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Chiral Technicolor and Precision Electroweak Measurements

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Abstract

I consider the possibility that electroweak symmetry is broken by a strongly interacting chiral gauge theory. I argue that some of the discrepancies between precision electroweak measurements and the predictions of QCD-like technicolor models can be resolved if technicolor is a chiral gauge theory. I present a toy technicolor model which demonstrates this idea, and gives $m_t \gg m_b$, with a small value for $\Delta\rho_* \equiv \alpha T$, and small corrections to $Z \to b\bar{b}$.

1 Introduction

The origin of electroweak symmetry breaking remains a basic problem of particle physics. There has recently been a small revival of interest in the technicolor (TC) approach to electroweak symmetry breaking [1], and in building realistic extended technicolor (ETC) models which reproduce the quark-lepton mass spectrum without phenomenological disasters like flavor changing neutral currents [2, 3]. Effort in this direction has focused on avoiding the pitfalls of TC models which relied on naively scaling-up the properties of QCD. This work has included examining theories possessing: smaller β functions (a.k.a. walking [4]), scalars [5], near-critical four-fermion interactions (strong ETC [6]), multiple scales of electroweak symmetry breaking (multiscale models [7]),and GIM symmetries (Techni-GIM [8]). Although there has been some measure of success along these lines, precision electroweak measurements continue to put tighter constraints on TC models, and if experiments converge near the current central values for S, T, m_t, and the partial width $\Gamma(Z \to b\bar{b})$, then life will be extremely difficult for model builders. The problems of compatibility with precision electroweak measurements, combined with the general awkwardness of obtaining a reasonable fermion spectrum seem to suggest that some new ingredient is required for a truly realistic ETC model.

In all the variants of TC mentioned above, there is one important common assumption: that TC is vector-like. That is (like QCD) left-handed and right-handed technifermions have identical TC gauge interactions. In this paper I will start to explore what can happen when this assumption is dropped. In the next section I describe in more detail the problems of vector-like TC models, and suggest how these problems could be ameliorated in chiral TC models. In the third and fourth sections I examine in some detail a toy chiral TC model which demonstrates some of the general features of this approach.

2 Problems with Vector-Like TC

There are three potential conflicts between the current crop of TC models and experiment that I would like to focus on:

• current measurements of the ratio of $\Gamma(Z \to b\bar{b})$ to $\Gamma(Z \to q\bar{q})$ are 2% above the standard model prediction (with an error around 1%), while simple TC models suggest that this ratio should be several percent below the standard model [9],

• the S parameter seems to be negative, or at least small, whereas simple QCD-like TC models prefer positive values [10] of order one,

• the t quark is very heavy, but in a TC framework it is difficult to make $m_t \gg m_b$, and it is even more difficult to do this while keeping $\Delta\rho_* \equiv \alpha T$ small [11, 12].

Each of these potential problems can be individually circumvented. Models where $SU(2)_L$ does not commute with ETC interactions can produce the opposite sign correction to the ratio of $\Gamma(Z \to b\bar{b})/\Gamma(Z \to q\bar{q})$ [13]. Alternatively, the correction can be reduced by fine-tuning ETC interactions [14]. There are several schemes which can produce negative contributions to S [15, 16]. One can arrange a model to make $m_t \gg m_b$ by fine-tuning ETC interactions [6] or by having different ETC scales [17] for the t_R and the b_R. However, these three problems are all intimately related to isospin breaking, and it is possible that there is a common solution to all three problems. It is this possibility that leads to the consideration of chiral TC models. In order to see why, it will be useful to consider the three problems in more detail

One possible solution to the $Z \to b\bar{b}$ problem is that the ETC corrections are actually much smaller than is expected in vector-like TC models. Recall that the size of the correction is proportional to

$$g^2_{ETC}\frac{f^2}{M^2_{ETC}} \approx \frac{m_t}{4\pi f} , \tag{1}$$

(where g_{ETC} is the ETC gauge coupling, M_{ETC} is the mass of the ETC gauge boson, and f is the technipion decay constant) which comes from assuming that the technifermions which couple to the b quark are approximately degenerate with those that couple to the t quark [9]. The implicit assumption being that if this were not the case $\Delta\rho_* \equiv \alpha T$ would be large. With this assumption the size of the correction is related to the t quark mass. Given that m_t is probably near 175 GeV, and $f < 246$ GeV, the correction is quite large.

If, however, the up-type and down-type technifermions were not degenerate, then there would be two different technipion decay constants: f_U and f_D. Then equation (1) would be replaced by:

$$g_{ETC}^2 \frac{f_D^2}{M_{ETC}^2} \ . \tag{2}$$

If the ETC scales are the same for the t and the b, then this suggests a smaller correction than is usually expected. Of course it remains to explain why T is small in such a model.

Next, consider the problem of negative S. A simple mechanism for producing negative contributions to S in a one-family model was suggested in ref. [16]. There a large mass splitting between the technielectron and the technineutrino gave a negative contribution to S. In order to keep T small, the technileptons had to be much lighter than the techniquarks, and the techniquarks had to be roughly degenerate. Since, naively, the contribution to the $W^\pm$ and Z masses (and hence to T) scales as the technifermion mass squared (or technipion decay constant squared) the degenerate techniquarks provided the bulk of the gauge boson masses, and kept T small. Note however that there are three times as many techniquarks as technileptons, and the techniquarks (not to mention the pseudo-Nambu-Goldstone bosons) give a positive contribution to S, so that although S could be reduced from naive expectations, it could not be made substantially negative. Now, if the up-type techniquark was much heavier than the down-type techniquark (as was suggested by the consideration of the $Z \to b\bar{b}$ problem), then the techniquark contribution to S would be reduced. Again, this solution, on the face of it, seems incompatible with a small value for T.

Finally, consider the problem of the large t quark mass. Obviously the simplest way to make $m_t \gg m_b$ is to (again) have an up-type techniquark that is much heavier than the down-type techniquark. Thus all three problems are at least partially alleviated by isospin breaking in the techniquark sector. The price of this solution seems to be an even bigger problem: the wrong masses for the $W^{\pm}$ and the Z, or in other words a very large value for T.

Traditional TC models have always done a beautiful job of getting the correct masses for the $W^{\pm}$ and Z. The problems with TC arise when one tries to use these same technifermions to give masses to the quarks and leptons. Once we assume that TC is vector-like, the $W^{\pm}$ and Z masses are guaranteed to be in the correct ratio by the unbroken $SU(2)_V$ custodial isospin group. When considering the problems of precision electroweak measurements above, I was lead to consider a spectrum of technifermions which is compatible with quark and lepton masses, but gives the wrong gauge boson masses. A resolution of this discrepancy is obvious, if perhaps inelegant. If there are two types of technifermions (that is technifermions in two different representations of the TC gauge group) then we can imagine that one type gives masses to the gauge bosons, while the other type primarily gives masses to the quarks and leptons. (This is what happens in multiscale TC models [7].) Since both types of technifermions must carry $SU(2)_L$ quantum numbers, they will in fact both contribute to the gauge boson masses. However it is the heavier technifermions that will provide the bulk of the gauge boson masses, and it is these technifermions that must have a good custodial isospin symmetry. Ensuring that only the light (isospin breaking) sector contributes to quark and lepton masses depends entirely on how the TC gauge group is embedded in the ETC gauge group.

The remaining problem is how to break isospin symmetries in the light technifermion sector. One could imagine putting the right-handed up-type and down-type technifermions in different ETC representations which decompose into equivalent TC representations [3, 7, 11, 18] (e.g. when $SU(3)_{ETC}$ breaks to $SU(2)_{TC}$, a $\mathbf{3}$ and $\overline{\mathbf{3}}$ both decompose to $\mathbf{1}+\mathbf{2}$). Then, if the ETC interactions are strong enough, they can produce a substantial splitting of technifermion masses. A more direct approach is to put the right-handed up-type and down-type technifermions in different TC representations, which implies that TC is a chiral gauge theory. In the next section I will present a toy model which exemplifies these ideas.

3 A Toy Model

The model is anomaly free and asymptotically free, so that although it is not complete (for example, there are numerous massless Nambu-Goldstone bosons), it is at least internally consistent. The gauge group of the model is $SU(5)_{ETC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with the fermion content taken to be[1]:

$$\begin{array}{ccc} (\mathbf{5},\mathbf{3},\mathbf{2})_{\frac{1}{3}} & (\overline{\mathbf{5}},\overline{\mathbf{3}},\mathbf{1})_{-\frac{4}{3}} & (\mathbf{5},\overline{\mathbf{3}},\mathbf{1})_{\frac{2}{3}} \\ (\overline{\mathbf{5}},\mathbf{1},\mathbf{2})_{-1} & (\mathbf{10},\mathbf{1},\mathbf{1})_{0} & (\overline{\mathbf{5}},\mathbf{1},\mathbf{1})_{2} \end{array} \tag{3}$$

$$(\mathbf{10},\mathbf{1},\mathbf{2})_{0} \quad (\mathbf{10},\mathbf{1},\mathbf{1})_{-1} \quad (\mathbf{10},\mathbf{1},\mathbf{1})_{1} \tag{4}$$

$$(\overline{\mathbf{15}},\mathbf{1},\mathbf{1})_{0} \quad (\mathbf{5},\mathbf{1},\mathbf{1})_{0}\ . \tag{5}$$

[1] I will use charge-conjugates of the right-handed fields.

The **10** is the familiar antisymmetric tensor representation of $SU(5)_{ETC}$, while the **15** is the symmetric tensor representation. With the fermions listed above, the $SU(5)_{ETC}$ gauge anomaly cancels since (in a certain normalization) the **5**'s and **10**'s each contribute $+1$ to the anomaly while the $\overline{\mathbf{15}}$ contributes[2] -9. The third generation of quarks and leptons, and their associated technifermions, are contained in the representations listed in (3). The fermions listed in (4) will turn out to contain the heavy technifermion sector.

This model is an example of a chiral gauge theory, and I expect that when the $SU(5)_{ETC}$ gauge interaction becomes strong it will break itself by forming a fermion condensate that is a gauge-non-singlet (this phenomena is referred to as "tumbling" [19] in the literature). Folklore suggests that the condensate will form in the most attractive channel (MAC) [19, 20], as determined by the examining the relative attractive strength of the exchange of one massless gauge boson in the various channels. In this approximation the strength of the channel $\mathbf{R_1} \times \mathbf{R_2} \to \mathbf{R}$ is proportional to

$$\Delta C_2(\mathbf{R_1} \times \mathbf{R_2} \to \mathbf{R}) \equiv C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\mathbf{R}) \ , \tag{6}$$

where $C_2(\mathbf{R})$ is the quadratic Casimir of representation $\mathbf{R}$. The MAC in this model is $\overline{\mathbf{15}} \times \mathbf{5} \to \overline{\mathbf{5}}$ with $\Delta C_2 = 28/5$. The next most attractive channels are $\mathbf{5} \times \overline{\mathbf{5}} \to \mathbf{1}$ and $\mathbf{10} \times \mathbf{10} \to \overline{\mathbf{5}}$, both with $\Delta C_2 = 24/5$. I will assume that condensation occurs in the MAC, which breaks $SU(5)_{ETC}$ down to $SU(4)_{TC}$.

Below the ETC scale (noting that under $SU(4)_{TC}$ the $\overline{\mathbf{15}}$ decomposes as $\mathbf{1}+\overline{\mathbf{4}}+\overline{\mathbf{10}}$) we have the following fermions (labeled by $SU(4)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes$

[2] For $SU(N)$, $N \geq 3$, the antisymmetric tensor contributes $N-4$ to the anomaly, while the symmetric tensor contributes $N+4$.

$U(1)_Y)$:

$$\begin{array}{ccc} (\mathbf{1},\mathbf{3},\mathbf{2})_{\frac{1}{3}} & (\mathbf{1},\overline{\mathbf{3}},\mathbf{1})_{-\frac{4}{3}} & (\mathbf{1},\overline{\mathbf{3}},\mathbf{1})_{\frac{2}{3}} \\ (t,b)_L & t^c_R & b^c_R \\ & & \\ (\mathbf{4},\mathbf{3},\mathbf{2})_{\frac{1}{3}} & (\overline{\mathbf{4}},\overline{\mathbf{3}},\mathbf{1})_{-\frac{4}{3}} & (\mathbf{4},\overline{\mathbf{3}},\mathbf{1})_{\frac{2}{3}} \\ (U,D)_L & U^c_R & D^c_R \end{array} \tag{7}$$

$$\begin{array}{ccc} (\mathbf{1},\mathbf{1},\mathbf{2})_{-1} & (\mathbf{6},\mathbf{1},\mathbf{1})_{0} & (\mathbf{1},\mathbf{1},\mathbf{1})_{2} \\ (\tau,\nu_\tau)_L & & \tau^c_R \\ & & \\ (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1} & (\mathbf{4},\mathbf{1},\mathbf{1})_{0} & (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{2} \\ (N,E)_L & N^c_R & E^c_R \end{array} \tag{8}$$

$$\begin{array}{ccc} (\mathbf{4},\mathbf{1},\mathbf{2})_{0} & (\mathbf{4},\mathbf{1},\mathbf{1})_{-1} & (\mathbf{4},\mathbf{1},\mathbf{1})_{1} \\ (\mathbf{6},\mathbf{1},\mathbf{2})_{0} & (\mathbf{6},\mathbf{1},\mathbf{1})_{-1} & (\mathbf{6},\mathbf{1},\mathbf{1})_{1} \end{array} \tag{9}$$

$$(\overline{\mathbf{10}},\mathbf{1},\mathbf{1})_0 \ . \tag{10}$$

Note that $SU(4)_{TC}$ is an asymptotically free chiral gauge theory. As we descend the energy scale, further fermion condensation should occur. I expect that the next condensate will form in the new MAC, which is $\mathbf{6}\times\mathbf{6}\to\mathbf{1}$, with $\Delta C_2 = 5$. This breaks $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{em}$, and makes some extra technineutrinos in line (8) heavy. I will refer to the technifermions which condense at this scale as the heavy technifermions. Below the electroweak symmetry breaking scale we have the following fermions (labeled according to $SU(4)_{TC}\otimes SU(3)_C \otimes U(1)_{em}$):

$$\begin{array}{cccc} (\mathbf{1},\mathbf{3})_{\frac{2}{3}} & (\mathbf{1},\mathbf{3})_{-\frac{1}{3}} & (\mathbf{1},\overline{\mathbf{3}})_{-\frac{2}{3}} & (\mathbf{1},\overline{\mathbf{3}})_{\frac{1}{3}} \\ t_L & b_L & t^c_R & b^c_R \\ & & & \\ (\mathbf{4},\mathbf{3})_{\frac{2}{3}} & (\mathbf{4},\mathbf{3})_{-\frac{1}{3}} & (\overline{\mathbf{4}},\overline{\mathbf{3}})_{-\frac{2}{3}} & (\mathbf{4},\overline{\mathbf{3}})_{\frac{1}{3}} \\ U_L & D_L & U^c_R & D^c_R \end{array} \tag{11}$$

$$\begin{array}{cccc} (\mathbf{1},\mathbf{1})_{0} & (\mathbf{1},\mathbf{1})_{-1} & & (\mathbf{1},\mathbf{1})_{1} \\ \tau_L & \nu_{\tau L} & & \tau^c_R \\ & & & \\ (\overline{\mathbf{4}},\mathbf{1})_{0} & (\overline{\mathbf{4}},\mathbf{1})_{-1} & (\mathbf{4},\mathbf{1})_{0} & (\overline{\mathbf{4}},\mathbf{1})_{1} \\ N_L & E_L & N^c_R & E^c_R \end{array} \tag{12}$$

$$(\mathbf{4},\mathbf{1})_{\frac{1}{2}} \quad (\mathbf{4},\mathbf{1})_{-\frac{1}{2}} \quad (\mathbf{4},\mathbf{1})_{-\frac{1}{2}} \quad (\mathbf{4},\mathbf{1})_{\frac{1}{2}} \tag{13}$$

$$(\overline{\mathbf{10}},\mathbf{1})_0 \ . \tag{14}$$

Even though the electroweak gauge symmetry has been broken, the effective $SU(4)_{TC}$ gauge theory is chiral, and further condensates are to be expected. The MAC is now $\overline{\mathbf{10}} \times \mathbf{4} \to \overline{\mathbf{4}}$ with $\Delta C_2 = 9/2$. This condensate would break $SU(4)_{TC}$ down to $SU(3)$. However, here I will assume that the MAC analysis is not correct. I imagine that the next condensation occurs in the channel $\mathbf{4} \times \mathbf{4} \to \mathbf{6}$, which breaks $SU(4)_{TC}$ down to $Sp(4)_{RTC}$. (See the Appendix for a justification of this assumption.) I will refer to this unbroken subgroup as the residual technicolor (RTC) group. With this pattern of condensation, all the light technifermions associated with the third family will become massive. Below the TC breaking scale we have (labeled according to $Sp(4)_{RTC} \otimes SU(3)_C \otimes U(1)_{em}$) the following fermion content:

$$\begin{array}{cccc}
(\mathbf{1},\mathbf{3})_{\frac{2}{3}} & (\mathbf{1},\mathbf{3})_{-\frac{1}{3}} & (\mathbf{1},\overline{\mathbf{3}})_{-\frac{2}{3}} & (\mathbf{1},\overline{\mathbf{3}})_{\frac{1}{3}} \\
t_L & b_L & t_R^c & b_R^c \\
 & & & \\
(\mathbf{1},\mathbf{1})_0 & (\mathbf{1},\mathbf{1})_{-1} & & (\mathbf{1},\mathbf{1})_1 \\
\tau_L & \nu_{\tau L} & & \tau_R^c
\end{array} \tag{15}$$

$$(\mathbf{10},\mathbf{1})_0 \ . \tag{16}$$

Finally, at this stage there is nothing to prevent condensation in the channel $\mathbf{10} \times \mathbf{10} \to \mathbf{1}$, and we are left with only the usual third family. We note that because of the chiral structure, the only third generation fermion that receives a mass is the t quark. This is, in fact, a good first approximation to the observed spectrum. To give masses to the other fermions, one would have to embed the $SU(5)_{ETC}$ gauge group in a larger group, but I will not pursue this possibility, since this is only a toy model. In the next section I will attempt to analyze the technifermion spectrum in some detail, not because this model is especially interesting as it stands, but in order to illustrate some generic features of chiral TC models.

4 The Technifermion Spectrum

Given the current state of field theory technology, we will have to be satisfied with the traditional analysis of the Schwinger-Dyson equation for the self-energy (in the traditional ladder approximation, for a discussion of the reliability of this approximation see ref. [21]). In the unbroken theory the self-energy graph of a fermion in representation $\mathbf{R}$ is proportional to $C_2(\mathbf{R})$, which comes from summing over the square of gauge generators. In chiral gauge theories the left-handed and right-handed-conjugate fermions can be in different representations ($\mathbf{R_1}$ and $\mathbf{R_2}$ say), and the condensate is not necessarily a singlet (call it $\mathbf{R}$). Neglecting the gauge boson masses for a moment, the factor $C_2(\mathbf{R})$ must be replaced by $\frac{1}{2}\Delta C_2(\mathbf{R_1} \times \mathbf{R_2} \rightarrow \mathbf{R})$ as given in equation (6). In a broken gauge theory (that is when the representation, $\mathbf{R}$, of the condensate is not a gauge singlet) we must separate the gauge bosons into their representations under the unbroken subgroup, since in general these different representations will have different masses. For the toy model I am discussing, the gauge bosons of $SU(5)_{ETC}$ decompose under $Sp(4)_{RTC}$ as $\mathbf{10}+\mathbf{5}+\mathbf{4}+\mathbf{4}+\mathbf{1}$, where the $\mathbf{10}$ are the massless gauge bosons of $Sp(4)_{RTC}$, the $\mathbf{4}$'s and $\mathbf{1}$ are the heavy ETC gauge bosons, and the $\mathbf{5}$ are the broken TC gauge bosons. Thus I will write

$$\Delta C_2(\mathbf{R_1} \times \mathbf{R_2} \rightarrow \mathbf{R}) = \sum_{\mathbf{G}} \Delta C_2^{\mathbf{G}} \ , \tag{17}$$

where the gauge boson representation, $\mathbf{G}$, runs over $\mathbf{1}$,$\mathbf{4}$,$\mathbf{5}$, and $\mathbf{10}$. For the sake of brevity the values for $\Delta C_2^{\mathbf{G}}$ in the various channels are presented in Table 1.

fermion	channel	$\Delta C_2^{\mathbf{10}}$	$\Delta C_2^{\mathbf{5}}$	$\Delta C_2^{\mathbf{4}}$	$\Delta C_2^{\mathbf{1}}$	ΔC_2
U	$\mathbf{5} \times \overline{\mathbf{5}} \to \mathbf{1}$	$\frac{5}{2}$	$\frac{5}{4}$	1	$\frac{1}{20}$	$\frac{24}{5}$
t	$\mathbf{5} \times \overline{\mathbf{5}} \to \mathbf{1}$	0	0	4	$\frac{4}{5}$	$\frac{24}{5}$
D	$\mathbf{5} \times \mathbf{5} \to \mathbf{10}$	$\frac{5}{2}$	$-\frac{5}{4}$	0	$\frac{1}{20}$	$\frac{6}{5}$
b	$\mathbf{5} \times \mathbf{5} \to \mathbf{15}$	0	0	0	$-\frac{4}{5}$	$-\frac{4}{5}$
N	$\overline{\mathbf{5}} \times \mathbf{10} \to \mathbf{5}$	$\frac{5}{2}$	$\frac{5}{4}$	0	$-\frac{3}{20}$	$\frac{18}{5}$
E	$\overline{\mathbf{5}} \times \overline{\mathbf{5}} \to \overline{\mathbf{10}}$	$\frac{5}{2}$	$-\frac{5}{4}$	0	$\frac{1}{20}$	$\frac{6}{5}$
τ	$\overline{\mathbf{5}} \times \overline{\mathbf{5}} \to \overline{\mathbf{15}}$	0	0	0	$-\frac{4}{5}$	$-\frac{4}{5}$

Table 1: Self-energy coefficients for the various fermions. The condensation channels are classified in terms of $SU(5)_{ETC}$ representations. The ν_τ is not listed since there no right-handed ν_τ.

It is worth pointing out several features in Table 1. First note that masses for the D and the b correspond to different condensation channels. This is a generic feature of chiral TC models. The b remains massless in this model, since there is no condensate which transforms as a **15** (a similar argument applies for the τ).

Secondly, note that as far as the massless RTC gauge bosons (in the **10**) are concerned, all the light technifermions are identical, so the critical coupling for this interaction is the same for all technifermions. However the broken TC gauge bosons have different couplings to different technifermions. Consider the U and D techniquarks at energies above the TC breaking scale. If the TC gauge coupling is close to critical for the U, it will be far below critical for the D. The repulsive interaction due to the broken TC gauge bosons will cause the self-energy of the D to fall much more rapidly than that of the U, which experiences attractive interactions, and hence the self-energy at the ETC scale will be larger for the U than the D. This effect will be enhanced in chiral TC models that enjoy walking. This is an important generic effect in chiral TC models since it reduces the amount of isospin breaking in techniquark masses that is needed (in a class of models) to explain the t-b mass splitting. The reasoning goes as follows. If the t and b have the same ETC scale, then scaling from QCD gives:

$$\frac{f_D}{f_U} = \left(\frac{m_b}{m_t}\right)^{\frac{1}{3}} \ . \tag{18}$$

This relation will *not* hold in chiral TC models, since the self-energies fall off at different rates. (Note that this ratio becomes even smaller in vector-like, walking models.) Since m_t and m_b are related to U and D self energies at the ETC scale, and the D self-energy falls faster, the b can be much lighter than the t, even though f_D and f_U are fairly close together.

Finally note that the U and t have especially strong ETC interactions (corresponding to $\Delta C_2^{\mathbf{4}}$ in Table 1), which is useful for enhancing the t mass. This feature seems to be more model dependent. Given that U has more attractive TC interactions and strong ETC interactions, I expect that the U can be substantially heavier than the D. In a model that could produce a reasonable t mass, one would naively expect the difference between the U and D masses to be of order m_t. Such a splitting is a severe problem in QCD-like TC models, because of the large contribution to T. Here, and in multiscale models in general, things are not as bad as in QCD-like models, since the heavy technifermions are providing the bulk of the $W^\pm$ and Z masses. A large value of T can still be a problem in multiscale models, if equation (18) holds. From Table 1, one can also see that the N has more attractive interactions than the E, but that they both have fairly weak ETC interactions, thus I expect that the N will be slightly heavier than the E. This is perhaps surprising. It suggests that the ETC corrections to the $Z\nu_\tau\overline{\nu_\tau}$ coupling could be larger than the ETC correction to the $Z\tau\overline{\tau}$ coupling.

In order to quantify how bad isospin splitting for techniquarks can be, I will resort to a crude approximation of using a single techniquark-loop (with constant masses) as an estimate of T. This is expected to be an overestimate, since technifermion masses should actually fall with increasing momentum. My hope-othesis (as opposed to hypothesis) is that this estimate is within a factor of 2 of the correct answer. The one-techniquark-loop graph gives [22]:

$$\Delta\rho_* \equiv \alpha T = \frac{d\, N_c\, a}{16\pi^2 f^2}\left(m_U^2 + m_D^2 - \frac{4m_U^2 m_D^2}{m_U^2 - m_D^2}\ln\left(\frac{m_U}{m_D}\right)\right) \ , \tag{19}$$

where d is the dimension of the technifermion representation, $f = 246$ GeV, $N_c = 3$ is the number of colors, and $1/2 < a < 2$ is the assumed factor of 2 uncertainty. Now, given a fixed mass ratio m_D/m_U and an upper bound on αT, we can find an upper bound on m_U (and m_D). For example for $m_D/m_U = 0.5$ and $\alpha T < 0.01$, we have $m_U < 156/\sqrt{a}\,\mathrm{GeV} < 221\mathrm{GeV}$. Thus there is substantial room for isospin breaking in this type of model, given that we expect $m_U > m_t$. As m_D/m_U approaches unity, the bound, of course, weakens.

The result that m_U is so close to m_t brings to light the central problem of multiscale models: how can the t quark be so heavy? In order to achieve $m_t \approx 175$ GeV would necessitate some very nasty fine-tuning of ETC interactions in this toy model. In vector-like multiscale models the isospin problem is much worse, even assuming that the correct value of m_t can be produced. Using equation (18) we expect that $m_D/m_U < 0.3$, which gives a stronger bound: $m_U < 114/\sqrt{a} < 161\mathrm{GeV}$. In order to avoid this absurdity, realistic, vector-like, multiscale models must have different ETC scales for the t and the b, which requires even more complicated models.

As for the techniquark contribution to S, using the methods of ref. [16] I find:

$$\begin{aligned} S_{TQ} \;\; &= \;\; \frac{-a'\, Y\, N_c\, d}{\pi} \int_0^1 dx \ln\left(\frac{m_U^2 - x(1-x)M_Z^2}{m_D^2 - x(1-x)M_Z^2}\right) x(1-x) \\ &\quad + \frac{a'\, N_c\, d}{2\pi}\frac{m_U^2}{M_Z^2} \int_0^1 dx \ln\left(\frac{m_U^2}{m_U^2 - x(1-x)M_Z^2}\right) \\ &\quad + \frac{a'\, N_c\, d}{2\pi}\frac{m_D^2}{M_Z^2} \int_0^1 dx \ln\left(\frac{m_D^2}{m_D^2 - x(1-x)M_Z^2}\right) \;, \end{aligned} \tag{20}$$

where a' is another factor of 2 uncertainty. For $m_D = 100$ GeV, and $m_U = 200$ GeV, $S_{TQ} = 0.35a'$, which is about half of the heavy, degenerate techniquark estimate: $S_{TQ} = a' N_c\, d/(6\pi)$.

5 Conclusions

Chiral TC models take the multiscale paradigm to an extreme. While the toy model I have discussed illustrates the possibility of a chiral TC model, it is far from satisfying. The most glaring defect is that isospin breaking is put in by hand, since there is no $SU(2)_R$ symmetry in the quark and lepton sector. One might have hoped for a dynamical explanation. Of course it is possible that at some higher scale dynamics could act in a theory with an $SU(2)_R$ symmetry and produce an effective theory where part of the $SU(2)_R$ doublet is essentially replaced by some other fermions in a different ETC representation. This is what is supposed to happen for neutrinos in the model of ref. [3].

On the positive side, chiral TC models offer a simple way to split the t and b quarks without fine-tuning, while at the same time reducing the required splitting between the U and D technifermions. In principle the ETC contribution to $\Gamma(Z \to b\bar{b})$ can be reduced by (up to) a factor of 4. The techniquark contribution to the S parameter can also be reduced. Since the phenomenology of chiral TC models is almost entirely unexplored, there may be more interesting chiral TC models, and possibly one that actually works.

Appendix on the inadequacy of the MAC analysis

While discussing the $SU(4)_{TC}$ gauge theory with the fermion content listed in lines (12) through (14), I assumed the MAC analysis did not yield the correct fermion condensate. The MAC is $\overline{\mathbf{10}} \times \mathbf{4} \to \overline{\mathbf{4}}$. This condensate would break $SU(4)_{TC}$ down to $SU(3)$. I assumed that the condensation occurs in the channel $\mathbf{4} \times \mathbf{4} \to \mathbf{6}$, which breaks[3] $SU(4)_{TC}$ down to $Sp(4)_{RTC}$. (I will leave consideration of the channel $\mathbf{4} \times \overline{\mathbf{4}} \to \mathbf{1}$ until the end of the appendix.) The first point is that while the assumed condensation channel has a smaller ΔC_2, it leaves a larger gauge symmetry than the MAC does. Perturbatively, generating masses for gauge bosons increases rather than decreases the vacuum energy. Furthermore, the MAC analysis may be going astray here since it counts all gauge bosons equally, whether the condensate gives them a mass or not. If we only count contributions from massless gauge bosons we find $\Delta C_2\,|_{\rm massless}(\overline{\mathbf{10}} \times \mathbf{4} \to \overline{\mathbf{4}}) = 8/3$, while $\Delta C_2|_{\rm massless}(\mathbf{4} \times \mathbf{4} \to \mathbf{6}) = 5/2$, which is almost as strong . However, there is an additional complication, which is probably the most important consideration. The channel $\mathbf{4} \times \mathbf{4} \to \mathbf{6}$ will give masses to 10 Dirac fermions while the MAC gives masses to only one Dirac fermion. While the dynamical mass produced by the $\overline{\mathbf{10}} \times \mathbf{4} \to \overline{\mathbf{4}}$ channel will be somewhat larger than

[3] A condensate in the $\mathbf{6}$ could also break the gauge group down to $SU(2) \otimes SU(2)$. If the gauge coupling were weak, then vacuum alignment [23] would prefer $Sp(4)$, since it is a larger group and keeps more $SU(4)$ gauge bosons massless. I am assuming that $Sp(4)$ is also preferred in the strong coupling problem at hand.

that produced by the $\mathbf{4}\times\mathbf{4}\rightarrow\mathbf{6}$, the contribution to the effective potential [24] will be far outweighed by the sheer number of fermions contributing in the later channel!

At first sight, one might expect that the channel $\mathbf{4}\times\overline{\mathbf{4}}\rightarrow\mathbf{1}$ would condense at a slightly higher scale than $\mathbf{4}\times\mathbf{4}\rightarrow\mathbf{6}$. If it did, it would not affect the arguments above regarding the pattern of gauge symmetry breaking. However we are dealing with strong gauge couplings here, so it is quite plausible that the broken TC gauge boson masses are larger than the mass scale associated with condensation. In particular, the naive critical coupling in the assumed channel is

$$\alpha_c(\mathbf{4}\times\mathbf{4}\rightarrow\mathbf{6})=\frac{2\pi}{3\Delta C_2}=\frac{8\pi}{15}\ , \tag{21}$$

which corresponds to a gauge coupling $g\approx 4.5$. Thus the broken TC gauge bosons should have a mass larger than the fermions which condense in the $\mathbf{4}\times\overline{\mathbf{4}}\rightarrow\mathbf{1}$ channel. Whether one says this channel condenses before or after TC breaks is largely a matter of definition.

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