MASS ENHANCEMENT AND CRITICAL BEHAVIOR IN TECHNICOLOR THEORIES

THOMAS APPELQUIST, JOHN TERNING*
Department of Physics, Yale University, New Haven, CT 06520

and

L.C.R. WIJEWARDHANA
Department of Physics, University of Cincinnati, Cincinnati, OH 45221

ABSTRACT

Quark and lepton masses in technicolor theories can be enhanced if the high energy, extended technicolor (ETC) interactions play an important role in electroweak symmetry breaking. This happens when the ETC coupling and the technicolor gauge coupling at high energies lie close to a certain critical line. The enhancement has been associated with the existence of composite scalars made mainly of technifermions, with masses small compared to the ETC scale. The initial study of these states was carried out with the technicolor gauge coupling neglected. In this paper we investigate the properties of such scalars including the technicolor gauge interactions. We find that for realistic values of the gauge coupling, the scalars will not be narrow resonances.

The recent revival of interest in technicolor theories of electroweak symmetry breaking has been stimulated partly by the observation that momentum components well above the confinement scale Λ_{tc} can play a more important role than they do in QCD. In particular the higher energy extended technicolor (ETC) interactions, which must be present to generate the masses of ordinary fermions, can play an important direct role, along with the technicolor interactions, in the electroweak breaking, leading to even larger fermion masses.[1,2] This can take place only if the combination of the ETC coupling and the technicolor coupling at the ETC scale is sufficiently close to a certain critical curve.[3] Here we summarize a study of the light composite scalars generated by near-critical high energy interactions. We conclude that unless the technicolor coupling at the ETC scale is unrealistically weak and the ETC coupling is very close to the critical curve, these light states will have large widths.

We consider a single doublet of technifermions $\Psi = (U, D)$ subject to a confining technicolor force and an additional attractive ETC interaction. The latter is approxi-

*Presented by John Terning

mated by an effective, $SU(2)_L \times U(1)$ invariant, four-fermion coupling

$$\mathcal{L}_{4f} = \frac{8\pi^2\lambda}{N_{tc}\Lambda^2}(\bar{\Psi}^i_L U_R)(\bar{U}_R \Psi^i_L) \ , \tag{1}$$

where i is a summed $SU(2)_L$ index, N_{tc} is the number of technicolors, Λ is the ETC mass scale, and λ is the interaction strength of the ETC interactions. Implicit technicolor indices are also summed in each fermion bilinear.

We begin by recalling some features of dynamical chiral symmetry breaking driven by the combination of gauge and four-fermion interactions. Suppose that the physics of interest takes place at energies well above the confinement scale Λ_{tc}. It should be possible to describe this physics in terms of λ and $\alpha \equiv \alpha(\Lambda)$ (the technicolor coupling at the ETC scale). With the running of α neglected, dynamical mass generation can be studied in linearized ladder approximation.[3,4] Analysis of the gap equation for the dynamical mass $\Sigma(p)$ of the U fermion in the α, λ plane[3] reveals that a critical curve separates the broken phase ($\Sigma \neq 0$) from the symmetric phase ($\Sigma = 0$). For $\lambda \leq 1/4$, the broken phase exists only for $\alpha > \alpha_c \equiv \pi/3C_2(R)$, where $C_2(R)$ is the Casimir of the fermion representation. For $\lambda \geq 1/4$, the critical curve separating the two phases is defined by

$$\lambda_\alpha = \left[\frac{1}{2} + \frac{1}{2}\eta\right]^2 \ , \tag{2}$$

where $\eta \equiv \sqrt{1-\alpha/\alpha_c}$. The broken phase then exists only for $\lambda > \lambda_\alpha$.

When the running of the technicolor gauge coupling is re-introduced, the distinction between broken and symmetric phases is blurred. The growth of the coupling at momenta near Λ_{tc} will in fact always break the chiral symmetry. The critical curve in λ and α is therefore, loosely speaking, the dividing line between the regime where the high energy interactions (four-fermion and technicolor combined) are able to break the symmetry, where typically $\Sigma(0) \sim \Lambda$, and the low energy breaking regime, where typically $\Sigma(0) \sim \Lambda_{tc} \ll \Lambda$. It is this regime, where the spontaneous breaking is dominated by the "low energy" technicolor interaction, that is of principal interest in this paper. There it has been shown that for a range of λ near λ_α, the high energy mass of the technifermion $\Sigma(\Lambda)$ takes the form[2]

$$\Sigma(\Lambda) \sim \frac{4\pi^2\lambda < \bar{\psi}\psi >_{\lambda=0}}{\Lambda^2(1-\lambda/\lambda_\alpha)} \ . \tag{3}$$

Here, $< \bar{\psi}\psi >_{\lambda=0}$ is the technifermion condensate in a pure technicolor theory normalized at Λ. If an ordinary fermion (quark or lepton) is coupled to the technifermion by an ETC interaction with strength of order λ/Λ^2, then its mass is also given by Eq. 3. This expression exhibits the ETC-driven mass enhancement as $\lambda \to \lambda_\alpha$. It will break down once $\Lambda(1-\lambda/\lambda_\alpha)^{1/2} \sim \Lambda_{tc}$.

In Ref. 5, it was suggested that this enhancement can be attributed to the existence of a light scalar particle of mass $M \sim \Lambda(1-\lambda/\lambda_\alpha)^{1/2}$. It couples to the technifermion and then develops a vacuum value due to the technicolor interactions, producing a "tadpole" diagram, thus leading to Eq. 3. The discussion of scalar formation in Ref. 5 was restricted to a pure four-fermion theory, i.e. a Nambu-Jona-Lasinio (NJL) model.[6] Here we include the technicolor gauge interactions and specifically address the issue of the existence of a light physical scalar.

The effective four-fermion interaction in Eq. 1 can be eliminated in favor of four auxiliary scalar fields,[7] each with mass Λ. The question of whether light scalars exist as narrow or even broad resonances can be addressed by constructing the inverse scalar propagator $\Delta^{-1}(p)$, for the auxillary field σ, which couples to $U\bar{U}$, for example. To compute $\Delta^{-1}(p)$, we will evaluate Feynman graphs with two scalars coupled to one fermion loop with any number of ladder gauge boson exchanges,[7] and with external momentum p on the scalar legs. This requires knowledge of $\Gamma(p,k)$, the 1PI $\sigma U\bar{U}$ vertex with momentum p flowing along the scalar line. In the full ladder approximation, and neglecting derivatives of $\Gamma(p,k)$ (details are given in ref. 7) $\Delta^{-1}(p)$ can be expressed in terms of $\Gamma(0,k)$. Using the known form[8] of $\Gamma(0,k)$:

$$\Gamma(0,k) = \frac{1}{\frac{1}{2}+\frac{1}{2}\eta}\left(\frac{k^2}{\Lambda^2}\right)^{-\frac{1}{2}+\frac{1}{2}\eta} , \tag{4}$$

we find

$$\Delta^{-1}(p) = -\Lambda^2\left(a\frac{p^2}{\Lambda^2} + b\left(\frac{p^2}{\Lambda^2}\right)^{\eta}[\cos(\eta\pi) - i\sin(\eta\pi)] + 1 - \frac{\lambda}{\lambda_\alpha}\right) , \tag{5}$$

where

$$a = \frac{\lambda}{2\lambda_\alpha(1-\eta)} , \tag{6}$$

$$b = \frac{\lambda}{\lambda_\alpha\eta(1-\eta^2)} . \tag{7}$$

In the regime $\lambda < \lambda_\alpha$ of special interest here, the zero-momentum limit of the scalar inverse propagator gives the "zero-momentum mass"

$$M(0) = \Lambda(1-\lambda/\lambda_\alpha)^{1/2} . \tag{8}$$

$M(0)$ will be small compared to Λ if nature provides us with a λ close to λ_α. Thus we recover the fermion mass enhancement formula (Eq. 3).

Before proceeding further, we examine some simple limiting cases. In the NJL limit ($\alpha \to 0$) it can be shown that Eq. 5 reproduces the usual logarithmically supressed width to mass ratio. For the more realistic case of finite α, we examine the location of the poles of the propagator as λ approaches λ_α. The poles occur for complex p^2 so we set $p^2 = p_0^2\exp(-i\theta)$. For $\eta < 1$, and λ very close to λ_α, we can neglect $\frac{p^2}{\Lambda^2}$ relative to $\left(\frac{p^2}{\Lambda^2}\right)^\eta$ in the real part of $\Delta^{-1}(p)$. We then find zeros of $\Delta^{-1}(p)$ at

$$p_0 \approx \Lambda\left(\frac{1-\frac{\lambda}{\lambda_\alpha}}{b}\right)^{\frac{1}{2\eta}} , \tag{9}$$

$$\theta \approx \left(\frac{m-\eta}{\eta}\right)\pi + \frac{a\sin(\frac{m-\eta}{\eta}\pi)}{b\eta}\left(\frac{1-\frac{\lambda}{\lambda_\alpha}}{b}\right)^{\frac{1-\eta}{\eta}} , \tag{10}$$

where m is an odd integer. We expect the physical pole to correspond to $m = 1$, since it is the closest pole to the physical region.

We next consider under what conditions the scalar resonance will be narrow. $\Delta(p)$ will describe a narrow resonance if θ is small. (In this case, the width to mass ratio is approximately equal to θ.) When is θ in fact small? We first observe that for finite α ($\eta < 1$), θ (Eq. 10) does not approach zero as $\lambda \to \lambda_\alpha$. Therefore, the width to mass ratio is not suppressed (as in the NJL case) as the critical curve is approached. For small but nonzero α, this expression gives $\theta \to \frac{\alpha}{2\alpha_c}\pi$ as $\lambda \to \lambda_\alpha$. Thus, as the mass scale of the scalar state is made small by approaching the critical curve, it is not described by a narrow Breit-Wigner resonance unless α is quite small.

Having considered these special limiting cases, we now consider more generic values of the coupling constants. A description of the resonance structure of the theory is provided by a plot of Im $\Delta(p)$ (Fig. 1). The two figures correspond to different values of $M(0)/\Lambda = (1-\lambda/\lambda_\alpha)^{1/2}$; in each case, a resonant curve exists for the smallest value of α/α_c, peaked at a momentum smaller than $M(0)$. As α/α_c is increased the curve shifts down (relative to $M(0)$) and broadens (relative to the position of the peak).

Fig. 1. Graphs of Im $\Delta(p)$ for different values of λ/λ_α and α/α_c. The curves in each graph are normalized so that the peak value of the $\alpha/\alpha_c = 0.01$ curve equals 1.

As a specific example, consider the case in which λ is tuned to within 1% of λ_α (Fig. 1a), giving $M(0)/\Lambda \approx 1/10$. If Λ is in a range between, say, 30 TeV and 1000 TeV, and if the technicolor coupling either runs normally or walks at a rate attainable in a realistic theory, α/α_c will be somewhere between roughly 0.2 and 0.5. This is a range within which a broad Breit-Wigner curve exists, peaked roughly around $0.3M(0)$, with a full width at half maximum of roughly the same order. Even with a great deal of fine tuning (Fig. 1b), the width to mass ratio will only be small if α is unrealistically small for a technicolor theory.

To understand the origin of these results, it is convenient to frame the discussion in terms of the wavefunction renormalization factor Z of the scalar. In the NJL limit, Z is sensitive to high momentum components and is proportional to $\ln \Lambda^2/p^2$. The scalar couplings to fermions are inversely proportional to $Z^{1/2}$. Therefore, these states are weakly coupled, so it is not surprising that they can be narrow. The effect of the gauge interaction is to shift the sensitivity of Z towards the infrared; $Z^{1/2}$ is then proportional to $(\Lambda^2/p^2)^{1/2-\eta/2}$. This large denominator factor, however, will be cancelled by $\Gamma(0,k)$ (Eq. 4), which also enters into the computation of the widths. There is therefore no reason for the resonances to be narrow.

To conclude, we have studied the properties of light composite scalars which are present in technicolor theories with near-critical ETC interactions. We have shown that these scalars will not be narrow resonances for realistic values of the technicolor gauge coupling.

Acknowledgements

We thank S. Chivukula, A. Cohen, D. Kosower, K. Lane, S. Macdowell, and V. Miransky for helpful discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and by the U.S. Department of Energy under contracts DE-AC-02-76ERO3075 and DE-FG-02-84ER40153.

References

1. T. Appelquist, M. Einhorn, T. Takeuchi, and L.C.R. Wijewardhana, *Phys. Lett.* **220B**, 1043 (1989).
2. T. Takeuchi, *Phys. Rev.* **D40**, 2697 (1989).
3. T. Appelquist, M. Soldate, T. Takeuchi, and L.C.R. Wijewardhana in *Proceedings of the 12th Johns Hopkins Workshop on Current Problems in Particle Theory*, ed. by G. Domokos and S. Kovesi-Domokos, World Scientific (1988); K.I. Kondo, H. Mino and K. Yamawaki, *Phys. Rev.* **D39**, 2430 (1989); M. Inoue, T. Muta, and T. Ochiumi, *Mod. Phys. Lett.* **A4**, 605 (1989).
4. T. Appelquist, K. Lane, and U. Mahanta, *Phys. Rev. Lett.* **61**, 1553 (1988); U. Mahanta, *Phys. Rev. Lett.* **62**, 2349 (1989).
5. R.S. Chivukula, A. Cohen, and K. Lane, *Nucl. Phys.* **B343**, 554 (1990).
6. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** , 345 (1961).
7. T. Appelquist, J. Terning, and L.C.R. Wijewardhana, to be published in *Phys. Rev.* **D44** (1991).
8. T. Appelquist, U. Mahanta, D. Nash, and L.C.R. Wijewardhana, *Phys. Rev.* **D43**, R646 (1991).