epoch, and thus *annihilation should not proceed.* Even if somehow the universe were to become radiation dominated at this time, either by monopole annihilation or some other mechanism, then we find instead

$$\ln\left(\frac{E_f}{E_i}\right) = \frac{0.03bM_{Pl}}{2m_M}\ln\left(\frac{t_i}{t_f}\right) . \tag{13}$$

If we take E_f to be the string energy (11) when $L = 2r$, i.e. the energy when the string has become a "bag", then even in this case the the time required to dissipate

[2]This calculation itself is probably an underestimate (unless the monopole couples to massless or light particles other than the photon), since it assumes the photon is massless, which it is not in this phase.

the initial string energy is O(50) t_i for $m_M \approx 10^{17} GeV$.[3] Unless the phase of broken $U(1)_{em}$ lasts for longer than this time (which depends sensitively upon the monopole mass) not all the string energy will be dissipated. We have ignored here possible transverse motion of the string. This energy must also be dissipated by friction, which may be dominated by Aharanov-Bohm type scattering[20].[4] In any case, even under the optimistic assumption that monopoles somehow annihilate sufficiently (reducing the initial abundance by a factor of 2) to result in a radiation dominated universe, this latter result is still a rather severe constraint on the temperature range over which the $U(1)_{em}$ breaking phase must last.

(3) Even if the string energy can dissipated so that the mean distance between monopole-antimonopole pairs is of order of the string width, they will be confined in a "bag", and one must estimate the actual time it takes for the pair to annihilate in such a "bag" state. (The monopole "crust", of characteristic size ${m_W}^{-1}$, is assumed to play a negligible role here. In any case, inside this "bag" it is quite possible that the electroweak symmetry may be restored, in which case such a crust would not be present.) In a low lying s-wave state, the annihilation time is very short. However, in an excited state, involving, for example, high orbital angular momentum (on the bag scale), this may not be the case, since the wave function at the origin will be highly suppressed. We provide here one approximate estimate for the annihilation time based on the observation that the Coulomb capture distance $a_c \approx 1/4\alpha E$ is 8 times smaller than the "bag" size, for a monopole whose "bag" energy is inferred from equation (11) with $L = 2r$. It is reasonable to suppose that annihilation might proceed via collapse into a tightly bound Coulomb state. Thus, for the sake of argument one might roughly estimate a lower limit on the annihilation time by utilizing the Coulomb capture cross section[3] inside the "bag". This capture time

[3]Note that even if the monopole annihilation were to keep the temperature relatively constant for some period while it remained matter dominated, the time at which the universe passes through any temperature could not be longer than that in a universe which had remained radiation dominated throughout.

[4]This latter issue has been raised in concurrent work by R. Holman, T. Kibble, and S.-J. Rey[21], which concentrates on this mechanism while pointing out the potential inefficiency of monopole annihilation. Our results suggest that dissipation of energy in the longitudinal modes of the string can take even longer, thus providing stronger constraints on models, and further support that monopole annihilation is not automatic in a confining phase.

is $\tau \approx (4e/3\pi T)(m_M/T)^{11/10}$, and is slightly longer than the lifetime of the universe at temperature $T \approx 300 GeV$, for $m_M = 10^{17} GeV$. Again, this suggests that the time during which the $U(1)_{em}$ breaking phase endures must be long compared to the lifetime of the universe when this phase begins[5]. If capture into a Coulomb state has not occured by the time the $U(1)_{em}$ breaking phase is over, previously confined monopole pairs separated by more than the Coulomb capture distance will no longer be bound. The annihilation rate for these previously confined pairs compared to the expansion rate will remain less than order unity, so that monopoles will again freeze out.

These considerations suggest that monopole-antimonopole annihilation by flux tube formation at the electroweak scale is far from guaranteed. In particular, monopole confinement triggered by monopole induced magnetic fields seems unworkable. More generally, in any confining scenario, dissipation of the initially large flux tube energies requires times which are generally long compared to the horizon time at the epoch of electroweak symmetry breaking. (If the universe remains matter dominated during this phase, it appears impossible to dissipate the string energy.) At the very least this places strong constraints on the minimum range of temperatures over which a confining phase for monopoles must exist.

[5]If one imagines that because of the monopole outer crust, emission of scalars is possible, the capture cross section may be increased[3] to $\approx (T_c)^{-2}$. This would decrease the capture time by a significant amount ($\approx 10^6$). However, once again, this requires that the scalars are light, otherwise phase space suppression might be important.

Figure Captions

Fig. 1. Phase diagram for W condensation as a function of external magnetic field and temperature assuming a second order electroweak phase transtion.

References

[1] G. 't Hooft, *Nucl. Phys.* **B79**, 276 (1974); A. Polyakov, *JETP Lett.* **20**, 194 (1974).

[2] T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976).

[3] J. Preskill, *Phys. Rev. Lett.* **43**, 1365 (1979).

[4] Ya.A. Zeldovich and M.Y. Khlopov, *Phys. Lett.* **79B**, 239 (1978).

[5] E.N. Parker, *Ap. J.* **160**, 383 (1970); Y. Raphaeli and M.S. Turner, *Phys. Lett.* **B121** 115 (1983); M.E. Huber et. al., *Phys. Rev. Lett.* **64** 835 (1990); S. Bermon et. al, *Phys. Rev. Lett.* **64** 839 (1990).

[6] A. Guth, *Phys. Rev.* **D23**, 347 (1981).

[7] P. Langacker and S.-Y. Pi, *Phys. Rev. Lett.* **45**, 1 (1980).

[8] A. Linde, *Phys. Lett.* **B 96**, 293 (1980)

[9] A. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Publishers (Chur, 1990)

[10] G. Lazarides and Q. Shafi, *Phys. Lett.* **B 94**, 149 (1980).

[11] E.I. Guendelman and D.A. Owen, *Phys. Lett.* **B 235**, 313 (1990).

[12] V.V. Dixit and M. Sher, *Phys. Rev. Lett.* **68**, 560 (1992); T.H. Farris et. al., *Phys. Rev. Lett.* **68**, 564 (1992).

[13] V.V. Skalozub, *Sov. J. Nucl. Phys.* **23**, 113 (1978); N.K. Nielsen and P. Olesen, *Nucl. Phys* **B144**, 376 (1978); J. Ambjørn, R.J. Hughes, and N.K. Nielsen, *Ann. Phys. (N.Y.)* **150**, 92 (1983).

[14] A. Linde, *Phys. Lett.* **B 62** 435 (1976).

[15] J. Ambjørn and P. Olesen, *Nucl. Phys.* **B 330**, 193 (1990).

[16] J. Ambjørn and P. Olesen, *Nucl. Phys.* **B 315**, 606 (1989).

[17] J. Ambjørn and P. Olesen, *Phys. Lett.* **B 257**, 201 (1991); J. Ambjørn and P. Olesen, *Phys. Lett.* **B 218**, 67 (1989); J. Ambjørn and P. Olesen, *Phys. Lett.* **B 214**, 565 (1991).

[18] E.B. Bogomol'nyi, *Sov. J. Nucl. Phys.* **24**, 449 (1977).

[19] A. Vilenkin, *Nucl. Phys.* **B196**, 240 (1982).

[20] M. Alford and F. Wilczek, *Phys. Rev. Lett.* **62**, 1071 (1989)

[21] R. Holman, T. Kibble, S.-J. Rey, preprint, YCTP-P06-92 (Yale), NSF-ITP-09 (Santa Barbara)