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Composite Quarks and Leptons from Dynamical Supersymmetry Breaking without Messengers

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Abstract

We present new theories of dynamical supersymmetry breaking in which the strong interactions that break supersymmetry also give rise to composite quarks and leptons with naturally small Yukawa couplings. In these models, supersymmetry breaking is communicated directly to the composite fields without “messenger” interactions. The compositeness scale can be anywhere between 10 TeV and the Planck scale. These models can naturally solve the supersymmetric flavor problem, and generically predict sfermion mass unification independent from gauge unification.

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1 Introduction

Supersymmetry is arguably the most attractive framework for physics beyond the standard model, but a truly satisfactory and attractive model for supersymmetry breaking has yet to emerge. One reason for dissatisfaction with present models is their “modular” structure: supersymmetry is assumed to be broken in some new sector, and the information that supersymmetry is broken is communicated to the observable fields via messenger interactions, which may be either (super)gravity [1] or standard-model gauge interactions [2, 3].¹

While there is in principle nothing wrong with such modular schemes, it is interesting to ask whether there exist simpler models in which supersymmetry is broken directly in the observable sector. An important obstacle in constructing such a model was pointed out by Dimopoulos and Georgi [5]. They showed that if one assumes (*i*) the gauge group is that of the standard model; (*ii*) no higher-dimension operators in the Kähler potential of the effective Lagrangian; and (*iii*) tree approximation, then there is always a colored scalar lighter than the down quark. Any realistic model of supersymmetry breaking must contain important effects that do not satisfy one of these assumptions. For example, gravity-mediated models violate (*ii*), and gauge-mediated models violate (*iii*). The effects that violate (*ii*) and (*iii*) are generally smaller than tree-level renormalizable effects, but the “modular” structure of these models guarantees that they are the *leading* effects that communicate supersymmetry breaking to the observable sector.

An interesting way to evade the “no go” theorem of Dimopoulos and Georgi without introducing modular structure is to make the observable fields composite, in the sense that they couple to new strong dynamics at a scale Λ above the weak scale.² If the strong dynamics also breaks supersymmetry, assumption (*iii*) will be violated (and the low-energy theory below the scale Λ will violate (*ii*)). We therefore look for a theory with a single sector that breaks supersymmetry dynamically and generates composite fermions.

More specifically, we have the following scenario in mind. Consider a model that breaks supersymmetry by strong interactions at the scale Λ , and suppose that the model has an unbroken global symmetry group G . If there are G^3 anomalies, the theory will have massless composite fermions in the low-energy spectrum to match

¹There has recently been important progress in simplifying models of gauge-mediated supersymmetry breaking [4].

²Supersymmetric composite models of quarks and leptons have been previously constructed [6, 7] but these models require separate sectors for supersymmetry breaking.

the anomalies [8]. It is easy to find such models where the standard-model gauge group G_{SM} can be embedded in G , either because there are no G_{SM}^3 anomalies, or because these anomalies are canceled by “elementary” states. In this case, some of the composite fermions will be charged under G_{SM} and may be identified with quarks and leptons.

If there is no unbroken $U(1)_R$ symmetry, standard model gaugino masses will be generated, suppressed compared to the mass of the composite scalars by a perturbative loop factor, and one must worry about gaugino masses being too small.³ One possibility is that the composite scalars are heavy enough that the gauginos are sufficiently heavy despite the loop suppression factor. This leads naturally to models with a low compositeness scale and a superpartner spectrum similar to that of the “more minimal” models [10]. Another alternative is to assume that the loop factor is compensated by a large multiplicity factor. In fact, in order to generate complete composite generations the global symmetry must be quite large, so a large multiplicity factor is hard to avoid. The large number of states also means that the standard-model gauge group is far from being asymptotically free, but the models can still accommodate perturbative gauge coupling unification if the scale Λ of non-perturbative composite dynamics is near the unification scale. A large value for Λ also helps avoid negative mass-squared terms for standard-model scalars, as we will explain in the text.

The composite nature of some of the standard-model fermions can also help in understanding the small Yukawa couplings for the first two generations. If there are no Yukawa couplings generated by the strong dynamics, all Yukawa couplings must arise from flavor-dependent higher-dimension operators in the fundamental theory suppressed by powers of a scale $M > \Lambda$. In the low-energy theory, these will become Yukawa couplings suppressed by powers of Λ/M .

This class of models makes two interesting generic predictions for the spectrum of superpartner masses. First, the gaugino masses will be lighter than the composite scalars. Second, the composite scalar masses are generated by the strong dynamics, and are therefore invariant under the global symmetry G at the scale Λ . This means that some or all of the soft masses for the composite fields unify at the scale Λ . If supersymmetry is discovered, this prediction can be tested if the scalar masses are accurately measured.

³This killed the models of Ref. [9], which were motivated by very similar considerations as those described above.

2 New theories of dynamical SUSY breaking

In this Section, we describe supersymmetric gauge theories that have local minima with dynamical supersymmetry breaking and composite fermions. These models are similar in some ways to the models considered in Ref. [11], but have some features that are more favorable to the kind of model-building we are interested in. The models have gauge and flavor symmetry group

$$SU(4) \times SU(N) \times [SU(N) \times U(1) \times U(1)_R] \quad (2.1)$$

where the group in brackets is a global symmetry group. The matter content is

$$\begin{aligned} Q &\sim (\square, \square) \times (\mathbf{1}; 1, -\frac{N}{4} - 1) , \\ L &\sim (\bar{\square}, \mathbf{1}) \times (\bar{\square}; -1, \frac{N}{4} + 3 - \frac{8}{N}) , \\ \bar{U} &\sim (\mathbf{1}, \bar{\square}) \times (\square; 0, \frac{8}{N}) , \\ A &\sim (\mathbf{1}, \square) \times (\mathbf{1}; \frac{4}{N-2}, 1) . \end{aligned} \quad (2.2)$$

The theory has a tree-level superpotential

$$W = \lambda L Q \bar{U} . \quad (2.3)$$

Here λ is a matrix that can be viewed as an adjoint spurion for the global $[SU(N)]$ symmetry. Note that there are 4 \square 's and N $\bar{\square}$'s under the global $[SU(N)]$, so the theory has a nonzero $[SU(N)]^3$ anomaly for $N \neq 4$. The analysis of this model is somewhat different depending on whether N is even or odd, so we consider both possibilities in turn.

2.1 Odd N Models

We first consider the case where $N = 2n + 1$ is odd. If we include the effects of the tree-level superpotential, the theory has a classical moduli space that can be parameterized by the gauge invariants (we indicate the $[SU(N)]$ quantum numbers)

$$\begin{aligned} L^4 &\sim \square , \quad (\text{for } N \geq 5) \\ A\bar{U}^2 &\sim \square , \\ \bar{U}^N &\sim \mathbf{1} . \end{aligned} \quad (2.4)$$

with the constraints

$$L^4 \cdot \bar{U}^N = 0, \quad L^4 \cdot A\bar{U}^2 = 0. \quad (2.5)$$

For $N \geq 5$, the classical moduli space has two branches: $\langle \bar{U}^N \rangle \neq 0$ with $\langle L^4 \rangle = 0$ (“baryon branch”), and $\langle L^4 \rangle \neq 0$ with $\langle \bar{U}^N \rangle = 0$ (“lepton branch”). For $N = 3$, only the baryon branch exists.

We first analyze the baryon branch. In terms of the elementary fields, the vacuum expectation values can be written (up to gauge and flavor transformations)

$$\langle Q \rangle = 0, \quad \langle L \rangle = 0 \quad (2.6)$$

$$\langle A \rangle = \sqrt{2} \begin{pmatrix} a_1 \epsilon_2 & & & 0 \\ & \ddots & & \vdots \\ & & a_n \epsilon_2 & 0 \\ 0 & \dots & 0 & 0 \end{pmatrix}, \quad \langle \bar{U} \rangle = \begin{pmatrix} b_1 \mathbf{1}_2 & & & 0 \\ & \ddots & & \vdots \\ & & b_n \mathbf{1}_2 & 0 \\ 0 & \dots & 0 & b \end{pmatrix}, \quad (2.7)$$

where $\mathbf{1}_2$ is the 2×2 identity matrix,

$$\epsilon_2 \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (2.8)$$

and the vacuum expectation values satisfy

$$|b_j|^2 = |a_j|^2 + |b|^2, \quad j = 1, \dots, n. \quad (2.9)$$

We begin by analyzing the theory in the regime where b, a_1, \dots, a_n are all nonzero and large, so that a classical description is valid. In that case, the gauge group $SU(N)$ is completely broken, while the $SU(4)$ gauge group remains unbroken. The fields Q and L get masses $\sim \lambda \langle \bar{U} \rangle$. Integrating out these massive fields gives an effective theory consisting of $SU(4)$ super-Yang–Mills theory and some singlets. $SU(4)$ gaugino condensation gives rise to a dynamical superpotential⁴

$$W_{\text{dyn}} \propto \det(\lambda \bar{U})^{1/4}. \quad (2.10)$$

(Alternatively, the anomaly-free $U(1) \times U(1)_R$ symmetries can be used to show that this is the most general superpotential allowed.) For large values of $\langle \bar{U} \rangle$, the Kähler potential is nearly canonical in \bar{U} , and so the potential slopes toward $\bar{U} = 0$ for

⁴In this Section, we do not include factors of 4π and N in our estimates for simplicity. These are included in the numerical estimates we give in the next Section.

$N \geq 5$. (For $N = 3$, the theory has a runaway supersymmetric vacuum.) We are therefore led to analyze the theory near the origin of the moduli space.⁵

We analyze the dynamics near the origin of the moduli space assuming that $SU(4)$ is weak at the scale Λ_N where the $SU(N)$ becomes strong. (Note that this includes large values of N where $SU(4)$ is not asymptotically free.) The $SU(N)$ theory is s-confining, and the effective theory can be written in terms of the fields [12]⁶

$$\begin{aligned}
(Q\bar{U}) &\sim \square \times \square, \\
(QA^n) &\sim \square \times \mathbf{1}, \\
(Q^3 A^{n-1}) &\sim \bar{\square} \times \mathbf{1}, \\
(A\bar{U}^2) &\sim \mathbf{1} \times \bar{\square}, \\
(\bar{U}^N) &\sim \mathbf{1} \times \mathbf{1}, \\
L &\sim \bar{\square} \times \bar{\square},
\end{aligned} \tag{2.11}$$

where we have given the transformation properties under $SU(4) \times [SU(N)]$. The parentheses indicate that these are elementary fields in the effective Lagrangian with the same quantum numbers as the composite operators inside the parentheses. The Kähler potential is smooth in terms of the effective fields, *e.g.*

$$K_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-2}} |(\bar{U}^N)|^2 + \dots \tag{2.12}$$

The effective superpotential is given by the sum of the tree superpotential and a dynamical superpotential [12]

$$\begin{aligned}
W_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-1}} &\left[(QA^n)(Q\bar{U})^3(A\bar{U}^2)^{n-1} + (Q^3 A^{n-1})(Q\bar{U})(A\bar{U}^2)^n \right. \\
&\left. + (\bar{U}^N)(QA^n)(Q^3 A^{n-1}) \right] + \lambda L(Q\bar{U}).
\end{aligned} \tag{2.13}$$

The trilinear term has become a mass term for L and $(Q\bar{U})$; integrating out these fields gives an $SU(4)$ gauge theory with 1 flavor (QA^n) , $(Q^3 A^{n-1})$ and singlets $(A\bar{U}^2)$,

⁵If $b = 0$, a_1, \dots, a_n large, the analysis is different. In that case, the $SU(4)$ gauge group has one light flavor that would run away if there were no further interactions. However, the would-be runaway direction is not D -flat, so there are no supersymmetric minima with $b = 0$.

⁶For a general analysis of s-confining theories, see Ref. [13].

(\bar{U}^N) , with a trilinear effective superpotential

$$W_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-1}} (\bar{U}^N) (QA^n) (Q^3 A^{n-1}). \quad (2.14)$$

If this were a theory of fundamental fields, it would have a runaway vacuum with $(\bar{U}^N) \rightarrow \infty$. This can be described by a superpotential of the form Eq. (2.10), but in the regime we are now considering the Kähler potential is smooth in terms of the field (\bar{U}^N) . But if $\langle \bar{U} \rangle$ is large compared to Λ_N , we can no longer treat (\bar{U}^N) as an elementary field; instead, we must use the analysis above, which shows that the potential slopes toward $\bar{U} = 0$ for $\langle \bar{U} \rangle \gg \Lambda_N$. We see that there is no supersymmetric vacuum for either large or small values of \bar{U} on the baryon branch, so there must be at least a local supersymmetry-breaking minimum for $\langle \bar{U} \rangle \sim \Lambda_N$. This is the mechanism for supersymmetry breaking found in the models of Refs. [11, 14]. Note that there is no unbroken $U(1)_R$ symmetry, so that when we gauge a subgroup of the global $[SU(N)]$ symmetry, gaugino masses can be generated.

We see that the baryon branch of this model has two descriptions. There is a ‘‘Higgs’’ description in which the gauge group $SU(N)$ is broken (valid for large $\langle \bar{U} \rangle$), and a ‘‘confining’’ description in which $SU(N)$ confines (valid near $\langle \bar{U} \rangle = 0$). Neither of these descriptions is under control near the local minimum found above, but both pictures are expected to be a reliable guide to the qualitative features of the low-energy dynamics [15]. We know that $b \sim \Lambda_N$ (using the Higgs description), but we cannot determine whether a_1, \dots, a_n are nonzero. In this paper, we will make the dynamical assumption that

$$\langle A \rangle = 0. \quad (2.15)$$

This corresponds to the largest possible unbroken global symmetry

$$SU(N) \times [SU(N)] \rightarrow [SU(N)]. \quad (2.16)$$

This is reasonable, since points of maximal symmetry are generically stationary points of the energy, but it is an assumption nonetheless. (The assumption is equivalent to the statement that certain mass-squared terms in the effective theory are positive.) With this assumption, we see that the fermionic components of A must remain massless in order to match the anomalies of the unbroken global $[SU(N) \times U(1)]$ symmetry. (In the Higgs description we are using, A is charged under the global symmetry due to the symmetry breaking in Eq. (2.16).) If we use the confined description, we find that the fermionic components of the composite field $(A\bar{U}^2)$ are massless. In either description, we find that there are massless fermions transforming as a \square under the

unbroken $[SU(N)]$ global symmetry. Later we will identify some of these fermions with composite quarks and leptons.

Note that if the dynamical assumption above is false, we can use this model to construct models of direct gauge-mediated supersymmetry breaking with composite messengers, along the lines suggested in Ref. [11]. In this case, we add higher-dimension terms to the superpotential that give a supersymmetric mass to the composite fields that stabilizes the vacuum at $\langle A \rangle = 0$, and gauge a subgroup of the $[SU(N)]$ global symmetry of this model with the standard-model gauge group. The negative supersymmetry-breaking mass-squared terms that result from the non-perturbative dynamics then induce *positive* mass-squared terms for the squarks and sleptons from gauge loops. We will not pursue this possibility further in this paper.

In the remainder of this Subsection, we will show that for $N > 5$ there is a runaway supersymmetric vacuum on the lepton branch of the classical moduli space. When we consider even N , we will find that the story is much the same: there are three branches of the classical moduli space, and there is a local supersymmetry-breaking minimum on the “baryon” branch whose description is identical to the one found for N odd, and there are runaway supersymmetric vacua on the other two branches. In the remainder of the paper, we will build models assuming that the universe lives in the false vacuum on the baryon branch.⁷ The rest of this Section is therefore not necessary to understand the main results of the paper. The reader interested primarily in model-building is strongly encouraged to skip to Section 3 at this point.

We now analyze the lepton branch of the classical moduli space. In terms of the elementary fields, the vacuum expectation values can be written (up to gauge and flavor transformations)

$$\langle Q \rangle = 0, \quad \langle L \rangle = \begin{pmatrix} 0 & \cdots & 0 \\ \ell \mathbf{1}_4 & & \vdots \\ & & 0 \cdots 0 \end{pmatrix}, \quad (2.17)$$

$$\langle A \rangle = \begin{pmatrix} \mathbf{0}_5 & & & \\ & a_1 \epsilon_2 & & \\ & & \ddots & \\ & & & a_{n-2} \epsilon_N \end{pmatrix}, \quad \langle \bar{U} \rangle = \sqrt{2} \begin{pmatrix} \mathbf{0}_5 & & & \\ & a_1 \mathbf{1}_2 & & \\ & & \ddots & \\ & & & a_{n-2} \mathbf{1}_2 \end{pmatrix}. \quad (2.18)$$

We begin by analyzing the theory in the regime where $\ell, a_1, \dots, a_{n-2}$ are all nonzero and large, so that a classical description is valid. In that case, the gauge group

⁷The rate for tunneling from the false to one of the supersymmetry vacua is shown to be negligibly small in Subsection 4.5.

$SU(4)$ is completely broken, and $SU(N)$ is broken down to $SU(5)$. $\langle L \rangle \neq 0$ gives mass to all of the Q 's and 4 flavors of \bar{U} 's, and most of the components of A and \bar{U} are eaten. The effective $SU(5)$ gauge theory has matter content $\bar{\square} \oplus \square$ plus singlets, with no superpotential. If this were a theory of fundamental fields, this would break supersymmetry [16], giving a vacuum energy proportional to $\Lambda_{5,\text{eff}}^4$, where $\Lambda_{5,\text{eff}}$ is determined by 1-loop matching to be

$$\Lambda_{5,\text{eff}} = \Lambda_N^{(4N+1)/13} \ell^{4/13} (a_1 \cdots a_{n-2})^{-(4N-8)/(13(n-2))}. \quad (2.19)$$

For $N > 5$, the vacuum energy goes to zero as the a 's go to infinity, and there are runaway vacua on the lepton branch. For $N = 5$, the classical constraints force $\langle A \rangle = 0$, and there are no runaway directions; in this case supersymmetry is broken on the lepton branch as well as the baryon branch.

For $N > 5$, we could lift the runaway directions by adding higher-dimension terms to the superpotential (see [17]). However, these will partially break the global symmetry, and can be shown to have lower energy than the local minima on the baryon branch.

2.2 Even N Models

We now consider the model for even $N = 2n$. The analysis closely parallels that of the odd N models, and the reader interested mainly in our models is encouraged to skip to Section 3. The classical moduli space can be parameterized by (again indicating the global $[SU(N)]$ quantum numbers)

$$\begin{aligned} L^4 &\sim \bar{\square}, & (\text{for } N \geq 4) \\ A\bar{U}^2 &\sim \square, \\ Q^4 A^{n-2} &\sim \mathbf{1}, \\ \bar{U}^N &\sim \mathbf{1}, \\ A^n &\sim \mathbf{1}, \end{aligned} \quad (2.20)$$

with the constraints

$$L^4 \cdot \bar{U}^N = 0, \quad L^4 \cdot A\bar{U}^2 = 0, \quad L^4 \cdot Q^4 A^{n-2} = 0, \quad \bar{U}^N \cdot Q^4 A^{n-2} = 0. \quad (2.21)$$

This moduli space has three branches: $\langle L^4 \rangle \neq 0, \langle \bar{U}^N \rangle, \langle Q^4 A^{n-2} \rangle = 0$ (“lepton branch”), $\langle \bar{U}^N \rangle \neq 0, \langle L^4 \rangle, \langle Q^4 A^{n-2} \rangle = 0$ (“baryon branch”), and $\langle Q^4 A^{n-2} \rangle \neq 0, \langle L^4 \rangle, \langle \bar{U}^N \rangle = 0$ (“mixed branch”).

We first analyze the baryon branch. On this branch, the vacuum expectation values can be written (up to gauge and flavor transformations)

$$\langle L \rangle = 0, \quad \langle Q \rangle = 0, \quad (2.22)$$

$$\langle A \rangle = \begin{pmatrix} a_1 \epsilon_2 & & \\ & \ddots & \\ & & a_n \epsilon_2 \end{pmatrix}, \quad \langle \bar{U} \rangle = \sqrt{2} \begin{pmatrix} b_1 \mathbf{1}_2 & & \\ & \ddots & \\ & & b_n \mathbf{1} \end{pmatrix}, \quad (2.23)$$

where

$$|a_j|^2 - |b_j^2| = c, \quad j = 1, \dots, n. \quad (2.24)$$

We begin by analyzing the theory in the region of moduli space where a_1, \dots, a_n are all nonzero and large, so that a classical description is valid. In that case, the gauge group $SU(N)$ is completely broken, while the $SU(4)$ gauge group remains unbroken. The fields Q and L get masses $\sim \lambda \langle \bar{U} \rangle$, and the low-energy theory is $SU(4)$ super-Yang–Mills with singlets. Gaugino condensation in this theory gives rise to a dynamical superpotential

$$W_{\text{dyn}} \propto \det(\lambda \bar{U})^{1/4}. \quad (2.25)$$

For large values of $\langle \bar{U} \rangle$, the Kähler potential is nearly canonical in \bar{U} , and so the potential slopes toward $\bar{U} = 0$ for $N > 4$. For $N = 2$, there is a runaway supersymmetric vacuum. For $N = 4$, the superpotential is linear in \bar{U} , and the location of the true vacuum depends on the form of the Kähler potential. For large values of $\langle \bar{U} \rangle$, the Kähler potential can be computed in perturbation theory, and one finds that 1-loop corrections involving the Yukawa coupling λ tend to push the the vacuum away from the origin, while 1-loop corrections involving the gauge couplings have the opposite sign. These effects can give rise to a local minimum for large values of $\langle \bar{U} \rangle$ for a range of parameters. (This is the inverted hierarchy mechanism [18].) For any $N \geq 4$, we see that there is no supersymmetric vacuum for large $\langle \bar{U} \rangle$, and we are led to analyze the theory near the origin of the moduli space.

We now analyze the dynamics near the origin of the moduli space assuming that $\Lambda_N \gg \Lambda_4$. The $SU(N)$ theory is s-confining, and the effective theory can be written

in terms of the fields [12]

$$\begin{aligned}
(Q\bar{U}) &\sim \square \times \square, \\
(A^n) &\sim \mathbf{1} \times \mathbf{1}, \\
(Q^2 A^{n-1}) &\sim \square \times \mathbf{1}, \\
(Q^4 A^{n-2}) &\sim \mathbf{1} \times \mathbf{1}, \\
(A\bar{U}^2) &\sim \mathbf{1} \times \square, \\
(\bar{U}^N) &\sim \mathbf{1} \times \mathbf{1}, \\
L &\sim \bar{\square} \times \bar{\square},
\end{aligned} \tag{2.26}$$

where we have given the transformation properties under $SU(4) \times [SU(N)]$. The superpotential is given by the sum of the tree superpotential and a dynamical superpotential [12]. The tree-level superpotential turns into a mass term for L and $(Q\bar{U})$. Integrating out these states gives an effective theory with gauge group $SU(4)$, a field $(Q^2 A^{n-1}) \sim \square$, and singlets, with effective superpotential

$$\begin{aligned}
W_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-1}} &\left[(Q^4 A^{n-2})(A\bar{U}^2)^n + (\bar{U}^N)(A^n)(Q^4 A^{n-2}) \right. \\
&\left. + (\bar{U}^N)(Q^2 A^{n-1})^2 \right].
\end{aligned} \tag{2.27}$$

For $\langle \bar{U}^N \rangle \neq 0$, $(Q^2 A^{n-1})$ is massive and $SU(4)$ gaugino condensation pushes (\bar{U}^N) away from the origin. If this were a theory of fundamental fields, it would have a runaway vacuum with $(\bar{U}^N) \rightarrow \infty$, but this description breaks down for large values of $\langle \bar{U} \rangle$. We see that this theory has a local supersymmetry-breaking minimum on the baryon branch through a mechanism identical to that in the odd N case. As before, we make the dynamical assumption that

$$\langle A \rangle = 0, \tag{2.28}$$

so that there is an unbroken $[SU(N)]$ global symmetry, and the theory has massless composite fermions transforming as a \square under $SU(N)$.

We now turn to the lepton branch. On this branch, the vacuum expectation values are

$$\langle L \rangle = \begin{pmatrix} 0 & \cdots & 0 \\ \ell \mathbf{1}_4 & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}, \quad \langle Q \rangle = 0, \tag{2.29}$$

$$\langle A \rangle = \begin{pmatrix} a\epsilon_2 & & & & \\ & a\epsilon_2 & & & \\ & & a_1\epsilon_2 & & \\ & & & \ddots & \\ & & & & a_{n-2}\epsilon_2 \end{pmatrix}, \quad \langle \bar{U} \rangle = \sqrt{2} \begin{pmatrix} \mathbf{0}_4 & & & & \\ & b_1\mathbf{1}_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & b_{n-2}\mathbf{1}_2 \end{pmatrix}. \quad (2.30)$$

with

$$|a|^2 = |a_j|^2 - |b_j|^2, \quad j = 1, \dots, n-2. \quad (2.31)$$

We analyze the theory in the region of moduli space where $\ell, a, a_1, \dots, a_{n-2}$ are all nonzero and large. In that case, the $SU(4)$ gauge group is completely broken, and the $SU(N)$ gauge group is broken down to $Sp(4)$. After taking into account the effects of the superpotential and the eaten fields, there are no charged fields under the unbroken $Sp(4)$. Gaugino condensation in $Sp(4)$ then pushes a, a_1, \dots, a_{n-2} away from the origin [17], and so there is a runaway supersymmetric vacuum in this branch.

Finally, we analyze the mixed branch. On this branch, the vacuum expectation values are

$$\langle L \rangle = 0, \quad \langle Q \rangle = \sqrt{2} \begin{pmatrix} 0 & \cdots & 0 \\ q\mathbf{1}_4 & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}, \quad (2.32)$$

$$\langle A \rangle = \begin{pmatrix} a\epsilon_2 & & & & \\ & a\epsilon_2 & & & \\ & & a_1\epsilon_2 & & \\ & & & \ddots & \\ & & & & a_{n-2}\epsilon_2 \end{pmatrix}, \quad \langle \bar{U} \rangle = \sqrt{2} \begin{pmatrix} \mathbf{0}_4 & & & & \\ & b_1\mathbf{1}_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & b_{n-2}\mathbf{1}_2 \end{pmatrix}, \quad (2.33)$$

where

$$|a|^2 + |q|^2 = |a_j|^2 - |b_j|^2, \quad j = 1, \dots, n-2. \quad (2.34)$$

As on the lepton branch, the low-energy theory is a pure $Sp(4)$ gauge symmetry, and gaugino condensation pushes a, a_j away from the origin, and so there are additional runaway vacua on this branch.

3 Numerical Estimates

We now consider the numerical estimates of various quantities of interest in these models. The models are non-calculable, but we can make estimates using dimensional

analysis, and also keep track of factors of 4π and N , which are potentially large. Neither the “Higgs” nor the “confining” descriptions of these theories is weakly coupled in the local supersymmetry-breaking vacuum we consider. We find it simplest to make estimates using the “Higgs” description that uses the elementary fields of the theory. We estimate the size of various effects by assuming that loop corrections are the same size as leading effects in perturbation theory. This is the philosophy of “naïve dimensional analysis” [19, 20].

We use these considerations to argue that the strong dynamics preserves an approximate $[SU(N)]$ flavor symmetry even if the Yukawa matrix λ in the tree-level superpotential is completely arbitrary.⁸ This is important for naturally suppressing flavor-changing neutral currents in the models we construct below.⁹ First of all, the dynamical superpotential Eq. (2.10) depends only on $\det(\lambda)$, and so has no flavor dependence. This means that all flavor dependence appears in the effective Kähler potential. In the Higgs description, the λ dependence in the Kähler potential comes from diagrams with λ vertices, and through the Dirac mass matrix of Q and L , which is proportional to λ . Diagrams with λ vertices are suppressed by $\lambda^2/(16\pi^2)$, so these give only small flavor violation. Internal Q and L loops without λ vertices do not contribute to flavor violation because they always involve traces of the mass matrix. (We are not interested in diagrams with external Q and L lines because the only light matter states correspond to $\text{tr } \bar{U}$ and A ; see below.) This shows that the flavor symmetry is preserved up to corrections of order $\lambda^2/(16\pi^2) \lesssim 10^{-2}$.

We now estimate $\langle \bar{U} \rangle$. Naïve dimensional analysis tells us that $\langle \bar{U} \rangle$ must be close to the value for which perturbation theory breaks down. The $SU(N)$ gauge dynamics becomes strong at the scale Λ_N , where

$$g_N(\mu \sim \Lambda_N) \sim \frac{4\pi}{\sqrt{N}}. \quad (3.1)$$

The perturbative description breaks down when the massive gauge bosons (and the states that get a mass due to the $SU(N)$ D -term potential) have masses of order Λ_N , which gives

$$\langle \bar{U} \rangle \sim \Lambda_N \frac{\sqrt{N}}{4\pi} \mathbf{1}_N. \quad (3.2)$$

⁸It is a consequence of our dynamical assumption that $\langle A \rangle = 0$, i.e. that the flavor symmetry is not spontaneously broken by the strong dynamics in the limit where λ is proportional to the identity.

⁹We thank M. Schmaltz for emphasizing this point.

The F component of \bar{U} is estimated to be¹⁰

$$\langle F_{\bar{U}} \rangle \sim \left\langle \frac{\partial W_{\text{eff}}}{\partial \bar{U}} \right\rangle \sim \frac{1}{4\pi} \frac{\sqrt{N}}{4} \left(\det(\sqrt{N}\lambda) \right)^{1/4} \Lambda_4^{3-N/4} \Lambda_N^{N/4-1} \mathbf{1}_N. \quad (3.3)$$

This shows that as long as $SU(4)$ is weak at the scale where $SU(N)$ becomes strong, we have $\langle F_{\bar{U}} \rangle \ll \langle \bar{U} \rangle^2$. If $N < 12$, $SU(4)$ is asymptotically free and the condition for $SU(4)$ to be weak at the scale Λ_N is $\Lambda_4 \ll \Lambda_N$. For $N \geq 12$, Λ_4 is the ultraviolet Landau pole of $SU(4)$, and so the condition that $SU(4)$ is weak at Λ_N is $\Lambda_4 \gg \Lambda_N$. As a consequence of our dynamical assumption, both $\langle \bar{U} \rangle$ and $\langle F_{\bar{U}} \rangle$ are proportional to the $N \times N$ unit matrix, so that the $SU(N) \times [SU(N)]$ symmetry is broken down to a global $[SU(N)]$.

The superpotential gives a supersymmetric mass to the fields Q and L of order

$$m_{Q,L} \sim \lambda \langle \bar{U} \rangle \sim \frac{\sqrt{N}\lambda}{4\pi} \Lambda_N. \quad (3.4)$$

(This mass does not become large compared to Λ_N for large N because the Yukawa coupling must be $\lambda \sim 1/\sqrt{N}$ in order to have a good large- N limit.) There are also supersymmetry breaking B -type mass terms of order $\langle F_{\bar{U}} \rangle$. Below the scale $m_{Q,L}$, the only light fields are the $SU(4)$ gauge bosons, $\text{tr } \bar{U}$ and A . (In the confined description, these fields correspond to (\bar{U}^N) and $(A\bar{U}^2)$, respectively.) The field $\text{tr } \bar{U}$ is a singlet, and A transforms as a \square under the unbroken $[SU(N)]$ global symmetry. The scalar and fermion components of $\text{tr } \bar{U}$ get masses of order

$$m_{\text{tr } \bar{U}} \sim \left\langle \frac{\partial^2 W_{\text{eff}}}{\partial \bar{U}^2} \right\rangle \sim \frac{\langle F_{\bar{U}} \rangle}{\langle \bar{U} \rangle} \equiv M_{\text{comp}}. \quad (3.5)$$

We will see that the scale M_{comp} sets the scale for all supersymmetry breaking masses in this model.

The scalar components of the field A receives (strong $SU(N)$ gauge-mediated) loop contributions both from the supersymmetry breaking in the Q, L spectrum and from the induced supersymmetry breaking in the fields at the scale Λ_N . These contributions can be most easily estimated using the method of Giudice and Rattazzi [21].¹¹ In this method, one computes the wavefunction renormalization factor Z_A as a function of the threshold m where heavy states are integrated out, and then makes the replacement $m \rightarrow 4\pi\sqrt{\bar{U}^\dagger \bar{U}}/\sqrt{N}$ to find the dependence on $\langle \bar{U} \rangle$ and $\langle F_{\bar{U}} \rangle$ to leading order in

¹⁰For a discussion of the factors of 4π in W_{eff} , see Ref. [20].

¹¹This method can be extended to all orders in perturbation theory [22].

$\langle F_{\bar{U}} \rangle / \langle \bar{U} \rangle$. The A scalar mass is then obtained from the $\theta^2 \bar{\theta}^2$ component of $\ln Z_A$. The quantity $\ln Z_A$ satisfies a renormalization group equation

$$\mu \frac{d}{d\mu} \ln Z_A = f \left(\frac{N g_N^2}{16\pi^2} \right), \quad (3.6)$$

where f is a function with no large parameters. (Note that there are N “flavors” of \bar{U} , so loops of \bar{U} fields are not suppressed for large N .) Since $N g_N^2 / (16\pi^2) \sim 1$, we obtain simply

$$m_{\phi_A}^2 \sim \left(\frac{\langle F_{\bar{U}} \rangle}{\langle \bar{U} \rangle} \right)^2 = M_{\text{comp}}^2. \quad (3.7)$$

If we identify the composite fermions with quarks and leptons, this gives the mass of the corresponding scalar superpartners.

We now assume that the standard-model gauge group is embedded into the $[SU(N)]$ global symmetry and estimate the standard-model gaugino and elementary scalar masses. We can compute these using the method of Ref. [21], or by simply estimating the corresponding perturbative diagrams. There are of order N messengers, so we obtain

$$m_{\lambda_{\text{SM}}} \sim N \frac{g_{\text{SM}}^2}{16\pi^2} M_{\text{comp}} \quad (3.8)$$

for the standard-model gaugino masses. In addition, the scalars will receive a gauge-mediated contribution

$$\delta m_{\phi, \text{gauge med}}^2 \sim N \left(\frac{g_{\text{SM}}^2}{16\pi^2} \right)^2 M_{\text{comp}}^2. \quad (3.9)$$

For the composite fields, this is a small correction; for the elementary fields, this is the dominant contribution to the scalar mass. (We will see below that there is also a flavor-dependent contribution to the scalar masses that can be comparable.)

In the models we consider, there is a scale of new physics M that is not far above the scale Λ_N . In the effective theory at the scale Λ_N , there will therefore be higher-dimension operators suppressed by powers of $1/M$. For example, the following terms in the Lagrangian are compatible with all symmetries:

$$\delta \mathcal{L} \sim \int d^4\theta \left[\frac{c_1}{M^2} \text{tr}(\bar{U}^\dagger \bar{U}) A^\dagger A + \frac{c_2}{M^2} \text{tr}(\bar{U}^\dagger \bar{U}) \Phi^\dagger \Phi \right], \quad (3.10)$$

where Φ is an elementary quark or lepton field. In the “Higgs” picture we are using, we can estimate the terms in the effective Lagrangian for the composite fields by simply

replacing \bar{U} by its vacuum expectation value.¹² We therefore obtain an additional contribution to the elementary and composite scalar masses of order

$$\delta m_{\phi, \text{new phys}}^2 \sim \frac{c_{1,2} N \langle F_{\bar{U}} \rangle^2}{M^2} \sim \frac{c_{1,2} N \langle \bar{U} \rangle^2}{M^2} M_{\text{comp}}^2. \quad (3.11)$$

On general grounds, we might expect $c_{1,2} \sim 1$; alternatively, if the model has a good large- N limit with M held fixed, we expect $c_{1,2} \sim 1/N$.

There are additional higher-dimension operators in the models we construct. We can easily estimate their effects on the composite fields in the Higgs description by simply replacing \bar{U} by appropriate scalar or F -component vacuum expectation values.

4 Composite Quarks and Leptons

We now build models of composite quarks and leptons using the models analyzed above as building blocks. Because the Yukawa couplings arise from high-dimension operators, they are naturally small compared to unity. This means that the top quark cannot be composite in the models we construct.¹³ In models of this type, the masses of the gauginos and elementary scalars are suppressed by a loop factor compared to the composite scalar masses:

$$\frac{m_{\lambda_{\text{SM}}}}{M_{\text{comp}}} \sim \frac{N g_{\text{SM}}^2}{16\pi^2}. \quad (4.1)$$

If N is not large, then this can be realistic only if the composite scalars are very heavy. As we will explain below, this leads naturally to models with a low compositeness scale. On the other hand, we can consider models where the loop suppression is overcome by the large multiplicity factor N , allowing models with a high compositeness scale.¹⁴

4.1 Embedding the Standard Model

Before turning to the models, we discuss some aspects of embedding the standard model gauge group into the global $[SU(N)]$ symmetry. Because we want to preserve

¹²When expressed in terms of the scale Λ_N , this gives results with 4π dependence in agreement with a “confined” description [20].

¹³It would be interesting to find supersymmetry-breaking models where the top-quark Yukawa coupling arises as a term in a dynamical superpotential. In that case, the top-quark Yukawa coupling is of order 4π at the compositeness scale, and runs down to a quasi-fixed point value at the weak scale [23]. The top-quark Yukawa coupling arises in this way in the models of Nelson and Strassler [7], but the composite dynamics does not break supersymmetry in these models.

¹⁴We do not consider the possibility that the gauginos may be ultra-light [24].

perturbative unification, we will consider only embeddings where the preons fall into complete $SU(5)_{\text{SM}}$ multiplets, even if only the standard-model subgroup is gauged.¹⁵

Because our models generate composite states transforming as a \boxplus of a global $[SU(N)]$ symmetry, it is tempting to generate a $\mathbf{10}$ of $SU(5)_{\text{SM}}$ from the antisymmetric product $(\mathbf{5} \otimes \mathbf{5})_{\text{asymm}}$. However, it is easy to see that there is no way of assigning baryon number to the preons to obtain the correct baryon numbers for the states of the the composite $\mathbf{10}$.¹⁶ Since baryon number is not a good quantum number of the strong dynamics, we expect baryon-number violating operators suppressed by powers of Λ_N in the low-energy theory, so this kind of embedding cannot be used in models where the compositeness scale is below the grand-unified theory (GUT) scale. It is not hard to construct baryon-number conserving as well as baryon-number violating embeddings, and we will consider both types below.

The first embedding we consider is based on the model with $N = 11$. $SU(5)_{\text{SM}}$ is embedded into $[SU(11)]$ so that the $\square = \mathbf{11}$ representation decomposes as

$$\square \rightarrow \mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}. \quad (4.2)$$

The composite states then decompose under $SU(5)_{\text{SM}}$ as

$$\boxplus \rightarrow \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1} \oplus [\mathbf{24} \oplus \bar{\mathbf{10}} \oplus \mathbf{5}]. \quad (4.3)$$

The composite states include a complete generation (including a right-handed neutrino), together with the exotic states in square brackets. Baryon number is violated at the scale Λ_N . We can remove the unwanted exotic states by adding an additional elementary generation $\mathbf{10} \oplus \bar{\mathbf{5}}$ to the theory and including higher-dimension operators of the form

$$\delta\mathcal{L}_{\text{eff}} \sim \int d^2\theta \left[\frac{1}{M}(A\bar{U}^2)_{\mathbf{5}}X_{\bar{\mathbf{5}}} + \frac{1}{M}(A\bar{U}^2)_{\bar{\mathbf{10}}}X_{\mathbf{10}} + \frac{1}{M^3}(A\bar{U}^2)_{\mathbf{24}}^2 \right] + \text{h.c.}, \quad (4.4)$$

which gives rise to masses

$$m_{\bar{\mathbf{5}},\mathbf{10}} \sim \frac{\langle \bar{U} \rangle^2}{M}, \quad m_{\mathbf{24}} \sim \frac{\langle \bar{U} \rangle^4}{M^3}. \quad (4.5)$$

One can obtain a model with two composite generations by considering a model with gauge group $[SU(4) \times SU(11)]^2/Z_2$. This may not be unnatural, since whatever explains the replication of families may also give rise to a replicated group structure.

¹⁵The interesting possibility that the preons fall into complete representations of the ‘‘trinification’’ group $SU(3)^3/Z_3$ will not be explored in this paper.

¹⁶It is possible to obtain a $\mathbf{10}$ from the antisymmetric product of two *different* $\mathbf{5}$ ’s, but this leads to rather uneconomical models.

A simple way to conserve baryon number is to have only composite $\bar{\mathbf{5}}$'s. The simplest such model is based on $N = 5 + k = 7$ with $SU(5)_{\text{SM}}$ acting on the preons as

$$\square \rightarrow \bar{\mathbf{5}} \oplus (k \times \mathbf{1}). \quad (4.6)$$

The composite states decompose as

$$\square \rightarrow (k \times \bar{\mathbf{5}}) \oplus \left[\bar{\mathbf{10}} \oplus \left(\frac{k(k-1)}{2} \times \mathbf{1} \right) \right]. \quad (4.7)$$

This gives rise to $k = 2$ composite $\bar{\mathbf{5}}$'s and some unwanted states that can be eliminated by adding higher-dimension operators similar to those described above.

Finally, we consider a more elegant embedding that naturally replicates generations and conserves baryon number. We consider the theory with $N = 15 + k = 18$, with $SU(5)_{\text{SM}}$ acting on the preons as

$$\square \rightarrow \mathbf{10} \oplus \bar{\mathbf{5}} \oplus (k \times \mathbf{1}). \quad (4.8)$$

Then the composite states decompose as

$$\square \rightarrow (k \times \mathbf{10}) \oplus (k \times \bar{\mathbf{5}}) \oplus \left[\mathbf{45} \oplus \bar{\mathbf{45}} \oplus \bar{\mathbf{10}} \oplus \mathbf{5} \oplus \left(\frac{k(k-1)}{2} \times \mathbf{1} \right) \right]. \quad (4.9)$$

If we now write down the most general superpotential involving the composite states, we will generate Dirac masses marrying $\mathbf{45}$ and $\bar{\mathbf{45}}$, as well as marrying one of the composite generations with the antigeneration, leaving us with $(k-1)$ complete composite generations.

We now address the question of Yukawa couplings. Yukawa couplings involving the composite fermions must arise from higher-dimension operators in the fundamental theory. We therefore assume that the new physics at the scale M induces terms in Lagrangian such as

$$\delta\mathcal{L} \sim \int d^2\theta \left[\frac{b_1}{M^4} (A\bar{U}^2)_{\bar{\mathbf{5}}} (A\bar{U}^2)_{\mathbf{10}} H + \frac{b_2}{M^2} (A\bar{U}^2)_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}} H + \dots \right] + \text{h.c.} \quad (4.10)$$

where H is a fundamental Higgs field and Φ is a fundamental matter field. This gives Yukawa couplings to the composite fields

$$\delta\mathcal{L}_{\text{eff}} \sim \int d^2\theta \left[\frac{b_1 \langle \bar{U} \rangle^4}{M^4} A_{\bar{\mathbf{5}}} A_{\mathbf{10}} H + \frac{b_2 \langle \bar{U} \rangle^2}{M^2} A_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}} H + \dots \right] + \text{h.c.} \quad (4.11)$$

Note that if the second generation quarks are to be composite, we require $\langle \bar{U} \rangle / M \sim \frac{1}{3}$, so the scale of new physics is not far above the scale of strong dynamics. This problem

appears particularly worrisome if we note that the scale $\langle \bar{U} \rangle$ is smaller than Λ_N (the scale of strong dynamics) for moderate N . However, the example of the charm quark in QCD suggests that it is not absurd to integrate out particles with masses near the scale Λ_N . (The charm quark mass and the scale Λ in QCD are both near 1 GeV.)

Note that, in all of the above models, an approximate flavor symmetry of the strong dynamics (the Z_2 in the $N = 11$ theory, $SU(k)$ in the $N = 5+k$ and $N = 15+k$ theories) guarantees equal soft masses for all the composite states. While this is somewhat artificial in the $N = 11$ case, it is quite natural in the $N = 5+k$ and $15+k$ cases. In particular, in the $N = 15+k$ case, *all* soft masses for the first two generation scalars are degenerate at leading order. Of course, the flavor physics responsible for generating the correct pattern of Yukawa couplings must distinguish between the first two generations and will necessarily break the flavor symmetry of the strong dynamics.¹⁷ The corrections to the soft masses induced by this flavor physics are model-dependent, but are at least suppressed by the same small parameters that control the small Yukawa couplings for the light generations. We will see that this suppression is already sufficient for marginal consistency with flavor-changing neutral current (FCNC) constraints, so the supersymmetric flavor problem is very mild in these models.

4.2 Low-scale Composite Models

If the multiplicity factor N is not large, then the composite scalars must have masses of order 10 TeV or more in order to have gaugino and elementary squark and slepton masses of order 100 GeV. In this case, there are negative 2-loop contributions to the elementary scalar mass-squared from the composite scalar masses [25]. These contributions are dangerous because they are enhanced by $\ln(\Lambda_N/M_{\text{comp}})$ compared to the usual gauge-mediated contributions. To avoid these, we must require that Λ_N is not far above $M_{\text{comp}} \sim 10\text{--}100$ TeV. Independently of these considerations, we are interested in the possibility of a low compositeness scale because it holds out the possibility of rich phenomenology.

One possibility is to use the $N = 5+k = 7$ model, which gives rise to the composite states $\bar{\mathbf{10}} \oplus (2 \times \bar{\mathbf{5}}) \oplus \mathbf{1}$. We identify the two composite $\bar{\mathbf{5}}$ fermions with quarks and leptons, and eliminate the unwanted composite fermions by combining them with elementary fields transforming as $\mathbf{10} \oplus \mathbf{1}$. In order to obtain sufficiently heavy masses for the elementary squarks and sleptons, we take the mass of the composite scalars

¹⁷Flavor violation in the the λ matrix does not break the chiral symmetries acting on the composite quarks and leptons, and therefore does not give rise to Yukawa couplings.

to be in the 10–100 TeV range. For purposes of running the standard-model gauge couplings, this model adds an equivalent of $6 \times (\mathbf{5} \oplus \bar{\mathbf{5}})$ to the theory above the scale Λ_7 of the strong $SU(7)$ dynamics, and so it is marginally compatible with unification if $\Lambda_7 \gtrsim 200$ TeV.

If we assume that the Yukawa couplings are generated by new physics at a scale M from operators of the form Eq. (4.10), we find that in order to generate Yukawa couplings of order 10^{-3} (for the composite s and μ), we require $\langle \bar{U} \rangle / M \sim 3 \times 10^{-2}$. This gives an explanation of the smallness of the down-type Yukawa couplings of the first two generations, but it does not explain why the up-type Yukawa couplings are also small.

We now discuss FCNC's in this model. Note that there is a global $SU(2)$ acting on the $SU(5)_{\text{SM}}$ singlet preons in this model, which becomes a $SU(2)$ flavor symmetry acting on the composite $\mathbf{5}$'s in the low-energy theory. We can therefore envision that the flavor breaking in the preon theory has a GIM mechanism acting on the first two generations that would align the flavor structure in the scalar and fermion sectors [26, 27, 28]. In the absence of such a mechanism, this model has FCNC's. Because the up-squarks are elementary, their mass arises dominantly from gauge-mediation, and this is not large enough to naturally suppress FCNC's. For example,

$$\frac{\delta m_{\bar{u}c}^2}{m_{\bar{u}}^2} \sim \left(\frac{\langle \bar{U} \rangle}{M} \right)^2 \left(\frac{g_3^2}{16\pi^2} \right)^{-2} \sim 1, \quad (4.12)$$

where we use $\langle \bar{U} \rangle^2 / M^2 \sim y_{uc} \sim \sqrt{y_u y_c}$. This is incompatible with the bound from D – \bar{D} mixing, which requires $\delta m_{\bar{u}c}^2 / m_{\bar{u}}^2 \lesssim 10^{-2}$. There are also problems with K – \bar{K} mixing.

We next consider a model based on the $N = 15 + k = 18$ embedding described above. With such a large value for N , it may not be necessary to have a low value for $\langle \bar{U} \rangle$ to avoid negative third generation scalar masses, but we can consider the possibility of a low compositeness scale nonetheless. This model produces 2 complete composite generations of quarks and leptons, but contains a large number of fields charged under the standard model gauge group above the scale Λ_{18} of the strong $SU(18)$ dynamics. The standard-model gauge couplings have a Landau pole at a few times Λ_{18} in this model; so it is certainly not compatible with perturbative unification. Since the Landau pole is so close to Λ_{18} is not clear that this model makes sense as an effective theory at the scale Λ_{18} . However, the strong dynamics at the Landau scale may have an interpretation in terms of a dual theory [29], and we expect such a theory to behave qualitatively the same as what we find here. We can also hope that models with a more favorable group-theory structure will be found.

The problems with perturbative unification lead us naturally to consider high-scale models with large values of Λ_N . The high-scale and low-scale models with two composite generations have a similar phenomenology, and we will discuss this after we have introduced the high-scale models.

4.3 High-scale Composite Models

We now discuss the possibility that the compositeness scale Λ_N is near or above $M_{\text{GUT}} \sim 10^{16}$ GeV, allowing perturbative unification even if N is large. If the scale Λ_N is large, we must address the dangerous negative contributions to the third generation scalar masses coming from the scalars of the first two generations [25]. These arise from the renormalization group equations

$$\mu \frac{dm_3^2}{d\mu} = \frac{8g^2}{16\pi^2} C_2 \left[\frac{3g^2}{16\pi^2} m_{1,2}^2 - m_\lambda^2 \right], \quad (4.13)$$

where we have assumed that one gauge group dominates and specialized to the case of two composite generations. (Here, m_3 is the third-generation scalar mass, $m_{1,2}$ are the scalar masses of the first two generations, and m_λ is the gaugino mass. C_2 is the quadratic Casimir, with the $U(1)_Y$ generator in $SU(5)$ normalization.) We see that the contribution to the gaugino mass dominates provided that

$$m_\lambda \gtrsim \frac{m_{1,2}}{10}, \quad (4.14)$$

which agrees with the detailed analysis of Ref. [25]. This condition is plausibly satisfied in our models if $N \gtrsim 10$.

Since dimension-6 B -violating operators suppressed by such high scales are safe, we consider both the B -violating “squared” $N = 11$ as well as the B -conserving $N = 18$ theories. Both of these theories give rise to two complete composite generations.

It is believed that new physics at the Planck scale will give rise to higher-dimension operators suppressed by the reduced Planck scale $M_* \sim 10^{18}$ GeV. It is therefore natural to consider the possibility that it is these effects that give rise to the higher-dimension operators that are required to make the theory realistic, and identify $M = M_*$. In this case, Λ_N will be *above* M_{GUT} , and even those extra charged states that become massive due to higher dimension operators are massive enough (within one or two decades of M_{GUT}) in order to leave perturbative gauge coupling unification (marginally) intact.

Finally, even though $N = 11$ or $N = 18$ is plausibly large enough to overcome the problem of negative third-generation scalar masses even in high-scale theories, we

note that the new physics at the scale M gives rise to third-generation scalar masses of order $\delta m_3 \sim \sqrt{cN}(\langle \bar{U} \rangle / M) M_{\text{comp}}$, which can be in the range 100 GeV–1 TeV. If this contribution is positive, it may improve the problem with the negative log-enhanced contributions to the third-generation scalar masses. The contributions of new physics at the scale M_* can give the gravitino a mass of order 100 GeV in high-scale models, so the gravitino need not be lightest supersymmetric particle (LSP) in these models. The LSP is most likely a neutralino with a mass in the 100 GeV range, which is a traditional favorite candidate for cold dark matter.

4.4 Implications for Flavor Physics

We now turn to the phenomenological implications of the models with two composite generations, concentrating mainly on flavor physics.¹⁸ If we assume that new physics at the scale M is responsible for the Yukawa couplings, then the Yukawa couplings will arise from operators of the form Eq. (4.10).¹⁹ This gives rise to Yukawa matrix with the skeletal form

$$y \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad \epsilon \sim \left(\frac{\langle \bar{U} \rangle}{M} \right)^2. \quad (4.15)$$

It is clear that additional structure is needed to construct fully realistic Yukawa matrices. However, this is certainly a good starting point for constructing a theory of flavor, and the automatic ϵ suppressions due to the composite nature of the first two generations leave a milder hierarchy in the coefficients of the higher-dimension operators that needs to be explained. For ϵ in the range 10^{-2} to 10^{-1} , realistic fermion masses can be obtained with simple textures and hierarchies of order 10 in the effective coupling constants.

Let us turn to the issue of FCNC's due to non-degeneracy of the scalar masses of the first two generations. We emphasize again that, due to approximate flavor symmetries of the strong dynamics, the leading contribution to the soft masses is equal for the first two generations, and the issue is whether sufficient degeneracy is maintained to avoid FCNC constraints after the effects of the higher dimension

¹⁸Theories of flavor exploiting compositeness (but not addressing supersymmetry breaking) have been constructed in [30].

¹⁹Models with dynamical supersymmetry breaking and composite states with large global symmetries were also found in Ref. [11]. However, the composite states were high-dimension baryons, and the Yukawa couplings for the composite generations are suppressed by the ratio of the compositeness scale to the higher scale M raised to the 30th power. Therefore, these models cannot naturally produce large enough Yukawa couplings even for the light generations.

operators are included.²⁰ The size of the corrections depends on the flavor physics at the scale M . For example, we have already pointed out that it is possible that the flavor physics has a GIM mechanism that suppresses FCNC's. We now analyze the possibility that there is no alignment mechanism at the scale M , so the off-diagonal scalar masses are suppressed only by the powers of Λ/M that suppress the corresponding Yukawa couplings. The mixing contributions to the soft mass matrices come from operators such as

$$\delta\mathcal{L} \sim \int d^4\theta \left[\frac{c}{M^2} (A\bar{U})^\dagger (A\bar{U}) \right], \quad (4.16)$$

which give

$$\frac{\delta m_{jk}^2}{M_{\text{comp}}^2} \sim c \left(\frac{\langle \bar{U} \rangle}{M} \right)^2 \sim c \sqrt{y_{jk}}. \quad (4.17)$$

(Note that the operator of Eq. (3.10) is enhanced by a factor of N , but is flavor-diagonal.) The most stringent FCNC bounds come from the $K-\bar{K}$ system, and can be summarized as

$$\text{Re} \left(\frac{\delta m_{d\bar{s}}^2}{M_{\text{comp}}^2} \right) \lesssim 10^{-1} \frac{M_{\text{comp}}}{10 \text{ TeV}}, \quad \text{Im} \left(\frac{\delta m_{d\bar{s}}^2}{M_{\text{comp}}^2} \right) \lesssim 10^{-2} \frac{M_{\text{comp}}}{10 \text{ TeV}}. \quad (4.18)$$

The constraint from $\text{Re}(\delta m_{d\bar{s}}^2)$ gives (using $y_{ds} \sim \sqrt{y_d y_s}$)

$$c \lesssim 5 \frac{M_{\text{comp}}}{10 \text{ TeV}}, \quad (4.19)$$

which is plausibly satisfied for M_{comp} as low as 1 TeV given the uncertainties. In order to also evade the bounds from $\text{Im}(\delta m_{d\bar{s}}^2)$, we must assume that the CP -violating phase in this quantity is somewhat small, of order $\frac{1}{10}$. Alternately, $M_{\text{comp}} \sim 10 \text{ TeV}$ is completely safe from all constraints.

Even if the induced non-degeneracies between the first two generation sfermions are small enough to avoid present FCNC constraints, there is still a rich spectrum of flavor changing signals due to the non-degeneracy between the first two and third generation sfermions. If the sfermion mixing angles are CKM-like, flavor-violating signals are expected at experimentally interesting levels in a wide variety of processes such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $B-\bar{B}$ mixing, and electron/neutron electric dipole moments [31].

²⁰Flavor symmetries have been used to constrain both the form of the Yukawa matrices and the scalar mass matrices, thereby addressing both the supersymmetric and usual flavor problems [26, 27]. In our case, however, the approximate flavor symmetry guaranteeing scalar degeneracy need not be respected by the higher dimension operators generating the Yukawa couplings.

Finally, we note that the new physics at the scale M may provide a solution [32] to the “ μ problem.” If the low-energy theory contains the terms

$$\delta\mathcal{L} = \int d^4\theta \frac{c'}{M} \text{tr}(\bar{U}^\dagger \bar{U})(H\bar{H} + \text{h.c.}), \quad (4.20)$$

where H, \bar{H} are the standard-model Higgs fields, then the low-energy theory contains μ and $B\mu$ terms of order

$$\mu \sim c'N \frac{\langle F_{\bar{U}} \rangle \langle \bar{U} \rangle}{M^2} \sim c'N \sqrt{y} M_{\text{comp}}, \quad B\mu \sim c'N \frac{\langle F_{\bar{U}} \rangle^2}{M^2} \sim c'N \sqrt{y} M_{\text{comp}}^2, \quad (4.21)$$

where $y \sim (\langle \bar{U} \rangle / M)^4$ is the magnitude of a Yukawa coupling generated at the scale M . If we want $\mu^2 \sim B\mu$, then we need $c'N \sim 1/\sqrt{y}$, which is plausible for large N . (The parameter c' is of order 1 or $1/N$, as in the discussion below Eq. (3.11).) In this case, both μ and $B\mu$ are naturally near M_{comp} , which is somewhat large for electroweak symmetry breaking even if $M_{\text{comp}} \sim 1$ TeV. However, given the large uncertainties and model-dependence in these estimates, this mechanism may work in a detailed model.

4.5 Decay of the False Vacuum

All of the models above require that the universe live in a false vacuum on the “baryon” branch, and so we must consider the possibility of the decay of the vacuum. All of the supersymmetric vacua occur at infinite field values on other branches of the moduli space. Therefore, the energy difference between the false vacuum and the true vacuum is small compared to the distance in field space to the classical escape point. We can therefore give a conservative bound on the tunneling rate by approximating the potential as completely flat. In that case, the Euclidean tunneling action is [33]

$$S_{\text{tunnel}} \simeq 2\pi^2 \frac{(\Delta\phi)^4}{V}, \quad (4.22)$$

where $\Delta\phi$ is the distance in field space to the classical escape point, and $V \sim \langle F_{\bar{U}} \rangle^2$ is the value of the energy density in the false vacuum. Since $(\Delta\phi)^2 \gg \langle F_{\bar{U}} \rangle$ in our models, this always gives a negligible tunneling rate.

5 Discussion and Conclusions

We have presented new models of dynamical supersymmetry breaking in which the same strong dynamics breaks supersymmetry and gives rise to massless composite

fermions that we identify with quarks and leptons of the first two generations. Since the corresponding composite squarks and sleptons arise directly from the supersymmetry breaking sector, they receive supersymmetry-breaking soft masses directly, without “mediation” via gravitational or SM gauge interactions. In this sense, these models provide an alternative to the “modular” structure of realistic models of supersymmetry breaking, where supersymmetry is broken in a separate sector of the model and communicated by messenger interactions to the observed particles.

It is also pleasing that the models we construct are quite simple. As an illustration, we write the complete $N = 18$ model below. The gauge group is

$$SU(4) \times SU(18) \times [SU(18)] \quad (5.1)$$

where $SU(5)_{\text{SM}}$ (the usual embedding of the standard-model group) is embedded into $[SU(18)]$ so that $\mathbf{18} \rightarrow \bar{\mathbf{5}} + \mathbf{10} + (3 \times \mathbf{1})$. The field content is

$$\begin{aligned} Q &\sim (\square, \square, \mathbf{1}) , \\ L &\sim (\bar{\square}, \mathbf{1}, \square) , \\ \bar{U} &\sim (\mathbf{1}, \bar{\square}, \bar{\square}) , \\ A &\sim (\mathbf{1}, \boxplus, \mathbf{1}) , \end{aligned} \quad (5.2)$$

together with a single (third) generation $\Phi_{\bar{\mathbf{5}}}$, $\Phi_{\mathbf{10}}$ and Higgs fields H , \bar{H} . The model has a superpotential of the form

$$\begin{aligned} W &\sim LQ\bar{U} + H\Phi\Phi + \frac{1}{M^2}(A\bar{U}^2)H\Phi \\ &+ \frac{1}{M^3}(A\bar{U}^2)(A\bar{U}^2) + \frac{1}{M^4}(A\bar{U}^2)(A\bar{U}^2)H \end{aligned} \quad (5.3)$$

where we have omitted indices for simplicity. The higher-dimension operators generate Yukawa couplings involving the composite states and eliminate unwanted composite fermions from the low-energy spectrum. This model generates two composite generations of quarks and leptons with small Yukawa couplings, breaks supersymmetry, communicates supersymmetry breaking directly to the composite squarks and sleptons, and gives sufficiently large gaugino masses through gauge loops.

It is striking that a simple model such as this can be completely realistic, with the compositeness scale ranging anywhere from 10 TeV to the Planck scale. The leading contribution to the scalar masses is naturally flavor-diagonal due to an approximate symmetry of the strong dynamics that is present even if λ has arbitrary flavor structure; this symmetry is violated only by “perturbative” corrections of order

$\lambda^2/(16\pi^2) \sim 10^{-4}$. These global symmetries also lead to the striking prediction that (depending on the model) some or all of the scalar masses of the first two generations unify at the compositeness scale, which need not be close to the GUT scale. (Models with flavor symmetries can also predict scalar unification at some level, but they cannot naturally explain unification between scalars with different gauge quantum numbers below the GUT scale.²¹) We emphasize that these features are present in our model without the need to impose any flavor symmetry on the underlying theory.

The Yukawa couplings are generated by new physics at a scale above the compositeness scale, naturally explaining why the fermion masses of the first two generations are small, while the corresponding scalar masses are large. In the absence of any flavor alignment mechanism, the off-diagonal terms are just compatible with existing constraints on CP -conserving FCNC's if the scalar masses are in the 1 TeV range. (Consistency with ϵ_K requires scalar masses of order 10 TeV.) In either case, one expects FCNC's that may be observed with increased experimental sensitivity. The models require a dynamical assumption regarding the sign of a dynamically-generated mass term. (If the sign is opposite to what is assumed here, one can use the dynamics to build a composite messenger model of direct gauge-mediated supersymmetry breaking along the lines of Ref. [11].)

We close with some speculations on how to build more attractive models based on the ideas presented here. The models discussed in this paper have a large number of states charged under the standard-model gauge group above the compositeness scale, resulting in a Landau pole close to the compositeness scale. Also, the scale of flavor physics must be very close to the compositeness scale in order to generate sufficiently large Yukawa couplings. Both of these potential difficulties may be alleviated if one could find models where the composite states correspond to dimension-2 “meson” operators of the form $P_1 P_2$, where $P_{1,2}$ are strongly-coupled preons. In that case, Yukawa couplings involving the composite states arise from terms in the Lagrangian of the form

$$\delta\mathcal{L} \sim \int d^2\theta \left[\frac{1}{M^2} (P_1 P_2)^2 H + \frac{1}{M} (P_1 P_2) \Phi H \right] + \text{h.c.}, \quad (5.4)$$

where H is an elementary Higgs field and Φ is an elementary third-generation quark or lepton field. This gives rise to Yukawa couplings for the composite fields of order

$$y_q \sim \frac{\langle P \rangle^2}{M^2} \quad (5.5)$$

²¹Even if the scalar unification scale is close to the GUT scale, the model above predicts that *all* squarks and sleptons from the first two generations unify. Even the degeneracy of squark and slepton masses within a generation is not easy to understand in $SO(10)$, since it is broken by D terms corresponding to broken generators.

compared to $\langle P \rangle^4/M^4$ in our models. This would allow the flavor scale to be larger compared to the compositeness scale. Off-diagonal scalar mass terms for the composite fields arise from

$$\delta\mathcal{L} \sim \int d^4\theta \frac{1}{M^2} (P_1^\dagger P_1)(P_2^\dagger P_2), \quad (5.6)$$

giving rise to off-diagonal scalar masses for the composite states

$$\frac{\delta m_{jk}^2}{M_{\text{comp}}^2} \sim \frac{\langle F_P \rangle^2}{M^2} \sim \frac{\langle P \rangle^2}{M^2} M_{\text{comp}}^2 \sim y_{jk} M_{\text{comp}}^2, \quad (5.7)$$

where y_{jk} is the corresponding off-diagonal Yukawa coupling. This is an extra suppression by $\sqrt{y_{jk}}$ compared to our models, which makes FCNC's completely safe. Finally, one might hope that the group-theory structure of such models allows more economical models with a higher Landau pole for the standard-model interactions. It is also interesting to see if models of this type can give rise to realistic theories of flavor. We believe that these are promising directions, and work along these lines is in progress [34].

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References

- [1] A. Chamseddine, R. Arnowitt, P. Nath, *Phys. Rev. Lett.* **49**, 970 (1982); R. Barbieri, S. Ferrara, C. Savoy, *Phys. Lett.* **119B**, 343 (1982); L. Hall, J. Lykken, S. Weinberg, *Phys. Rev.* **D27**, 2359 (1983).
- [2] M. Dine, W. Fischler, M. Srednicki, *Nucl. Phys.* **B189**, 575 (1981); S. Dimopoulos, S. Raby, *Nucl. Phys.* **B192**, 353 (1981); L. Alvarez–Gaumé, M. Claudson, M.B. Wise, *Nucl. Phys.* **B207**, 96 (1982).
- [3] M. Dine, A.E. Nelson, *Phys. Rev.* **D48**, 1277 (1993), hep-ph/9303230; M. Dine, A.E. Nelson, Y. Shirman, *Phys. Rev.* **D51**, 1362 (1995), hep-ph/9408384; M. Dine, A.E. Nelson, Y. Nir, Y. Shirman, *Phys. Rev.* **D53**, 2658 (1996), hep-ph/9507378.
- [4] H. Murayama, *Phys. Rev. Lett.* **79**, 18 (1997), hep-ph/9705271; S. Dimopoulos, G. Dvali, R. Rattazzi, G.F. Giudice, hep-ph/9705307; M.A. Luty, hep-ph/9706554, to appear in *Phys. Lett.* **B**.
- [5] S. Dimopoulos, H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
- [6] M.A. Luty, R.N. Mohapatra, *Phys. Lett.* **396B**, 161 (1997), hep-ph/9611343; N. Okada, hep-ph/9711342; N. Kitazawa, hep-ph/9712223.
- [7] M. Strassler *Phys. Lett.* **376B**, 119 (1997), hep-ph/9510342; A.E. Nelson, M. Strassler, *Phys. Rev.* **D56**, 4226 (1997), hep-ph/9607362.
- [8] G. ‘t Hooft, *Recent Developments in Gauge Theories* (Plenum Press, 1980) 135; reprinted in *Unity of Forces in the Universe Vol. II*, A. Zee ed. (World Scientific 1982) 1004.
- [9] M.A. Luty, hep-ph/9611387.
- [10] S. Dimopoulos, G.F. Giudice, *Phys. Lett.* **357B**, 573 (1995); A.G. Cohen, D.B. Kaplan, A.E. Nelson, *Phys. Lett.* **388B**, 588 (1996), hep-ph/9607394.
- [11] M.A. Luty, J. Terning, hep-ph/9709306.
- [12] P. Pouliot, *Phys. Lett.* **367B**, 151 (1996), hep-th/9510148.
- [13] C. Csáki, M. Schmaltz, W. Skiba, *Phys. Rev. Lett.* **78**, 799 (1997), hep-th/9610139; *Phys. Rev.* **D55**, 7840 (1997), hep-th/9612207.

- [14] K. Intriligator, S. Thomas, hep-th/9608046; Y. Shirman, *Phys. Lett.* **389B**, 287 (1996).
- [15] E. Fradkin, S.H. Shenker, *Phys. Rev.* **D19**, 3682 (1979); S. Dimopoulos, S. Raby, L. Susskind, *Nucl. Phys.* **B173**, 208 (1980).
- [16] I. Affleck, M. Dine, N. Seiberg, *Phys. Lett.* **137B**, 187 (1984); Y. Meurice, G. Veneziano, *Phys. Lett.* **141B**, 69 (1984); D. Amati, K. Konishi, Y. Meurice, G.C. Rossi, *Phys. Rep.* **162**, 169 (1988).
- [17] I. Affleck, M. Dine, N. Seiberg, *Nucl. Phys.* **B256**, 557 (1985).
- [18] E. Witten, *Phys. Lett.* **105B**, 267 (1981).
- [19] A.V. Manohar, H. Georgi, *Nucl. Phys.* **B234**, 189 (1984); H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin/Cummings, (Menlo Park, 1984); H. Georgi, L. Randall, *Nucl. Phys.* **B276**, 241 (1986).
- [20] M.A. Luty, hep-ph/9706235, to appear in *Phys. Rev.* **D**; A.G. Cohen, D.B. Kaplan, A.E. Nelson, hep-ph/9706275.
- [21] G.F. Giudice, R. Rattazzi, hep-ph/9706540.
- [22] N. Arkani-Hamed, G.F. Giudice, M.A. Luty, R. Rattazzi, in preparation.
- [23] B. Pendleton, G.G. Ross, *Phys. Lett.* **98B**, 291 (1981); C.T. Hill, *Phys. Rev.* **D24**, 691 (1981).
- [24] See G. Farrar, *Nucl. Phys. Proc. Suppl.* **62**, 485 (1998), hep-ph/9710277, and references therein.
- [25] N. Arkani-Hamed, H. Murayama, *Phys. Rev.* **D56**, 6733 (1997), hep-ph/9703259.
- [26] M. Dine, R. Leigh, A. Kagan, *Phys. Rev.* **D48**, 4269 (1993), hep-ph/9304307; R. Barbieri, L. Hall, S. Raby, A. Romanino, *Phys. Lett.* **493B**, 3 (1997), hep-ph/9610449; L. Hall, H. Murayama, *Phys. Rev. Lett.* **75**, 3985 (1995), hep-ph/9508296.
- [27] Y. Nir, N. Seiberg, *Phys. Lett.* **309B**, 337 (1993), hep-ph/9304299.
- [28] S. Dimopoulos, H. Georgi, S. Raby, *Phys. Lett.* **127B**, 101 (1983); S.-C. Chao, K. Lane, *Phys. Lett.* **159B**, 135 (1985); R.S. Chivukula, H. Georgi, *Phys. Lett.* **188B**, 99 (1987).

- [29] N. Seiberg, *Nucl. Phys.* **B435**, 129 (1995), hep-th/9411149.
- [30] D.B. Kaplan, F. Lepeintre, M. Schmaltz, *Phys. Rev.* **D56**, 7193 (1997), hep-ph/9705411.
- [31] R. Barbieri, L. Hall, A. Strumia, *Nucl. Phys.* **B445**, 219 (1995), hep-ph/9501334.
- [32] G.F. Giudice, A. Masiero, *Phys. Lett.* **206B**, 480 (1988).
- [33] K. Lee, E.J. Weinberg, *Nucl. Phys.* **B267**, 181 (1986); M.J. Duncan, L.G. Jensen, *Phys. Lett.* **291B**, 109 (1992).
- [34] N. Arkani-Hamed, M.A. Luty, J. Terning, work in progress.