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Precision Electroweak Constraints on Top-Color Assisted Technicolor

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Abstract

Using precision electroweak data, we put limits on "natural" top-color assisted technicolor models. Generically the new $U(1)$ gauge bosons in these models must have masses larger than roughly 2 TeV, although in certain (seemingly unrealistic) models the bound can be much lower.

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1 Introduction

Recently Hill [1] introduced top-color assisted technicolor (TC^2) in order to explain electroweak symmetry breaking via a dynamical mechanism and to allow for a large top quark mass. In this model a top-condensate is driven by the combination of a strong isospin-symmetric top-color interaction and an additional (possibly weak, but probably strong) isospin-breaking $U(1)$ interaction. He argued that the extreme fine-tuning that was required in pure top-condensate models can be done away with if the scales of the critical top-color and $U(1)$ interactions are brought down to a TeV. Given a top quark mass of around 175 GeV, such top-color interactions would produce masses for the W and Z that are far too small, hence there must be further strong interactions (technicolor) that are primarily responsible for breaking electroweak symmetry.

The authors of ref. [2] argued that the isospin breaking $U(1)$ gauge interactions, which are necessary in order to split the top and bottom quark masses, are likely to have isospin-violating couplings to technifermions. In this case they produce a significant shift in the W and Z masses (i.e. contribute to $\Delta\rho_* = \alpha T$). In order to satisfy experimental constraints on $\Delta\rho_*$, they found that either the effective top quark coupling or the top-color coupling must be tuned to 1%.

Subsequently, Lane and Eichten [3] showed that it was possible to construct models in which the technifermions had only isospin symmetric charges under the new $U(1)$. They referred to such models as "natural" TC^2 models. In order to produce mixing among heavy and light generations, such models seem to require direct couplings of the new $U(1)$ to fermions in the first two generations [5], and thus predict a variety of effects which are potentially observable in precision electroweak measurements[1].

In this paper we perform global fits to precision data for three examples of natural TC^2 scenarios: a simple "baseline" scenario with universal couplings to quarks and leptons (of a sort in which it does not seem too difficult to allow for mixing), an "optimal" scenario where it is assumed that there is no direct coupling to the first two generations, and an explicit model due to Lane [5] where new $U(1)$ charges have been assigned to all the known fermions in such a way as to allow for intergenerational mixing and gauge anomaly cancelation. For the "baseline" and "optimal" scenarios, we find that the Z' must typically weigh more than 2 TeV, although for the "optimal" scenario the Z' mass can be substantially smaller for a range of gauge couplings. For Lane's model [5], in which there are couplings of $\mathcal{O}(10)$, we find that the Z' must weigh more than approximately 20 TeV.

[1]Since both top quarks and technifermions condense, these models also contain "top-pions" [4] which, if light, can produce observable effects. Here we ignore "top-pion" corrections; since "top-pion" loops further reduce R_b, including them could only worsen the fit to data.

2 Electroweak Phenomenology of Top-Color Assisted Technicolor

The electroweak gauge symmetry in TC^2 models is $SU(2)_L \times U(1)_1 \times U(1)_2$. Here $U(1)_2$ is the, presumably strong, interaction with isospin-violating quark couplings (that allows for top-quark, but not for bottom-quark, condensation) and $U(1)_1$ is a weak gauge interaction. The pattern of electroweak gauge symmetry breaking that is required is more complicated than that in ordinary technicolor models; it generally involves two scales (rather than just one) to break the $SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry down to $U(1)_{em}$. The required pattern of breaking is:

$$
\begin{array}{c}
SU(2)_L \otimes U(1)_1 \otimes U(1)_2 \\
\downarrow \quad u \\
SU(2)_L \otimes U(1)_Y \\
\downarrow \quad v \\
U(1)_{em},
\end{array}
$$

where hypercharge, $Y = Y_1 + Y_2$, is equal to the sum of the generators of the two $U(1)$'s.

The gauge covariant derivative is given in the canonical basis by

$$\partial^\mu + ig\, T^a\, W_a^\mu + ig_1'\, Y_1\, B_1^\mu + ig_2'\, Y_2 B_2^\mu, \tag{2.1}$$

where the T^a, $a = 1$ to 3, are the generators of $SU(2)$. The gauge couplings may be written

$$g = \frac{e}{\sin\theta}\,, \quad g_1' = \frac{g'}{\cos\phi} = \frac{e}{\cos\phi\cos\theta}\,, \quad g_2' = \frac{g'}{\sin\phi} = \frac{e}{\sin\phi\cos\theta}\,, \tag{2.2}$$

in terms of the usual weak mixing angle θ and a new mixing angle ϕ. It is convenient to rewrite the neutral gauge bosons in terms of the photon,

$$A^\mu = \cos\theta\,(\cos\phi\, B_1^\mu + \sin\phi\, B_2^\mu) + \sin\theta\, W_3^\mu, \tag{2.3}$$

which couples to electric charge Q with strength e, a field

$$Z_1^\mu = -\sin\theta\,(\cos\phi\, B_1^\mu + \sin\phi\, B_2^\mu) + \cos\theta\, W_3^\mu, \tag{2.4}$$

which couples as the standard model Z would couple, to $T_3 - Q\sin^2\theta$ with strength

$$\frac{e}{\sin\theta\cos\theta}\,, \tag{2.5}$$

and the field

$$Z_2^\mu = -\sin\phi\, B_1^\mu + \cos\phi\, B_2^\mu, \tag{2.6}$$

which couples to the current $Y' = Y_2 - \sin^2\phi Y$ with strength

$$\frac{g'}{\sin\phi\cos\phi} = \frac{e}{\cos\theta\sin\phi\cos\phi} \,. \tag{2.7}$$

In this basis, using the relation

$$Q = T_3 + Y \tag{2.8}$$

and the fact that Q is conserved, the mass-squared matrix for the Z_1 and Z_2 can be written as:

$$M_Z^2 = \left(\frac{e}{2\sin\theta\cos\theta}\right)^2 \begin{pmatrix} < T_3T_3 > & \frac{\sin\theta}{\sin\phi\cos\phi} < T_3Y' > \\ \frac{\sin\theta}{\sin\phi\cos\phi} < T_3Y' > & \frac{\sin^2\theta}{\sin^2\phi\cos^2\phi} < Y'Y' > \end{pmatrix} \,, \tag{2.9}$$

where, from the charged-W masses we see that $< T_3T_3 >= v^2 \approx (250\,\mathrm{GeV})^2$.

In natural TC^2 models (where the strong $U(1)$ couplings to technifermions are isospin symmetric) the expectation value leading to $Z_1 - Z_2$ mixing, $< T_3Y' >$, can be calculated *entirely* in terms of the gauge couplings, v, and the Y_2 charges of the left- and right-handed top quark. Using the definition of Y', we see that

$$< T_3Y' > = < T_3Y_2 > - \sin^2\phi < T_3Y > \ . \tag{2.10}$$

Since $Y = Q - T_3$ and Q is conserved, the last term is equal to $+\sin^2\phi < T_3T_3 >$. Furthermore in natural TC^2 models, since the technifermion Y_2-charges are assumed to be isospin symmetric, the technifermions do not contribute to the first term. The only contribution to the first term comes from the top-quark condensate

$$\frac{< T_3Y_2 >}{< T_3T_3 >} = 2(Y_2^{t_L} - Y_2^{t_R})\frac{f_t^2}{v^2} \,, \tag{2.11}$$

where f_t is the analog of f_π for the top-condensate and is equal to

$$f_t^2 \approx \frac{N_c}{8\pi^2}\, m_t^2 \log\left(\frac{M^2}{m_t^2}\right) \tag{2.12}$$

in the Nambu—Jona-Lasinio [6] approximation, and M is the top-gluon mass. For $m_t \approx 175$ GeV and $M \approx 1$ TeV, we find $f_t \approx 64$ GeV.

If we define

$$x \equiv \frac{\sin^2\theta}{\sin^2\phi\cos^2\phi}\,\frac{< Y'Y' >}{< T_3T_3 >} \propto \frac{u^2}{v^2} \,, \tag{2.13}$$

and

$$\epsilon \equiv 2\,\frac{f_t^2}{v^2}\left(Y_2^{t_L} - Y_2^{t_R}\right) \,, \tag{2.14}$$

the $Z_1 - Z_2$ mass matrix can be written as

$$M_Z^2 = M_Z^2|_{\rm SM} \begin{pmatrix} 1 & \tan\phi\sin\theta\left(1+\frac{\epsilon}{\sin^2\phi}\right) \\ \tan\phi\sin\theta\left(1+\frac{\epsilon}{\sin^2\phi}\right) & x \end{pmatrix} . \tag{2.15}$$

In the large-x limit the mass eigenstates are

$$Z \approx Z_1 - \frac{\tan\phi\sin\theta}{x}\left(1+\frac{\epsilon}{\sin^2\phi}\right) Z_2 \tag{2.16}$$

$$Z' \approx \frac{\tan\phi\sin\theta}{x}\left(1+\frac{\epsilon}{\sin^2\phi}\right) Z_1 + Z_2 \tag{2.17}$$

The shifts in the Z coupling to $f\overline{f}$ (with $e/(\cos\theta\sin\theta)$ factored out) are therefore given by:

$$\delta g^f \approx -\frac{\sin^2\theta}{x\cos^2\phi}\left(1+\frac{\epsilon}{\sin^2\phi}\right)\left[Y_2^f - \sin^2\phi Y^f\right] . \tag{2.18}$$

Mixing also shifts the Z mass, and gives a contribution to T equal to:

$$\alpha T \approx \frac{\tan^2\phi\,\sin^2\theta}{x}\left(1+\frac{\epsilon}{\sin^2\phi}\right)^2 . \tag{2.19}$$

The shifts in the Z-couplings and mass are sufficient to describe electroweak phenomenology on the Z-peak. For low-energy processes, in addition to these effects we must also consider the effects of Z'-exchange. To leading order in $1/x$, these effects may be summarized by the four-fermion interaction

$$-\mathcal{L}_{\rm NC}^{Z'} = \frac{4G_F}{\sqrt{2}}\,\frac{\sin^2\theta}{x\sin^2\phi\cos^2\phi}\left(J_{Y_2} - \sin^2\phi J_Y\right)^2 , \tag{2.20}$$

where J_{Y_2} and J_Y are the Y_2- and hypercharge-currents, respectively. It is useful to note that if ϵ is negative, then all the Z pole mixing effects (equations (2.18) and (2.19)) vanish when $\sin^2\phi = -\epsilon$, although the low-energy effects of Z' exchange do not.

The result of all these corrections is that the predicted values of many electroweak observables are altered from those given by the standard model[2] [12, 13, 14]. The shifts in the predictions depend on the charge assignments of the quarks and leptons under the new $U(1)$. The "baseline" scenario we consider in this paper has universal couplings of the strong $U(1)$ to quarks and leptons given by $Y_2 = Y$ (and $Y_1 = 0$ for the ordinary fermions). This results in a shift in the total Z width given by:

$$\Gamma_Z = (\Gamma_Z)_{SM}\left(1 + 0.250\frac{1}{x\cos^2\phi} - 0.017\frac{1}{x\cos^2\phi\sin^2\phi} + 0.039\frac{\tan^2\phi}{x}\right) . \tag{2.21}$$

[2] We are using $\alpha_{em}(M_Z)$, G_F, and M_Z as the tree-level inputs.

The full list of changes to the electroweak observables for this "baseline" scenario appears in the Appendix.

For the "optimal" scenario we take the generation-dependent charge assignments of Y_2 to have the values of ordinary hypercharge on the third generation, and zero on the first two generations. This results, for example, in a shift in the total Z width:

$$\Gamma_Z = (\Gamma_Z)_{SM} \left(1 + 0.056 \frac{1}{x \cos^2\phi} - 4.0 \times 10^{-3} \frac{1}{x \cos^2\phi \sin^2\phi} + 0.039 \frac{\tan^2\phi}{x}\right) . \tag{2.22}$$

For Lane's model we use the charge assignments given in the Appendix of ref. [5]. These charges are listed in Table 1, with a normalization implied by equation (2.8). The shift in the total Z width in Lane's model is given by:

$$\Gamma_Z = (\Gamma_Z)_{SM} \left(1 + 9.4 \times 10^{-3} \frac{1}{x \cos^2\phi} - 8.4 \times 10^{-3} \frac{1}{x \cos^2\phi \sin^2\phi} + 0.039 \frac{\tan^2\phi}{x}\right) . \tag{2.23}$$

1st, 2nd	Y_2	3rd	Y_2
$(u,d)_L$, $(c,s)_L$	-10.5833	$(t,b)_L$	8.7666
u_R, c_R	-5.78333	t_R	11.4166
d_R, s_R	-6.78333	b_R	10.4166
$(\nu,e)_L$, $(\nu,\mu)_L$	-1.54	$(\nu,\tau)_L$	-1.54
e_R, μ_R	2.26	τ_R	2.26

Table 1: The charges for Lane's model. The first two columns refer to the first two generations, while the last two columns refer to the third generation.

3 Comparison with Data

3.1 Data and the Standard Model

Before describing the details of the fit, we briefly discuss higher-order corrections. Beyond tree-level, the predictions of the standard model (as well as predictions of extended models like those we are discussing) depend on the values of $\alpha_s(M_Z)$ and the top-quark mass m_t. Given the success of the standard model, we expect that, for the allowed range of $1/x$, the changes in the predicted values of physical observables due to radiative corrections in the standard model or extended models will be approximately the same for the same values of $\alpha_s(M_Z)$ and m_t.

The best-fit standard model predictions [7] which we use are based on a top quark mass of 173 GeV (taken from a fit to precision electroweak data) which is consistent with the mass range preferred by CDF (176 ± 13 GeV) and D0 (170 ± 25 GeV)[8].

The treatment of $\alpha_s(M_Z)$ is more problematic: the LEP determination for $\alpha_s(M_Z)$ comes from a *fit* to electroweak observables *assuming* the validity of the standard model. For this reason it is important [9, 13, 14] to understand how the bounds vary for different values of $\alpha_s(M_Z)$. We present bounds both for $\alpha_s(M_Z) = 0.124$ (which is the LEP best-fit value assuming the standard model is correct [7]) and for $\alpha_s(M_Z) = 0.115$ (which is the most precisely measured value, determined from lattice results [10] and consistent with deep-inelastic scattering [7, 11]). To the accuracy to which we work, the α_s dependence of the standard model predictions only appears in the Z partial widths and we use [7]

$$\Gamma_q = \Gamma_q|_{\alpha_s=0}\left(1 + \frac{\alpha_s}{\pi} + 1.409\left(\frac{\alpha_s}{\pi}\right)^2 - 12.77\left(\frac{\alpha_s}{\pi}\right)^3\right) \tag{3.1}$$

to obtain the standard model predictions for $\alpha(M_Z) = 0.115$.

With the above caveats, we have performed a global fit for the "baseline" TC^2 scenario, the "optimal" TC^2 scenario, and for Lane's model to all precision electroweak data: the Z line shape, forward backward asymmetries, τ polarization, and left-right asymmetry measured at LEP and SLC; the W mass measured at FNAL and UA2; the electron and neutrino neutral current couplings determined by deep-inelastic scattering; the degree of atomic parity violation measured in Cesium; and the ratio of the decay widths of $\tau \to \mu\nu\bar{\nu}$ and $\mu \to e\nu\bar{\nu}$.

3.2 Results of Fitting

Using the current experimental values[3] we have fit the "baseline" and "optimal" TC^2 scenarios and Lane's TC^2 model to the data. Figure 1 summarizes the fit to the "baseline" scenario (with universal couplings to quarks and leptons) by displaying the 95% confidence level[4] lower bound on the Z' mass for different values of $\sin^2\phi$ ($\alpha_s(M_Z) = 0.115$ corresponds to the solid line and $\alpha_s(M_Z) = 0.124$ to the dashed line). The plot was created as follows: for each value of $\sin^2\phi$ we fit to $1/x$; we then found the upper bound on $1/x$ and translated it into a lower bound on the Z' mass. The resulting bounds are quite sensitive to $\alpha_s(M_Z)$, with $\alpha_s(M_Z) = 0.124$ giving a much tighter constraint on the mass for most values of $\sin^2\phi$. The plot clearly shows a dip in the bound at $\sin^2\phi = -\epsilon \approx 0.07$, where the $Z - Z'$ mixing vanishes. For generic values of $\sin^2\phi$ the 95 % bound is roughly 2 TeV. While the results shown are for the case where $Y_2 = Y$, we expect that *any* model where the Y_2 couplings to the first and second generations are $\mathcal{O}(1)$ will give similar results.

[3]The experimental data and standard model predictions are given in Table 4 in the Appendix.

[4]This corresponds to $\Delta\chi^2 = 4.0$ relative to the best-fit point, since we are fitting to one parameter $1/x$, holding $\sin^2\phi$ fixed.

Figure 1: The solid line is the 95% confidence lower bound for the mass of the Z' as a function of $\sin^2\phi$ for the "baseline" TC^2 scenario, using $\alpha_s(M_Z) = 0.115$. The dashed line is for $\alpha_s(M_Z) = 0.124$.

Figure 2 summarizes the fit to the "optimal" model with generation dependent couplings by displaying the 95% confidence level lower bound (solid line) on the Z' mass for different values of $\sin^2\phi$ (using $\alpha_s(M_Z) = 0.115$; for this model the bounds are not very sensitive to the value of $\alpha_s(M_Z)$). Also shown is the 68% confidence level lower bound (dashed line) which displays the sensitivity of the fit for different values of $\sin^2\phi$.

For Lane's model the fit results are summarized in Figure 3, which displays the 95% and 68% confidence lower bounds on the Z' mass (solid and dashed lines) as a function of $\sin^2\phi$. The lowest possible Z' mass at the 95% confidence level is roughly 20 TeV. The bound is significantly larger in this model since the new $U(1)$ charges (as listed in Table 1) are $\mathcal{O}(10)$ rather than $\mathcal{O}(1)$. As anticipated by Lane [5], the fit for this model is very sensitive to the atomic parity violation measurement in Cesium, as can be seen from the predicted value of the $Q_W(Cs)$ in this model:

$$Q_W(Cs) = (Q_W(Cs))_{SM} - 324\frac{1}{x\cos^2\phi} - 24887\frac{1}{x\cos^2\phi\sin^2\phi} + 16.7\frac{\tan^2\phi}{x}\ . \quad (3.2)$$

The large coefficients result from the large quark charges in this model, and the large number of quarks in a Cesium atom. In a different version of Lane's model [17], where the new $U(1)$ has vectorial couplings to leptons, the 95% confidence level bound drops to 2.7 TeV, in accord with the results of the "baseline" scenario fit.

The summary of the quality of the fits is given in Table 2 for two different values

Figure 2: The solid line is the 95% confidence lower bound for the mass of the Z' as a function of $\sin^2\phi$ for the "optimal" scenario (using $\alpha_s(M_Z) = 0.115$). The dashed line is the 68% confidence lower bound.

Figure 3: The solid line is the 95% confidence lower bound for the mass of the Z' as a function of $\sin^2\phi$ for Lane's TC^2 model (using $\alpha_s(M_Z) = 0.115$). The dashed line is the 68% confidence lower bound.

of $\sin^2\phi$ for both the "baseline" and "optimal" scenarios. We first show the result of allowing both $\sin^2\phi$ and $1/x$ to vary. For the "baseline" model the fit prefers $\sin^2\phi$ to be close to 1, however this makes the coupling g_1' diverge. Requiring $g_1'/4\pi \leq 1$ gives a best fit at $\sin^2\phi = 0.99$, which corresponds to a Z' mass of $M_{Z'} = 11.6^{+3.4}_{-1.8}$ TeV. For the "optimal" model the fit yields $\sin^2\phi = 0.62$ which corresponds to $M_{Z'} = 2.1^{+0.6}_{-0.3}$ TeV.

Such large values of $\sin^2\phi$ imply that the new $U(1)$ gauge coupling is quite weak, and therefore such models require a fine-tuning to arrange for a top condensate but not a bottom condensate [1]. Thus we show a second fit where we required that $g_2'/4\pi > 0.1$ (which corresponds to $\sin^2\phi < 0.1$). Given this constraint we chose $\sin^2\phi$ such that the best-fit value for $1/x$ corresponded to the lightest possible mass for the Z'. For the "baseline" model this occurs at $\sin^2\phi = 0.036$ which corresponds to $M_{Z'} = 4.4^{+\infty}_{-1.8}$ TeV, while for the "optimal" model the minimum is at $\sin^2\phi = 0.07$, where there is no Z - Z' mixing, which corresponds to $M_{Z'} = 290^{+\infty}_{-110}$ GeV. A fit for Lane's model [5] is not shown since there is no value of $\sin^2\phi$ that has a best fit with a physical Z' mass (i.e. $1/x > 0$).

Table 2 shows the fit to the standard model for comparison. The percentage quoted in the Table is the probability of obtaining a χ^2 as large or larger than that obtained in the fit, for the given number of degrees of freedom (df), assuming that the model is correct. Thus a small probability corresponds to a poor fit, and a bad model. Note that the standard model gives a poor fit to the data, while the fine-tuned TC^2 scenarios are only modestly better, and in the absence of fine-tuning the TC^2 scenarios are slightly worse.

Model	$\sin^2\phi$	χ^2	df	χ^2/df	probability
standard model	-	43.7	23	1.90	0.6%
"baseline"	0.99	37.5	21	1.79	1.5%
"optimal"	0.62	37.3	21	1.78	1.6%
"baseline", no tuning	0.036	43.4	22	1.97	0.4%
"optimal", no tuning	0.069	43.3	22	1.97	0.4%

Table 2: The best fits for the standard model, the "baseline" TC^2 scenario, and the "optimal" TC^2 scenario, for different values of $\sin^2\phi$, assuming $\alpha_s(M_Z) = 0.115$. χ^2 is the sum of the squares of the difference between prediction and experiment, divided by the error, the number of degrees of freedom, df, is the number of experiments minus the number of fitted parameters. The phrase "no tuning" corresponds to the requirement $\sin^2\phi < 0.1$ as discussed in the text.

For comparison we have also performed the fits using $\alpha_s(M_Z) = 0.124$; the quality of the fit is summarized in Table 3. The quality of all the fits improves, but there are only small changes in the relative goodness of fit. The best fit for

Model	$\sin^2\phi$	χ^2	df	χ^2/df	probability
standard model	-	33.9	22	1.47	6.7%
"baseline", no tuning	0.01	31.9	21	1.52	5.9%
"optimal"	0.99	30.3	21	1.38	11.2%
"optimal", no tuning	0.071	33.9	22	1.54	5.1%

Table 3: The best fits for the standard model, the "baseline" TC^2 scenario, and the "optimal" TC^2 scenario, for different values of $\sin^2\phi$, assuming $\alpha_s(M_Z) = 0.124$. For this value of $\alpha_s(M_Z)$ the best fit for the "baseline" scenario with a physical Z' mass requires no fine-tuning. See Table 2 for an explanation of the notation.

the "baseline" model corresponds to an unphysical Z' mass (i.e. $1/x < 0$). If we require a physical Z' mass then the fit prefers $\sin^2\phi$ close to zero, so no fine-tuning is required. For $\sin^2\phi = 0.01$ we find $M_{Z'} = 5.9^{+5.2}_{-1.4}$ TeV. The best fit for the "optimal" model corresponds to $\sin^2\phi = 0.99$ and $M_{Z'} = 18.9^{+8.6}_{-3.6}$ TeV. We also show a fit for the "optimal" model where we require the absence of fine-tuning ($\sin^2\phi < 0.1$), and the lightest possible Z' mass. We found this occurs for $\sin^2\phi = 0.07$, the value where there is no Z - Z' mixing, which corresponds to $M_{Z'} = 1.1^{+\infty}_{-0.75}$ TeV. Again we do not show a fit for Lane's model since there is no value of $\sin^2\phi$ that has a best fit with a physical Z' mass. Here the "optimal" scenario provides a better fit than the standard model if fine-tuning is allowed. In the absence of fine-tuning the TC^2 scenarios give fits to the data that are slightly worse than the standard model.

4 Conclusions

In this paper we have used precision electroweak measurements to constrain "natural" TC^2 models. The models we examined can provide a better fit to data than the standard model if fine-tuning is allowed, however in the absence of fine-tuning they generally provide a fit that is slightly worse than the poor fit given by the standard model. Generically we have found that the mass of the new Z' in such models has to be above roughly 2 TeV. In models with no couplings of the new $U(1)$ to the first two generations, the bound on the Z' mass can be lower, but it seems difficult to allow for mixing between generations in such models. In the explicit model due to Lane [5], which allows for inter-generational mixing, the bounds are much more stringent, the couplings to the first two generations are $\mathcal{O}(10)$, requiring a Z' mass above 20 TeV.

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A Appendix: Data and the "baseline" TC^2 scenario predictions

Quantity	Experiment	SM
Γ_Z	2.4964 ± 0.0022	2.4925
R_e	20.801 ± 0.058	20.717
R_μ	20.795 ± 0.043	20.717
R_τ	20.815 ± 0.061	20.717
σ_h	41.490 ± 0.078	41.492
R_b	0.2215 ± 0.0017	0.2156
R_c	0.1596 ± 0.0070	0.1720
A^e_{FB}	0.0152 ± 0.0027	0.0155
A^μ_{FB}	0.0163 ± 0.0015	0.0155
A^τ_{FB}	0.0203 ± 0.0022	0.0155
$A_\tau(P_\tau)$	0.1394 ± 0.0069	0.1440
$A_e(P_\tau)$	0.1429 ± 0.0079	0.1440
A^b_{FB}	0.1002 ± 0.0028	0.1010
A^c_{FB}	0.0756 ± 0.0051	0.0720
A_{LR}	0.1551 ± 0.0040	0.1440
M_W	80.17 ± 0.18	80.34
M_W/M_Z	0.8813 ± 0.0041	0.8810
$g_L^2(\nu N \to \nu X)$	0.3003 ± 0.0039	0.3030
$g_R^2(\nu N \to \nu X)$	0.0323 ± 0.0033	0.0300
$g_{eA}(\nu e \to \nu e)$	-0.503 ± 0.018	-0.507
$g_{eV}(\nu e \to \nu e)$	-0.025 ± 0.019	-0.037
$Q_W(Cs)$	-71.04 ± 1.81	-72.88
$R_{\mu\tau}$	0.9970 ± 0.0073	1.0

Table 4: Experimental [7, 15, 16] and predicted values of electroweak observables for the standard model for $\alpha_s(M_Z) = 0.115$. The standard model values correspond to the best-fit values (with $m_t = 173$ GeV, $m_{\mathrm{Higgs}} = 300$ GeV) in [7], with $\alpha_{em}(M_Z) = 1/128.9$, and corrected for the change in $\alpha_s(M_Z)$.

The following equations are the predictions of the "baseline" scenario as functions of ϕ and x. For brevity we use the notation $c = \cos\phi$, $s = \sin\phi$, and $t = \tan\phi$.

$$\Gamma_Z = (\Gamma_Z)_{SM}\left(1 + 0.250\frac{1}{xc^2} - 0.017\frac{1}{xc^2s^2} + 0.039\frac{t^2}{x}\right) \tag{A.1}$$

$$R_e = (R_e)_{SM}\left(1 - 0.160\frac{1}{xc^2} + 9.9\times10^{-3}\frac{1}{xc^2s^2} + 0.201\frac{t^2}{x}\right) \tag{A.2}$$

$$R_\mu = (R_\mu)_{SM}\left(1 - 0.160\frac{1}{xc^2} + 9.9\times10^{-3}\frac{1}{xc^2s^2} + 0.201\frac{t^2}{x}\right) \tag{A.3}$$

$$R_\tau = (R_\tau)_{SM}\left(1 - 0.160\frac{1}{xc^2} + 9.9\times10^{-3}\frac{1}{xc^2s^2} + 0.201\frac{t^2}{x}\right) \tag{A.4}$$

$$\sigma_h = (\sigma_h)_{SM}\left(1 + 0.012\frac{1}{xc^2} - 7.3\times10^{-4}\frac{1}{xc^2s^2} - 0.015\frac{t^2}{x}\right) \tag{A.5}$$

$$R_b = (R_b)_{SM}\left(1 + 0.036\frac{1}{xc^2} - 2.2\times10^{-3}\frac{1}{xc^2s^2} - 0.044\frac{t^2}{x}\right) \tag{A.6}$$

$$R_c = (R_c)_{SM}\left(1 - 0.073\frac{1}{xc^2} + 4.5\times10^{-3}\frac{1}{xc^2s^2} + 0.094\frac{t^2}{x}\right) \tag{A.7}$$

$$A^e_{FB} = (A^e_{FB})_{SM} - 0.375\frac{1}{xc^2} + 0.023\frac{1}{xc^2s^2} + 0.477\frac{t^2}{x} \tag{A.8}$$

$$A^\mu_{FB} = (A^\mu_{FB})_{SM} - 0.375\frac{1}{xc^2} + 0.023\frac{1}{xc^2s^2} + 0.477\frac{t^2}{x} \tag{A.9}$$

$$A^\tau_{FB} = (A^\tau_{FB})_{SM} - 0.375\frac{1}{xc^2} + 0.023\frac{1}{xc^2s^2} + 0.477\frac{t^2}{x} \tag{A.10}$$

$$A_\tau(P_\tau) = (A_\tau(P_\tau))_{SM} - 1.57\frac{1}{xc^2} + 0.097\frac{1}{xc^2s^2} + 2.00\frac{t^2}{x} \tag{A.11}$$

$$A_e(P_\tau) = (A_e(P_\tau))_{SM} - 1.57\frac{1}{xc^2} + 0.097\frac{1}{xc^2s^2} + 2.00\frac{t^2}{x} \tag{A.12}$$

$$A^b_{FB} = \left(A^b_{FB}\right)_{SM} - 1.12\frac{1}{xc^2} + 0.069\frac{1}{xc^2s^2} + 1.42\frac{t^2}{x} \tag{A.13}$$

$$A^c_{FB} = (A^c_{FB})_{SM} - 0.877\frac{1}{xc^2} + 0.054\frac{1}{xc^2s^2} + 1.11\frac{t^2}{x} \tag{A.14}$$

$$A_{LR} = (A_{LR})_{SM} - 1.57\frac{1}{xc^2} + 0.097\frac{1}{xc^2s^2} + 2.00\frac{t^2}{x} \tag{A.15}$$

$$M_W = (M_W)_{SM}\left(1 - 0.022\frac{1}{xc^2} + 7.6\times10^{-4}\frac{1}{xc^2s^2} + 0.166\frac{t^2}{x}\right) \tag{A.16}$$

$$M_W/M_Z = (M_W/M_Z)_{SM}\left(1 - 0.022\frac{1}{xc^2} + 7.6\times10^{-4}\frac{1}{xc^2s^2} + 0.166\frac{t^2}{x}\right) \tag{A.17}$$

$$g_L^2(\nu N\to\nu X) = \left(g_L^2(\nu N\to\nu X)\right)_{SM} + 0.117\frac{1}{xc^2} - 2.6\times10^{-3}\frac{1}{xc^2s^2} + 0.058\frac{t^2}{x} \tag{A.18}$$

$$g_R^2(\nu N\to\nu X) = \left(g_R^2(\nu N\to\nu X)\right)_{SM} - 0.040\frac{1}{xc^2} + 0.055\frac{1}{xc^2s^2} - 0.020\frac{t^2}{x} \tag{A.19}$$

$$g_{eA}(\nu e\to\nu e) = (g_{eA}(\nu e\to\nu e))_{SM} - 1.2\times10^{-3}\frac{1}{xc^2} - 0.101\frac{1}{xc^2s^2} - 7.3\times10^{-4}\frac{t^2}{x} \tag{A.20}$$

$$g_{eV}(\nu e\to\nu e) = (g_{eV}(\nu e\to\nu e))_{SM} - 0.311\frac{1}{xc^2} + 0.324\frac{1}{xc^2s^2} - 0.153\frac{t^2}{x} \tag{A.21}$$

$$Q_W(Cs) = (Q_W(Cs))_{SM} + 34.3\frac{1}{xc^2} - 51.3\frac{1}{xc^2s^2} + 16.7\frac{t^2}{x} \tag{A.22}$$

$$R_{\mu\tau} = (R_{\mu\tau})_{SM} \tag{A.23}$$

References

[1] C. Hill, *Phys. Lett.* **B345** (1995) 483, hep-ph/9411426.

[2] R.S. Chivukula, B.A. Dobrescu, and J. Terning, *Phys. Lett.* **B353** (1995) 289, hep-ph/9503203.

[3] K. Lane and E. Eichten, *Phys. Lett.* **B352** (1995) 382, hep-ph/9503433.

[4] D. Kominis *Phys. Lett.* **B358** (1995) 312, hep-ph/9506305; G. Buchalla et. al. *Phys. Rev.* **D53** (1996) 5185, hep-ph/9510376; D. Kominis, Technische Universitat Muchen preprint TUM-HEP-248/96, hep-ph/9605272.

[5] K. Lane, BUHEP-96-2, to be published in *Phys. Rev.* **D**, hep-ph/9602221.

[6] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961), see also H. Pagels and S. Stokar, *Phys. Rev.* **D20** (1979) 2947.

[7] P. Langacker, hep-ph/9408310; see also P. Langacker and J. Erler, http://www-pdg.lbl.gov/rpp/book/page1304.html, *Phys. Rev.* **D50** (1994) 1304.

[8] F. Abe et. al. (CDF collaboration), *Phys. Rev. Lett.* **74** (1995) 2626, hep-ex/9503002; S. Abachi et. al. (D0 collaboration), *Phys. Rev. Lett.* **74** (1995) 74 (1995) 2632, hep-ex/9503003; J. Butler private communication.

[9] B. Holdom, hep-ph/9407311, *Phys. Lett.* **B339** (1994) 114; T. Takeuchi, A.K. Grant and J.L. Rosner, *The Albuquerque Meeting*, S. Seidel ed., (World Scientific, 1995), hep-ph/9409211; J. Erler and P. Langacker, *Phys. Rev.* **D52** (1995) 441, hep-ph/9411203.

[10] C.T.H. Davies et. al., *Phys. Lett.* **B345** (1995) 42, hep-ph/9408328.

[11] I. Hinchliffe, http://www-pdg.lbl.gov/rpp/book/page1297.html, *Phys. Rev.* **D50** (1994) 1297; M. Virchaux, Saclay preprint, DAPHNIA/SPP 92-30, presented at the "QCD 20 years later" workshop, Aachen Germany, (1992); R. Voss, in *Proceedings of the 1993 International Symposium on Lepton and Photon Interactions at High Energies*, Ithaca NY (1993).

[12] C.P. Burgess, et. al., hep-ph/9312291, *Phys. Rev.* **D49** (1994) 6115.

[13] R.S. Chivukula, E.H. Simmons, and J. Terning, hep-ph/9412309, *Phys. Lett.* **B346** (1995) 284.

[14] R.S. Chivukula, E.H. Simmons, and J. Terning, *Phys. Rev.* **D53** (1996) 5258, hep-ph/9506427.

[15] LEP Electroweak Working Group, P. Antilogu et. al., LEPEWWG/9601, http://www.cern.ch/LEPEWWG/stanmod/l9601.ps.gz.

[16] Particle Data Group 1994; See also A. Pich and J.P. Silva hep-ph/95-5327, FTUV/95-21, IFIC/95-21.

[17] K. Lane, unpublished.