A GIM Mechanism from Extra Dimensions

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Abstract

We explore how to protect extra dimensional models from large flavor changing neutral currents by using bulk and brane flavor symmetries. We show that a GIM mechanism can be built in to warped space models such as Randall-Sundrum or composite Higgs models if flavor mixing is introduced via UV brane kinetic mixings for right handed quarks. We give a realistic implementation both for a model with minimal flavor violation and one with next-to-minimal flavor violation. The latter does not suffer from a CP problem. We consider some of the existing experimental constraints on these models implied by precision electroweak tests.

1 Introduction

Extra dimensional theories offer new avenues for flavor physics. Following the suggestion of Arkani-Hamed and Schmaltz [1], the standard approach is to use the overlaps of wave functions in extra dimensions to generate the fermion mass hierarchy. Since the fermions are physically located in different places this is referred to as the split-fermion approach. If implemented in Randall-Sundrum-type [2] warped space (as suggested in [3, 4]) it has the added benefit that unwanted 4-fermi operators leading to flavor changing neutral currents (FCNC's) are suppressed by a high UV scale (assuming that the fermions are mostly localized around the UV brane). The main advantage of this approach is that it generates the fermion mass hierarchy without flavor symmetries. The price to pay for the absence of the flavor symmetries is that the structure of quark mixing is much more complex than in the usual Cabibbo–Kobayashi–Maskawa (CKM) picture [5], in particular one would expect more mixing angles and CP phases to be physical [6, 7]. However, after many years of running B-factories the experimental results seem to suggest that flavor mixing is well described by the standard CKM picture with a single CP violating phase as in the standard model (SM). For the RS model these constraints would imply that the mass of the lightest KK gauge bosons would have to be of order ~ 8 TeV for generic choices of the parameters [9]. Given these constraints one should revisit the question of how to introduce flavor physics in extra dimensional scenarios that will reproduce CKM. An added motivation for this study comes from the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [10, 11]. We have learned over the past few years that some simple extra dimensional setups behave like weakly coupled duals of approximately conformal (walking) technicolor theories [12]. Technicolor models are notorious for their problems with FCNC's and the simplest known technicolor models incorporating a Glashow-Illiopoulos-Maiani (GIM) mechanism [13] are terrifyingly complex even for trained model builders [14].

In this paper we suggest an alternative general approach to flavor physics in extra dimensions based on flavor symmetries. This approach can be applied to warped space models including traditional Randall-Sundrum 1 (RS1), composite Higgs models or higgsless models, and also to flat extra dimensional models (for example for gauge-Higgs unification models in flat space). The main feature of this construction is that the bulk has a large flavor symmetry, while the IR brane where the SM Yukawa couplings are localized still preserves a large diagonal subgroup of the symmetry. Flavor mixing is then introduced via kinetic mixing terms of the right-handed (R) fields on the UV brane. Therefore, higher dimensional flavor violating operators are forbidden by the flavor symmetries everywhere but on the UV brane. However the suppression scale for flavor violating 4 fermi operators on the UV brane is very large, so higher dimensional operators will never pose a flavor problem (contrary for example to UED models where there is a GIM mechanism for the lowest dimensional operators [15] but there is no suppression mechanism against flavor violation from higher dimensional operators).

We will show that this approach can incorporate a GIM mechanism and reproduce the SM CKM picture. Thus it can be viewed as the simplest implementation of minimal flavor violation[1] (MFV) [16] in extra dimensional theories. The downside of these constructions is that we are no longer trying to *explain* the fermion mass hierarchy, rather we want to accommodate it with the least amount of flavor structure.

In order to find a realistic implementation of this idea we also need to make sure that the traditional precision electroweak bounds are satisfied beyond the flavor constraints. Incorporating a heavy top quark will require us to slightly modify the simplest toy example, by making sure that the large top quark mass does not feed into the electroweak precision constraints of the light quarks (but still leaving sufficient flavor symmetries). This can be achieved by using the modified representations under the custodial symmetry for the quarks [17]. We present two examples of this sort, one an example of minimal flavor violation (MFV) [16], and one of next-to-minimal flavor violation (NMFV) [8] with no CP problem.

This paper is organized as follows. In Sec. 2 we introduce the basic construction for flavor mixing via UV brane kinetic terms. We use symmetry arguments to show that such a setup indeed has a GIM mechanism built in, and then also explain the origin of the GIM mechanism in 5D language using wave function orthogonality. We also show how the CKM matrix emerges in this picture. In Sec. 3 we give the CFT interpretation of this setup, and also consider some of the bounds on the extra gauge bosons. In Sec. 4 we show how a realistic model incorporating a heavy top can be obtained. We focus on models with a light Higgs on the IR brane (such as the RS1 model [2, 18] or models with a composite Higgs [19]) and a Kaluza-Klein (KK) mass scale $\gtrsim$ 3 TeV. The first model we present has a full GIM mechanism built in, and we check explicitly that it can also be consistent with the other precision electroweak bounds. The second model is an implementation of next-to-minimal flavor violation (NMFV) (by which we mean that all additional flavor violation has to go through the third generation), but we argue that it does not have a CP problem. We conclude in Sec. 5.

2 Symmetry considerations

We will be using the following setup: there will be an exact flavor symmetry in the bulk separately for left-handed (L) and right-handed fermions[1]. To be concrete, in this section we will consider three generations of quarks, and we will leave to the reader the straighforward extension to leptons. In order to incorporate a custodial symmetry necessary to be in agreement with precision electroweak tests we assume that the bulk electroweak gauge group is $SU(2)_L \times SU(2)_R \times U(1)_X$, broken to the SM gauge group by boundary conditions on the UV brane, $SU(2)_R \times U(1)_X \to U(1)_Y$. It will be further broken to $U(1)_{em}$ on the IR brane, either by higgsing or boundary conditions. The largest global flavor symmetry that we can impose in the bulk is a $U(3)_Q \times U(3)_{uR} \times U(3)_{dR}$: this would be broken to $U(3)_Q \times U(3)_{qR}$ by gauge interactions if the right-handed quarks are embedded in the same multiplet of

[1] Which posits that that all flavor violation and the CKM matrix arise from the same source.

[1] We will use L to describe the 5D Dirac fermion field that contains a Weyl (chiral) left-handed zero-mode in the absence of Higgs Yukawa couplings, and similarily with R for right-handed. When we need to distinguish between left and right-handed components of the Dirac fermion we will use χ and ψ respectively.

$SU(2)_R$. This symmetry implies for example that the bulk masses of the fermions with a given quantum number are precisely equal and that there is no kinetic mixing. On the IR brane we assume that the quark Yukawa couplings (or mass terms in Higgsless scenarios) break the flavor symmetry to the family symmetry $U(3)_V$, the vector (diagonal) subgroup of the three U(3)'s. This is achieved by flavor independent Yukawa couplings proportional to the identity matrix in family space for both the up-type and down-type quarks.[2] Finally, both the splitting among the quark masses of different generations and flavor mixing are achieved via kinetic mixing terms among the right handed up-type and down-type quarks on the UV-brane where the gauge group is broken to $SU(2)_L\times$ $U(1)_Y$, thus allowing kinetic mixing that distinguish between up and down. The UV mixing terms will generically break the flavor symmetry $U(3)_u\times U(3)_d \to U(1)_{uR}\times U(1)_{dR}$, where all R up-type quarks transform by the same phase under $U(1)_{uR}$. Such a breaking pattern implies that, in order to avoid massless Goldstone bosons or massless flavor gauge bosons in the bulk, the largest flavor group that we can have in the bulk is $SU(3)_Q\times SU(3)_{qR}$, independently of the fermion representations. Thus if uR and dR are in different representations there must be an additional source of breaking in the bulk to reduce the symmetry to $SU(3)_{qR}$.

We will now argue that this setup, together with the assumption that only two kinetic mixing terms are allowed on the UV brane for up and down quarks, in fact results in an extra dimensional GIM mechanism, that is there will be no tree-level FCNC's generated, and MFV via the CKM matrix only.

To show this, let us first turn off the charged current interactions. In the neutral current sector in the bulk we have a bigger global symmetry, $U(3)_{uL}\times U(3)_{dL}\times U(3)_{uR}\times U(3)_{dR}$. This is due to the fact that neutral currents can not mix and up- and down-type quarks. On the IR brane the Yukawa couplings break the symmetry to the vector (diagonal) subgroup $U(3)_{uV}\times U(3)_{dV}$. So these are the symmetries we can use to diagonalize the kinetic terms on the UV brane. If we only have a kinetic mixing in the R quarks on the UV brane, then we can use the $SU(3)_{uV}\times SU(3)_{dV}$ symmetry to diagonalize the kinetic terms. Note, that the extra U(1) factors leave the kinetic mixing terms invariant, so they cannot be used to further simplify the kinetic matrix. While these kinetic terms will be diagonal, they will not be proportional to the unit matrix, and so the diagonal (non-equal) components clearly break the $U(3)_u\times U(3)_d$ symmetry to $U(1)_u\times U(1)_c\times U(1)_t\times U(1)_d\times U(1)_s\times U(1)_b$. This symmetry is sufficient to eliminate FCNC's not just via the ordinary Z-boson, but also through the Z', g', etc. KK modes. However, non-universalities in the diagonal couplings will still be generated at higher order in quark masses.

If there are additional kinetic mixings, for instance for the L fields, on the UV brane, then (unless the kinetic matrices for the L and R fields are simultaneously diagonalized) one will break the flavor symmetry completely, and there will be FCNC's. This may also happen if the multiplets containing the right-handed quarks also contain extra exotic quarks, made heavy via boudary conditions, that mix with the SM ones via the Yukawa interactions (IR mass terms).

[2]Note, that if we started with a single right handed bulk multiplet for up and down at this stage the unbroken flavor symmetry would be identical.

In the charged current (CC) sector there are no residual diagonal U(1) symmetries, leading to the possibility of flavor mixing in CC's. The reason behind the flavor violation in this setup is that the U(1) flavor symmetries are misaligned in the bulk and on the UV brane due to the kinetic mixing terms for the R fermions. In fact, one needs to do a different rotation on the up and down-type fields, which is not an invariance of the bulk. This misalignment becomes physical in the CC interactions.

The origin of the GIM mechanism and the emergence of the CKM matrix can be seen explicitly if we consider the bulk fields χ_L, an $\mathrm{SU}(2)_L$ doublet that contains the left-handed doublet, and $\psi_R^{u,d}$, the fields containing the right-handed quarks. As already mentioned, it is not crucial which representation of $\mathrm{SU}(2)_R$ $\psi_R^{u,d}$ are embedded in, since this symmetry is broken on the UV brane where the relevant flavor mixings are introduced. In the following, we will use χ and ψ to indicate the left and right-handed helicity components of the bulk fields. On the UV brane, we can write two kinetic terms of the form:

$$\mathcal{L}_{UV} = i\psi_R^\alpha \sigma_\mu D^\mu \mathcal{K}^{\alpha\beta} \bar{\psi}_R^\beta \Big|_{z=z_{UV}} , \tag{2.1}$$

both for up and down quarks, with different mixing matrices $\mathcal{K}_u$ and $\mathcal{K}_d$. To simplify the notation, we will suppress the weak isospin index as the following discussion can be separately applied both to up and down quarks. $\mathcal{L}_{UV}$ determines the BC's on the UV brane for the R fields:

$$\chi_R(z_{UV}) = m\, \mathcal{K} \cdot \psi_R(z_{UV}) . \tag{2.2}$$

The key point is that this is the *only* source of flavor mixing: in fact both the bulk equation of motion and the remaining boundary conditions are flavor diagonal. We can therefore solve the equations of motion for all the fields and impose the IR BC's (and remaining BC's on the UV brane): this is enough to determine uniquely the wave functions up to an overall normalization. The solutions will look like:

$$\begin{array}{ll} \chi_L^\alpha = A^\alpha f_L(m,z) , & \chi_R^\alpha = A^\alpha f_R(m,z) , \\ \psi_L^\alpha = A^\alpha g_L(m,z) , & \psi_R^\alpha = A^\alpha g_R(m,z) . \end{array} \tag{2.3}$$

It is crucial here that the functions $f_{L,R}$ and $g_{L,R}$ do not carry any flavor index: all the flavor information is in the normalization vectors A. The specific form of the functions f, g depends on the detail of the bulk physics and will not play any role for our argument.

The remaining BC's in Eq. (2.2) determine the masses of the SM fermions and their KK excitations:

$$\begin{array}{c} f_R(m, z_{UV}) A = m\, g_R(m, z_{UV})\, \mathcal{K} \cdot A \\ \Downarrow \\ \mathcal{K} \cdot A = \frac{f_R(m,z_{UV})}{m\, g_R(m,z_{UV})}\, A \end{array} \tag{2.4}$$

This implies that the A's are the eigenvectors of $\mathcal{K}$ and the eigenvalues of $\mathcal{K}$ will determine the fermion masses. If the unitary matrix U diagonalizes $\mathcal{K}$, that is $\mathcal{K} = U\mathcal{K}^{diag}U^\dagger$, then the normalized eigenvectors are given by

$$A^\alpha_{(i)} = U^\alpha_i\,, \tag{2.5}$$

where the lower index on A indicates which mass eigenstate we are considering, and the upper index is the index in flavor space. Thus the U matrix determines in which direction in flavor space the various mass eigenstates are pointing. The solutions of (2.4) will thus consist of 3 distinct towers of fermions (that include the light SM fermions) corresponding to the KK towers of the three generations. The actual spectrum is then determined by the equations

$$\frac{f_R(m_i, z_{UV})}{m_i\, g_R(m_i, z_{UV})} = k_i\,, \quad i = 1\ldots 3 \tag{2.6}$$

where k_i are the eigenvalues of the matrix $\mathcal{K}$. These equations will determine the masses of the light quarks, and their KK states.

It is now simple to verify our claims: in the neutral sector, all the couplings are diagonal. In fact, they will either come from bulk or IR brane kinetic terms and thus be proportional to $U^\dagger U = 1$, or from the UV brane kinetic terms and thus will be proportional to $U^\dagger\mathcal{K}U = \mathcal{K}^{diag}$. Let us stress here that this conclusion can be applied not only to the SM light particles, but also to fermion and gauge resonances. For the charged W and its resonances, the couplings are diagonal in flavor space. However, if the matrices $\mathcal{K}_u$ and $\mathcal{K}_d$ are misaligned, the couplings to the mass eigenstates will be proportional to $U_u^\dagger U_d = V_{CKM}$: this defines the CKM mixing matrix in this scenario. Note that this conclusion can be applied to not just to the W KK states, but also to extra charged vectors arising from SU(2)$_R$ since they vanish on the UV brane, so their couplings are necessarily flavor diagonal.

Finally, the model may also contain extra exotic massive quarks that can couple to the SM ones via the W: in this case, such couplings may be proportional to a different mixing matrix, for instance $U_u^\dagger U_{q'} \neq V_{CKM}$. However, their effect is model dependent and will only enter at loop level.

Let us now count how many mixing parameters and CP violating phases one has in this setup. We assume that we have N generations, and we are allowing a separate kinetic mixing matrix for the right-handed up and down quarks. These kinetic mixing terms are described by two hermitian $N \times N$ matrices, and therefore in total there are $2N^2$ real parameters. The parameters of a general hermitian matrix can be divided into the real diagonal components, the number of off-diagonal components and the phases of the off-diagonal components. Thus in total we have $2 \times (N + N(N-1)/2) = N(N+1)$ real parameters, and $N(N-1)$ phases. We are free to make single SU(N) unitary transformation on both up-type and down-type quarks simultaneously, since this is an unobservable redefinition of flavor, that leaves the physics invariant. This SU(N) symmetry accounts for $N(N-1)/2$ real parameters and $(N-1)(N+2)/2$ phases. Thus we are left with $2N + N(N-1)/2$ observable real parameters, however $2N$ of these correspond to the quark masses. So there will be $N(N-1)/2$ mixing angles. We are also left with $(N-1)(N-2)/2$ physical phases. This exactly reproduces the usual CKM picture of CP violation.

3 Holographic interpretation in warped space

The setup used in this paper has a natural four dimensional explanation in terms of the AdS/CFT correspondence. The conjecture is that a 5D theory in AdS space is equivalent to a 4D conformal field theory. In our case we are considering a finite slice of AdS_5. The UV (or Planck) brane would correspond to the CFT having a UV cutoff, and the IR (or TeV) brane to spontaneous breaking of conformal invariance by strong dynamics. Here we are in addition requiring that there are some additional *global symmetries* in the 5D theory. This is somewhat unusual, since the usual lore about AdS/CFT is that a global symmetry of the CFT corresponds to a *gauge* symmetry in the bulk. If we accept that this is the only reasonable interpretation, we can still make this bulk gauge symmetry behave almost like a global symmetry by taking the bulk gauge coupling to be very small.

The CFT interpretation is the following: there is a CFT, which has a global symmetry $\mathrm{U}(3)_Q \times \mathrm{U}(3)_{qR}$. This global symmetry is then spontaneously broken by the CFT interactions that become strong in the IR (which is related to the breaking of the conformal invariance and also of electroweak symmetry) to $\mathrm{U}(3)_V$. The SM fermions are linear combinations of elementary fermions and of composite states. The elementary fermions do not feel electroweak symmetry breaking directly, only through the mixing with the composite modes. The elementary left handed fields respect the same $\mathrm{U}(3)_Q$ flavor symmetry as the conformal sector. However, due to the misalignment of the kinetic terms of the elementary right-handed fermions, the $\mathrm{U}(3)_{uR} \times \mathrm{U}(3)_{dR}$ symmetry of the elementary sector and the $\mathrm{U}(3)_{qR}$ symmetry of the conformal sector are broken down to $\mathrm{U}(1)_{qR}$. Since the CFT also spontaneously breaks $\mathrm{U}(3)_Q \times \mathrm{U}(3)_{qR}$ to $\mathrm{U}(3)_V$ in the end overall there is no flavor symmetry left unbroken (except of $\mathrm{U}(1)_V$ which is identified with overall baryon number), leading to the possibility of quark mixing. However, as explained in Sec. 2 this global symmetry breaking pattern is sufficient to ensure that in the neutral current sector there is a $\mathrm{U}(1)_u \times \mathrm{U}(1)_c \times \mathrm{U}(1)_t \times \mathrm{U}(1)_d \times \mathrm{U}(1)_s \times \mathrm{U}(1)_b$ symmetry unbroken protecting the theory from FCNC's.

Finally, we need to discuss the fate of the bulk gauge bosons (which are the consequence of the global symmetry of the CFT). In the CFT language these will just be a towers of spin 1 modes. As already discussed, we can reduce the global symmetries of the CFT (i.e. the gauged symmetries in the bulk) to $\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_{qR}$ without affecting our symmetry argument: this minimal choice ensures the absence of massless degrees of freedom (like scalar goldstone bosons and/or massless gauge bosons). Since the elementary sector breaks $\mathrm{SU}(3)_{qR}$, the only gauge symmetry that survives on the UV brane is $\mathrm{SU}(3)_Q$. Therefore, in addition to the usual tower of KK states with masses proportional to the IR scale R', there will be a lighter adjoint of $\mathrm{SU}(3)_Q$. The mass of this flavor gauge bosons is model dependent, but will generically be suppressed with respect to the IR scale by $\sqrt{\log R'/R}$. Numerically, it will be roughtly a factor of 10 lighter than the first KK state, and as low as the W mass in Higgsless models.

One may worry that these new gauge bosons whose masses can be quite low will themselves mediate flavor changing interactions and therefore impose an incredibly tight bound on the gauge couplings. However, as we will show, this is not the case. In fact, the only source for flavor violation here is the misalignment between the up and down type quarks. Such misalignment is given by the matrices U_u and U_d that diagonalize the UV kinetic terms $\mathcal{K}_u$ and $\mathcal{K}_d$. Therefore, the mass eigenstates will couple to different combinations of flavor gauge bosons: for example, in the left-handed sector we have couplings like:

$$i g_Q \bar{u}_\ell^i \gamma_\mu u_\ell^j \, [U_u^\dagger \cdot T^a \cdot U_u]_{ij} \, W^a_{Q\;\mu}, \quad i g_Q \bar{d}_\ell^i \gamma_\mu d_\ell^j \, [U_d^\dagger \cdot T^a \cdot U_d]_{ij} \, W^a_{Q\;\mu}\,; \tag{3.1}$$

where T^a are the generators of $\mathrm{SU}(3)_Q$. We can immediately see that no flavor changing operator will involve only up or down type quarks (which would in fact correspond to highly constrained FCNC's). Therefore, the only flavor violating 4-fermion operators must involve the exchange of a W_Q gauge boson between up and down currents, where the misalignment has a physical effect:

$$(\bar{u}_\ell^i \gamma_\mu u_\ell^j)\,(\bar{d}_\ell^k \gamma^\mu d_\ell^l)\,. \tag{3.2}$$

The explicit calculation shows that the coefficient of this operator is given by

$$U_u^{\dagger\;ii'} U_u^{jj'} U_d^{\dagger\;kk'} U_d^{nn'} \frac{g_Q^2}{M_{W_Q}^2} \sum_a T^a_{i'j'} T^a_{k'n'} \tag{3.3}$$

Using an SU(3) and a Dirac Fierz identity this operator is equivalent to (up to flavor conserving terms)

$$-\frac{g_Q^2}{2M_{W_Q}^2} \left[V_{in} \bar{u}_\ell^i \gamma_\mu d_\ell^n\right] \left[V_{kj}^\dagger \bar{d}_\ell^k \gamma_\mu u_\ell^j\right] \tag{3.4}$$

This is exactly equivalent to the effect of the ordinary W-boson, with a suppressed gauge coupling g_Q replacing the standard $\mathrm{SU}(2)_L$ coupling g. Thus we conclude that the MFV prescription also applies to the flavor gauge bosons and there is no bound on such operators from flavor physics. The actual bounds will come from the traditional electroweak precision bounds on flavor conserving operators. The induced four fermi operators will be of the form $qqqq$ and thus are not very strongly constrained by precision electroweak measurements. A few TeV suppression scale should be sufficient. In the $\mathrm{SU}(3)_{uR}$, $\mathrm{SU}(3)_{dR}$ sector this can be ensured if the effective 4D gauge coupling is smaller by a factor of 2-3 than the ordinary SM weak couplings. This can be achieved by choosing $g_{5,u} \sim g_{5d} \leq 0.3 g_{5L}$. However, as we discussed the gauge boson corresponding to the $\mathrm{SU}(3)_Q \times \mathrm{U}(1)_{uR} \times \mathrm{U}(1)_{dR}$ symmetry can be much lighter than the others, due to the fact that it is not broken explicitly in the elementary sector (but only spontaneously by the CFT). Thus its coupling must be much smaller than that of $\mathrm{SU}(3)_{uR}$, $\mathrm{SU}(3)_{dR}$. One may worry that as we take the gauge coupling to zero the mass of this gauge boson will vanish. However, we know from higgsless models that this is not the case, and we expect that these gauge bosons could have a mass at least that of the ordinary W, Z (and in general of order $1/(R'\sqrt{\log R'/R})$). To suppress contributions to 4-fermi operators by a few TeV one should make sure that the gauge coupling in this sector

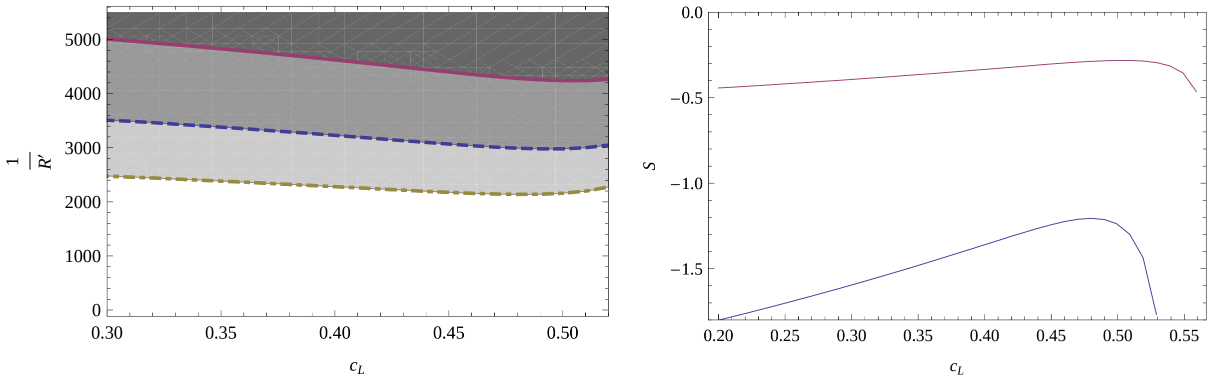

Figure 1: Left panel: We show the lower bounds on $1/R'$ as a function of c_L for which $|S| < 1, 0.5, 0.25$ (bottom to top). Right panel: S in RS1 as a function of the bulk mass c_L for $1/R' = 2$ and 4 TeV. In both cases we have set $c_R = 0$.

(assuming $1/R' \sim 1$TeV) is at most $g_{5Q} \leq 0.1 g_{5L}$. In this case all 4 fermi operators induced by this very weakly coupled gauge boson will be negligible.

4 Applications to models

In this section we show how the general ideas explained above can be applied to obtain concrete realistic warped space models with flavor symmetry. We will be focusing on the RS1 model [2, 18] and the Minimal Composite Higgs (MCH) [19] model. In these models there is a light Higgs localized on or around the TeV brane with the usual SM VEV of size ~ 246 GeV. In both cases unitarization of WW scattering happens via Higgs exchange, and the KK resonances of the W, Z, g are heavy $m_{KK} \geq 2$ TeV, corresponding to $1/R' \geq 1$ TeV. The main difference is that in RS1 there is a little hierarchy problem (i.e. there is no understanding of why $vR' \ll 1$), while in the MCH model this is explained since the Higgs is also a pseudo-Goldstone boson of an approximate global SO(5) symmetry (which also incorporates custodial SU(2)). From the point of view of fermion representations the main difference is that in RS1 the bulk fermions are in $(2, 1) + (1, 2)$ of $\mathrm{SU(2))}_L \times \mathrm{SU(2)}_R$, while in the MCH they are in the spinor 4 of SO(5).

The GIM mechanism outlined in the previous sections can be applied without any complication to the first two generations of quarks for both models. The reason is that if the heaviest quark mass is the charm then any overall deviation of fermion wave functions from those of zero modes will be proportional to $(m_c R')^2 \sim 10^{-6}$. This is no longer true if one also includes the top quark. Then $(m_t R')^2 \sim 10^{-2}$, so a percent-level overall shift in the fermionic gauge couplings would follow. Since these couplings are measured at the $\sim 0.1\%$ level this would be inconsistent with the precision electroweak observables. If all fermions (including leptons) had a universal c_L and Dirac mass on the TeV brane, then this could be reinterpreted as a large negative shift in the effective S-parameter, which is unacceptable.

This is illustrated in the right panel of Fig. 1, where we show the effective S-parameter as a function of c_L and of $1/R'$ for the RS1 case. Note that one striking difference of this plot vs. that for traditional RS1 with bulk fermions is that the S-parameter does not cross through zero around $c_L \sim 1/2$. A simple explanation of this overall shift in S is the following: the large top mass usually accompanies an unacceptably large shift in the left handed couplings of the bottom. By introducing the flavor symmetry we eliminate the *relative* shifts in the couplings, at the price of introducing an *overall* shift that is S. To avoid this we clearly need to treat the third generation separately. A nice solution for avoiding a large correction in the $Zb\bar{b}$ coupling has been proposed by Agashe et al. [17]. The main idea is to use different embeddings of the SM fermions into the custodial symmetry. The simplest possibility for a single generation is the following:

$$\begin{array}{cccc} & SU(2)_L & SU(2)_R & U(1)_X \\ Q_L & \square & \square & \frac{2}{3} \\ t_R & 1 & 1 & \frac{2}{3} \\ b_R & 1 & \square\!\square & \frac{2}{3} \end{array} \tag{4.1}$$

Zero-modes for the additional fields are projected out using the BC's. We can then eliminate/reduce FCNC's in two separate ways using these representations.

Model 1.

In this setup there are no FCNC's (thus it corresponds to a MFV scenario, i.e. a model with a built-in GIM mechanism), no large corrections to the $Zb\bar{b}$ coupling, and no large vertex corrections (or S-parameter). The proposal is to use the representations in (4.1) for all three generations, and to also break the flavor symmetry in the up-sector from U(3) to $\mathrm{U}(1)^3$ by adding different bulk and TeV brane localized masses for the 3 up-type quarks. This will allow us to avoid a large overall correction for the light quark couplings, since the light quark wave functions will not be modified strongly by the Dirac masses. The use of the unconventional representations will protect the bottom quark from large vertex corrections (but not the top or other up-type quarks). The Dirac masses on the TeV brane are assumed to be of the form (in flavor space)

$$Q_L^s \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} t_R + m_b \, Q_L^t \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} b_R \tag{4.2}$$

where $Q_L^{s,t}$ are the singlet and triplet components under $\mathrm{SU}(2)_D$ of the bidoublet Q_L and we have inserted a bifundamental Higgs VEV. The key observation is that the breaking of the up-type flavor symmetry to $\mathrm{U}(1)^3$ does not get communicated into the bottom sector due to the fact that t_R is a singlet under the custodial symmetry and thus this mass term does not involve any quarks with the quantum numbers of the bottom that could potentially mix with the bottom (this is unlike the case using the standard representations where the (t_R, b_R) are in a doublet of $\mathrm{SU}(2)_R$). Thus in the NC sector the full flavor symmetry in the down-type quark sector is unbroken on the TeV brane. Then if we introduce all flavor mixing via UV brane localized kinetic mixing terms for the down-type quarks only, all FCNC's will be avoided, so that this setup also has a GIM mechanism.

What needs to be checked is that this setup is also consistent with other precision electroweak constraints. In particular the main worry is that since we are still requiring an $\mathrm{SU}(3)_L$ flavor symmetry for the left handed quarks the requirement for a large top mass may still result in a large vertex corrections. Generic electroweak precision bounds on models using the new representations under custodial symmetry were presented in [20]. Note however, that the choice of the new representations under the custodial symmetry is designed such that the down-type left handed quarks do not get much of a vertex correction, while there is no protection mechanism for the up-type quarks. Therefore by construction the corrections are non-oblique and one should not use the the oblique parameter formalism to estimate the sizes of the corrections. A simple scheme [21] to estimate the bounds from the precision electroweak observables is to simply fix the W, Z masses and the electromagnetic couplings as the input parameters, and shift all corrections into vertex corrections to Z and W couplings. The leptons are not a problem: they can be either in the bulk or localized near the Planck brane, depending on how low $1/R'$ is. Concerning the quarks, right handed up and down-type quarks are near the Planck brane (except for the top, whose couplings are not strongly constrained), thus the deviation of their couplings to the Z is under control. Since there is an explicit protection mechanism for the left handed down-type quarks, the only potentially dangerous corrections are those to the light left handed up-type quarks. To find the experimental bounds on this model from this effect we have first calculated the maximal value of c_L for which a sufficiently heavy top mass can still be obtained for perturbative values of the Yukawa couplings. This bound turns out to be relatively insensitive to the values of R' and the bound is around $c_L \leq 0.47$. However, this bound is somewhat sensitive to the localization parameter c_R of the right handed top. What one then needs to check is the correction to the vertex corrections to $g_{Zu_Lu_L}$ are not too large. Note, that the main constraints on this coupling come from the measurements involving hadronic final states at LEP, for example $\Gamma(Z)$ and $\sigma(\text{hadron})$. Using the method of [22] we estimate that a reasonable 3 sigma bound on the deviation of this coupling is about ± 0.4 percent. In addition we also need to make sure that the couplings to the KK Z' bosons or KK gluons will not be too large to generate flavor invariant 4 fermi operators. Our results are summarized in Fig. 2 where we show that unless $1/R'$ is very small the shift in $g_{Zu_Lu_L}$ is acceptably small, while the coupling of the light fermions to the KK Z' always remain within an acceptable range. A bound on $1/R'$ of order $\sim 1 - 1.5$ TeV follows from these constraints. In Table 1 we show an example for the shifts in the couplings for an allowed point in the parameter space for a low value of $1/R'$. Another electroweak precision bound that one may consider (depending on the exact treatment of the right handed up quarks) are four fermi operators of the form eeu_Ru_R. These will be generated if the bulk $\mathrm{SU}(3)_{u_R}$ flavor symmetry is maintained (i.e. we use the same c_R for all u_R's). In this case the light u_R's will have an enhanced coupling to the KK Z' mode. This operator is only constrained from the LEP2 $e^+e^- \to q\bar{q}$ cross section measurement and its coefficient is not very strongly constrained. Also, the enhancement of the u_R couplings is partly offset by the suppression of the electron couplings due to them being localized on the UV brane. Using the χ^2 provided in [23] we have checked that these operators do not further reduce the allowed parameter space in Fig. 2.

u d	$\gamma_L^u = -3.1$ $\gamma_L^d = 1.4$	$\omega_L = -0.48$	$\gamma_R^u = 0.76$ $\gamma_R^d = -0.012$	$\omega_R < 10^{-7}$
c s	$\gamma_L^c = -3.1$ $\gamma_L^s = 1.4$	$\omega_L = -0.48$	$\gamma_R^c = 0.76$ $\gamma_R^s = -0.016$	$\omega_R < 10^{-3}$
t b	$\gamma_L^t = -3.9$ $\gamma_L^b = 1.4$	$\omega_L = -0.85$	$\gamma_R^t = 20$ $\gamma_R^b = -7.1$	$\omega_R = -2.2$

Table 1: Per mille relative deviations of the effective couplings to the SM values of the fermion gauge coupling strengths for a particular allowed point from Fig. 2, chosen to correspond to $1/R' = 1.5$ TeV, $c_L = 0.47$, $c_R = -0.51$, and $c_{tR} = 1$. We parameterize the deviation by $g^Z_{f_L} = (1+\gamma^f_L)\frac{g}{\cos\theta_W}(T_3 - \sin^2\theta_W\, Q)$, $g^W_{f_L} = (1+\omega^f_L)g$ and similarly for the right-handed couplings.

Next we can check that the number of flavor parameters really agree with that expected from the ordinary CKM mechanism. The mixing is described now by a single hermitian kinetic mixing matrix for the right handed down quarks, which is parametrized by $N(N+1)/2$ real numbers (angles) and $N(N-1)/2$ phases. Of these real numbers N correspond to the quark masses in the down sector, and we can still use the $\mathrm{U}(1)^N$ unbroken flavor symmetry to remove $N-1$ phases (one overall is baryon number). Thus we again end up with $N(N-1)/2$ mixing angles and $(N-1)(N-2)/2$ CP violating phases as expected.

Finally, we comment on the possible presence of familons and alignment issues in this model. The symmetry breaking pattern is the following. On the UV brane we have and $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_{u_R}$ symmetry, in the bulk we have $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_{d_R}\times\mathrm{U}(1)^3_{u_R}$ while on the TeV brane just $\mathrm{U}(1)^3_{diag}$. Since the coupling of the u_R quarks turns out to be quite insensitive to the value of c_R, we are not necessarily forced to break the $\mathrm{SU}(3)_{u_R}$ symmetry in the bulk to $\mathrm{U}(1)^3_{u_R}$. In the case of a bulk $\mathrm{SU}(3)_{u_R}$ symmetry we would not have to deal with the question of why the $\mathrm{U}(1)^3$ symmetries in the bulk and the brane are aligned. If we do break the symmetry in the bulk, we have to insist that the same spurion is used to break the bulk and the brane symmetries.

In the CFT interpretation there is a gauged $\mathrm{SU}(3)_L\times\mathrm{U}(1)^3_{u_R}$ symmetry (or $\mathrm{SU}(3)_L\times \mathrm{SU}(3)_R$) and an additional $\mathrm{SU}(3)_{d_R}$ global symmetry, all of which are broken by the CFT to a $\mathrm{U}(1)^3_{diag}$ global symmetry. Thus there would be familons corresponding to the breaking of $\mathrm{SU}(3)_{d_R}$. In order to make these appropriately heavy there has to be an additional explicit breaking of this symmetry, which in the 5D picture corresponds to a bulk Higgs for $\mathrm{SU}(3)_{d_R}$ which is however not coupled to the bulk fermion fields. Such a bulk Higgs can also make bulk gauge bosons arbitrarily heavy and effectively decouple them while leaving a global symmetry in the fermion sector.

Model 2.

In this scenario, we treat the third generation differently from the two light generations,

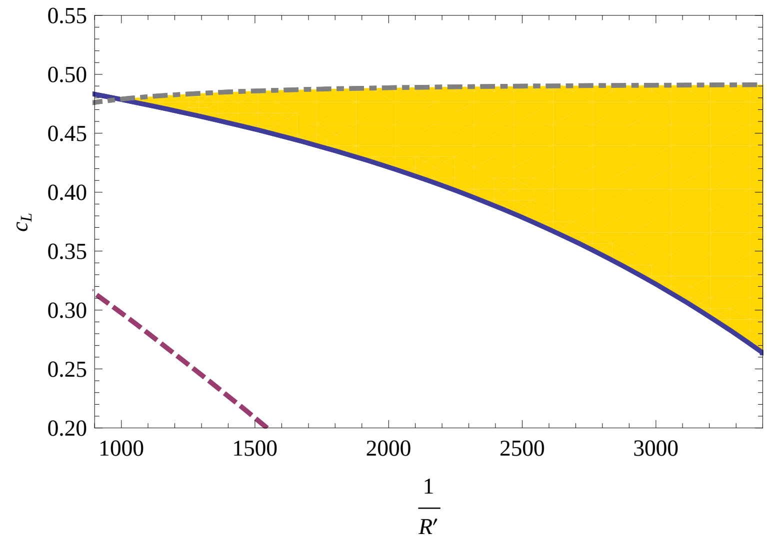

Figure 2: The allowed region of the parameter space in Model 1 with MFV. The upper dash-dotted curve shows the perturbativity bound on the parameter c_L assuming that the right handed top is strongly localized on the TeV brane (we chose $c_R = 1$, and required that the Yukawa coupling is less than four). The middle solid curve shows the region where the deviation of the coupling of the left handed up-type quark is below 0.5%. The allowed region is the shaded one between these two curves. The lowest dashed curve shows the bound from the coupling of the KK mode of the Z coupling to light left handed quarks. Such a coupling would generically induce four fermi operators involving leptons. The bound obtained in this plot is requiring that the $q/M_{Z'}$ ratio of coupling to mass is less than 1/(5 TeV) [22]. We can see that this bound is irrelevant for the allowed region.

and therefore we only impose a SU(2)×U(1) flavor symmetry. The symmetry breaking pattern will be the same as in Sec. 2 with SU(3) replaced by this reduced symmetry. The new feature of this scenario is that the third generation can have different bulk and IR masses, and even be in different representations of the bulk gauge symmetries. We will leave the light generations in the usual representations and localized towards the UV brane, while the third generation is in the new custodial representation and can be localized near the IR brane. As a result we will clearly not have a problem with a large shift in the S-parameter and the deviation of the $Zb\bar{b}$ coupling will be sufficiently reduced due to the use of the representations (4.1). The drawback of this scenario is that FCNC's will be generated, not only involving the top and bottom, but also for the light generations. However, due to the SU(2) symmetry, those FCNC's will have a particular form and may result in weaker bounds than in the usual RS1 case [6, 7].

Let us first note that due to the heaviness of the top quark, the right handed top should be strongly localized on the TeV brane: therefore the effective kinetic mixing terms in the right handed up-sector only involve the first two generations (since the wave function of the right handed top is negligible on the UV brane). This matrix can be diagonalized using the

SU(2) symmetry, so that all the off-diagonal terms are in the down sector. As a consequence, FCNCs in the up-quark sector are negligible.

In the down sector the situation is more complicated, since the R bottom is also localized towards the UV brane and we do need to generate a mixing with the third generation. The difference with respect to the case analysed in Sec. 2 is that the wave function of the third generation is different from the wave function of the light generations. This means that the elements of the mixing matrix $\mathcal{K}$ will acquire different coefficients, depending on the quantity we are interested in: for instance, the matrices entering the boundary conditions and the couplings of the right-handed down quarks to the Z will be different. However, due to the SU(2) symmetry, the effect of the different wave functions will preserve a block structure as follows:

$$\begin{pmatrix} a\mathcal{K}_{(12)} & b\mathcal{K}_{(13)} \\ b\mathcal{K}_{(13)}^T & c\mathcal{K}_{33} \end{pmatrix} \tag{4.3}$$

where $\mathcal{K}_{(12)}, \mathcal{K}_{(13)}, \mathcal{K}_{33}$ are the appropriate blocks of the UV kinetic mixing matrix, and the coefficients a, b, c depend on the details of the wave functions and the quantity we are calculating. If we diagonalize the 2×2 block $\mathcal{K}_{(12)}$ with an SU(2) rotation, we only generate the Cabibbo angle in the couplings of the W, but no FCNC in the couplings with neutral gauge bosons, as in Sec. 2. We still have off-diagonal terms proportional to $\mathcal{K}_{(13)}$ that cannot be diagonalized in the same way due to the effect of the different wave functions. However, in this basis every flavor changing effect involving the down quark must be proportional to the top component of the vector $\mathcal{K}_{(13)}$ since this is the only flavor violating matrix element involving the down quark. The CKM matrix connecting the down to the third generation must also be directly proportional to this matrix element, since there is no flavor mixing involving the top directly. Applying a similar argument for the strange quark we find that the flavor changing couplings must always be suppressed by the appropriate CKM matrix elements:

$$g_{Zds} \sim V_{td} V_{ts} \delta \tag{4.4}$$

$$g_{Zdb} \sim V_{td} \delta \tag{4.5}$$

$$g_{Zsb} \sim V_{ts} \delta \tag{4.6}$$

This pattern will show up for the couplings of the Z as well as other neutral massive bosons, and for both L and R fermions. This means that this model is a simple implementation of NMFV, where all flavor violation is due to the flavor violating interactions with the third generation. The main difference compared to the traditional RS models is that here the NMFV structure of the corrections (by which we mean that all additional flavour violation must proceed through the third generation) is due to symmetries (rather than smallness of wave function overlaps) and thus here there will never be any direct contributions to $K\bar{K}$ mixing that are not proportional to the CKM matrix elements involving the third generation.

We can also easily understand the origin of the parameter δ. There are two sources for gauge boson couplings: bulk kinetic terms and the brane localized kinetic mixing terms. The bulk kinetic terms are diagonal but not necessarily universal due to the different wave function for the bottom, thus take the form $\mathrm{diag}(g_1, g_1, g_3)$ in flavor space before diagonalizing the localized mixing terms. The localized mixing terms have an arbitrary structure in the down sector, but the main point is that once the kinetic terms are diagonalized so will the resulting contributions to the neutral gauge boson couplings as well. Therefore, the only source of off-diagonal couplings is the difference $\delta \propto g_3 - g_1$. For the case of left handed fields there is no localized kinetic term so $g_3 - g_1$ is directly proportional to $g_{Zb_Lb_L} - g_{Zd_Ld_L}$. However for the right handed fields there can also be a contribution to δ from hierarchies in $\mathcal{K}_d$. Thus the precise value of δ for the right handed fields is likely to be quite model dependent.

Now we can estimate the bounds on the model using the above structure for the corrections. For simplicity we will apply a bound of the form [24]

$$\frac{g_{Zds}}{g}\frac{m_Z}{m_{Z'}} < 10^{-5} \tag{4.7}$$

where Z' stands for an arbitrary neutral gauge boson in the model.

If we apply this for the Z in the left handed sector we find that the requirement is

$$\frac{\delta_{Zbb}}{g} < 3\% \tag{4.8}$$

which is weaker than the direct bound from precision electroweak observables. For the effects of the Z' in the L sector we find (assuming that there is not much suppression in the coupling of the light L quarks to the Z' compared to the SM value)

$$m_{Z'} > 10^5\, V_{td} V_{ts}\, m_Z \;\sim (3\,\mathrm{TeV}) \tag{4.9}$$

A slightly stronger bound would be obtained by considering the KK mode of the gluon field. For the right-handed quarks, the presence of non-diagonal kinetic terms makes the analysis more involved: however, we can observe that all the right-handed down quarks are very localized on the UV brane, so that their couplings to the KK gauge bosons are suppressed by a log factor compared to the SM ones:

$$\delta \sim \frac{g}{\sqrt{\log R'/R}} \tag{4.10}$$

The bound on the mass of the Z' is therefore weaker than from the L sector considered above. However, we do not have a reliable estimate for the suppression of the flavor violating right handed couplings of the ordinary Z, which may be still be a problem and needs to be calculated numerically.

We can see one additional advantage of this setup versus the traditional split-fermion approach to RS flavor, and that is when considering the additional CP phases. In the traditional setup there is a CP problem, i.e. there are additional physical CP phases that have no reason to be small, and which will tighten the bounds on the KK scale to ~ 8 TeV [9]. However, in this setup the symmetries will forbid the appearance of an additional physical CP phase. As we have seen in this setup all flavor violating parameters originate from the mixing matrices $\mathcal{K}_{u,d}$.[1] For the model under consideration $\mathcal{K}_d$ is a generic 3×3 hermitian matrix and $\mathcal{K}_u$ a generic 2×2 hermitian matrix. Thus $\mathcal{K}_d$ contains 3 complex phases, while $\mathcal{K}_u$ contains 1. However, the remaining symmetry of the bulk + IR brane sector is SU(2)×U(1)×U(1)$_B$. The U(1)$_B$ corresponding to baryon number has no effect on the phases in $\mathcal{K}_{u,d}$, however the remaining SU(2)×U(1) can still be used to eliminate non-physical phases. Since SU(2)×U(1) contains 3 phases, we conclude that in total there is just a single CP violating phase in this setup, which should be identified with the CP phase from the CKM mechanism. Thus there is no possibility for an additional CP problem to emerge here.

5 Conclusions

In this work we have explored how to implement an alternative realization of the SM flavor structure in warped extra dimensional models. The accepted approach is to use the split fermion scenario where hierarchies are obtained from overlaps of wave functions. Here we asked how a more traditional picture based on flavor symmetries can be implemented. We found that if there is a sufficiently large bulk+IR brane flavor symmetry, a GIM mechanism can be incorporated preventing the generation of FCNC's. In this case all the mixing is obatined from UV brane localized kinetic mixings. Inclusion of a large top mass (together with electroweak precision constraints) forces us to modify this minimal setup. Models with MFV can be obtained by putting all SM quarks into new representations under the custodial SU(2) symmetry, while models with NMFV are obtained by using the new representations only for the third generation quarks.

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