

The AdS/CFT/Unparticle Correspondence

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Abstract

We examine the correspondence between the anti-de Sitter (AdS) description of conformal field theories (CFTs) and the unparticle description of CFTs. We show how unparticle actions are equivalent to holographic boundary actions for fields in AdS, and how massive unparticles provide a new type of infrared cutoff that can be simply implemented in AdS by a soft breaking of conformal symmetry. We also show that processes involving scalar unparticles with dimensions $d_s < 2$ or fermion unparticles with dimensions $d_f < 5/2$ are insensitive to ultraviolet cutoff effects. Finally we show that gauge interactions for unparticles can be described by bulk gauge interactions in AdS and that they correspond to minimal gauging of the non-local effective action, and we compute the fermion unparticle production cross-section.

1 Introduction

Recently Georgi [1, 2] proposed a new way of dealing with conformal field theories (CFTs). The exact propagator of a gauge invariant field is fixed up to a normalization by the scaling dimension of the field. The corresponding final state phase space looks like that of a non-integer number of particles, and so Georgi dubbed them unparticles. Weakly coupled field theories in anti-de Sitter (AdS) space provide another way of approaching large N CFTs by means of the AdS/CFT correspondence [3]. Since unparticles and AdS theories can in principle both describe the same CFT there must be a way to translate results from one approach into the other. As is often the case, this is not merely a trivial exercise, since problems that are difficult to solve in one language turn out to be easy to solve in the other. In this paper we will work through several cases where insights from AdS provide new clarity to the unparticle description and also where insights from the unparticle description lead to new developments in the AdS description.

In [4] we proposed using an effective (non-local) action to work with unparticles. This action is completely fixed by requiring that it gives the correct unparticle propagator. Using this effective action it is a relatively straightforward exercise to gauge the global symmetries and to derive the Feynman vertices for gauge interactions [5–7]. However the resulting inclusive scalar unparticle production cross-section via gauge bosons only makes sense for scaling dimensions below two. If the cross-section is extrapolated to scaling dimensions above two it turns negative. Clearly the effective action cannot be used for scaling dimensions above two, but it would be nice to have a simple physics explanation. Such an explanation will arise in this paper.

Another issue related to gauge interactions is the question of whether it is theoretically consistent to consider minimal coupling of an unparticle to an ordinary gauge boson, neglecting the possible mixing of the gauge bosons with spin one CFT states with the same gauge quantum numbers. In the AdS description this mixing typically results in the absence of a normalizable zero mode gauge boson corresponding in 4D to the gauge coupling being driven to zero in the infrared (IR) by the large N CFT. Finally, unparticles with standard model gauge charges are compatible with low energy experiments in the real world only if their effects are somehow cut off at low energies. Some phenomenological proposals for how to break the unparticle conformal invariance at low energies exist [4, 8], however it is not clear if it is possible to do so while preserving a continuum of states above the threshold and without introducing light particle resonances, whose presence would completely spoil the unparticle approximation. Naively one might suspect that introducing an IR cutoff in AdS corresponds to confinement in the CFT and thus produces towers of Kaluza-Klein (KK) modes rather than just cutting off the bottom of a continuum. In [4] we suggested that solving the problem of the IR cutoff can also solve the problem of the gauge coupling being driven to zero. The idea is most simply illustrated by considering a Banks-Zaks model [9]. A large N Yang-Mills theory can be tuned to have a weak IR fixed point by adjusting the number of fermions so that the one-loop β function coefficient is very small and the two-loop term has the opposite sign. We can weakly gauge a subgroup of the global symmetry by letting two of the (Weyl) fermions have opposite perturbative $U(1)$ gauge charges. Since the

fermions are massless they will drive the $U(1)$ gauge coupling to zero in the IR. If we give the fermions with the $U(1)$ charges a Dirac mass, then they will no longer contribute to the running below this mass and the $U(1)$ gauge coupling will not run to zero. The absence of this fermion in the IR will change the value of the Banks-Zaks IR fixed point by an amount of $\mathcal{O}(1/N)$. Since the massive fermion has a finite anomalous dimension it will look like an unparticle above its mass but the phase space will be cutoff below the mass. There is no possibility of any confining behavior in this model, so it is an existence proof that we should be able to introduce an IR cutoff for unparticles that does not correspond to confinement.

In this paper we will show how such an IR cutoff can be introduced in AdS and that it reproduces exactly the simple IR cutoff used for unparticles and does not give a spectrum like a confining theory. We will also show how the zero mode gauge boson can be normalizable in AdS without an IR brane.

The outline of the paper is as follows. In Section 2 we extend the unparticle formalism to fermions and show that the results correspond exactly with those of the AdS/CFT correspondence. In Section 3 we show how holographic boundary actions are related by a Legendre transformation to unparticle actions. In Section 4 we describe a new type of IR cutoff for AdS that reproduces the massive unparticle ansatz without KK resonances. The case of scalar unparticle thresholds is discussed completely while some of the details of the fermion unparticle thresholds are left for an appendix. In Section 5 we show how the bulk gauge action must be modified so that the corresponding 4D β function behaves correctly and how this leads to a normalizable zero mode. In Section 6 we discuss inclusive production cross-sections, what happens when the scalar dimension goes above two, and how the results are extended to unfermions, summarizing the Feynman rules for scalar and fermion unparticles in an appendix. Finally we give our conclusions in Section 7.

2 Fermion Unparticles and AdS

As a warm up, we will first analyze the relation between fermion unparticles and bulk fields in AdS₅. The correspondence between scaling dimensions for fermions in AdS₅ with an ultraviolet (UV) cutoff has been analyzed in Ref. [10] and in AdS₅ with supersymmetry in [11], so we will simply sketch the analysis and refer to these references for further details. We focus on fermions here because the case of scalar fields and unparticles has been already considered in some detail in the literature (see [14, 15] and [16]); furthermore in the scalar case there is a non trivial transition to dimensions less than two [14], which is absent in the fermion case. We will comment on this issue in the next section because dimensions between one and two are actually the most interesting range for scalar unparticle physics. Finally, as it was pointed out in [11], using a supersymmetric formalism with off-shell auxiliary fields, fermions and scalars can be treated in a similar fashion and in particular the subtleties for scalar dimensions below two can be removed.

Let us first define fermion unparticles in analogy to the scalar case [1]: we take the

propagator of a left-handed fermion operator Θ with scaling dimension d_f to be given by ¹

$$\begin{aligned}
\Delta_f(p, d_f) &\equiv \int d^4x e^{ipx} \langle 0 | T \Theta(x) \Theta^\dagger(0) | 0 \rangle \\
&= \frac{A_{d_f-1/2}}{2\pi i} \int_0^\infty (M^2)^{d_f-5/2} \frac{\sigma^\mu p_\mu}{p^2 - M^2 + i\epsilon} dM^2 \\
&= \frac{A_{d_f-1/2}}{2i \cos d_f \pi} (\sigma^\mu p_\mu) (-p^2 - i\epsilon)^{d_f-5/2} + \dots .
\end{aligned} \tag{2.1}$$

This is, of course, is the form one finds for a massless fermion propagator after resumming collinear emissions of a massless gauge boson [12, 13]. We can fix the normalization [17]

$$A_{d_f-1/2} = \frac{16\pi^{5/2}}{(2\pi)^{2d_f-1}} \frac{\Gamma(d_f)}{\Gamma(d_f - 3/2) \Gamma(2d_f - 1)} \tag{2.2}$$

so that in the limit $d_f \rightarrow 3/2$ we find

$$\Delta_f(p, 3/2) = \frac{i\sigma^\mu p_\mu}{p^2 + i\epsilon} , \tag{2.3}$$

and we recover the usual particle propagator for a left-handed fermion. Note that $d_f - 1/2 = d_f - \text{spin}$ is what is conventionally known as the ‘‘twist’’ of the operator. Let us finally recall that for a right-handed unparticle we simply replace $\sigma^\mu p_\mu$ with $\bar{\sigma}^\mu p_\mu$. Taking the discontinuity across the cut from Eq. (2.1) we find the phase space factor

$$d\Phi_f(p, d_f) = A_{d_f-1/2} \theta(p^0) \theta(p^2) (\sigma^\mu p_\mu) (p^2)^{d_f-5/2} \tag{2.4}$$

which, in the limit $d_f \rightarrow 3/2$, becomes the usual phase space factor for a massless fermion:

$$d\Phi_f(p, 3/2) = 2\pi \theta(p^0) (\sigma^\mu p_\mu) \delta(p^2) . \tag{2.5}$$

Note however that the propagator in Eq. (2.1) is not well defined for $d_f > 5/2$: in fact, for large scaling dimensions, the integral is divergent and the propagator becomes UV sensitive. Therefore (just as with scalars with $d_s > 2$) other terms, that depend on the UV completion of the theory, must be added to the propagator (and are represented by the dots in Eq. (2.1)). Another way to formulate the problem is that for $d_f > 5/2$, Georgi’s propagator is dominated by large momenta. On the other hand, the phase space derived from such propagator, is well behaved for all dimensions ($d_f > 3/2$). We will comment more on this point by the end of this section.

Let us now compare the fermion unparticle analysis to fermions in AdS₅. We will use the conformally flat metric

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) , \tag{2.6}$$

¹Note that we could have chosen $\sigma \cdot p + M$ in the numerator of Eq. (2.1): this would have generated a non-local Dirac (or Majorana) mass that scales like $(-p^2 - i\epsilon)^{d_f-3/2}$ and whose coefficient vanishes in the $d_f \rightarrow 3/2$ limit. Our choice is the simplest one, and also what we obtain from a single bulk field in AdS [10].

with a brane at $z = \epsilon$, which represents a UV cutoff $\Lambda_{UV} \sim 1/\epsilon$, and the space extending to infinity [18]. We write the 5D Dirac fermion Ψ in terms of two Weyl spinors χ and $\bar{\psi}$

$$\Psi = \begin{pmatrix} \chi \\ \bar{\psi} \end{pmatrix}. \quad (2.7)$$

The 5D Lagrangian is

$$\mathcal{L}_{5D} = \int_{\epsilon}^{\infty} dz \left(\frac{R}{z} \right)^4 \left(-i\bar{\chi}\bar{\sigma}^{\mu}\partial_{\mu}\chi - i\psi\sigma^{\mu}\partial_{\mu}\bar{\psi} + \frac{1}{2}(\psi\overleftrightarrow{\partial}_z\chi - \bar{\chi}\overleftrightarrow{\partial}_z\bar{\psi}) + \frac{c}{z}(\psi\chi + \bar{\chi}\bar{\psi}) \right), \quad (2.8)$$

where $\overleftrightarrow{\partial}_z = \overrightarrow{\partial}_z - \overleftarrow{\partial}_z$ with the convention that the differential operators act only on the spinors and not on the metric factors. The bulk equations of motion (EOMs) can be solved by

$$\chi = g(p, z)\chi_4 \quad \text{and} \quad \bar{\psi} = f(p, z)\bar{\psi}_4, \quad (2.9)$$

where the 4D plane-wave spinors χ_4 and $\bar{\psi}_4$ satisfy the usual 4D Dirac equation with mass $p = \sqrt{p^2}$:

$$-i\bar{\sigma}^{\mu}\partial_{\mu}\chi_4 + p\bar{\psi}_4 = 0 \quad \text{and} \quad -i\sigma^{\mu}\partial_{\mu}\bar{\psi}_4 + p\chi_4 = 0. \quad (2.10)$$

The solutions are linear combinations of Bessel functions

$$g(p, z) = A z^{\frac{5}{2}} \left(c_{\alpha} J_{c+\frac{1}{2}}(pz) + s_{\alpha} J_{-c-\frac{1}{2}}(pz) \right), \quad (2.11)$$

$$f(p, z) = A z^{\frac{5}{2}} \left(c_{\alpha} J_{c-\frac{1}{2}}(pz) - s_{\alpha} J_{-c+\frac{1}{2}}(pz) \right); \quad (2.12)$$

where $c_{\alpha}^2 + s_{\alpha}^2 = 1$, and we are assuming, for simplicity, that $c \pm 1/2$ is not an integer. The ratio s_{α}/c_{α} is determined by the boundary conditions (BCs) in the IR, $z \rightarrow \infty$, and we will leave it undetermined for the moment. In fact, the properties of the CFT operator do not depend on this choice. The normalization A is determined by the BCs on the UV brane: this depends on which source we couple the CFT operator to, and also determines the chirality of the CFT operator.

For instance, a left-handed source coupled to a right-handed CFT operator corresponds to fixing the value of the left-handed bulk field on the UV brane:

$$\chi(p, \epsilon) = \chi_0 = g_0(p)\chi_4, \quad (2.13)$$

where χ_0 is the source field. This BC fixes the normalization factor A :

$$A = \frac{g_0 \epsilon^{-5/2}}{c_{\alpha} J_{c+\frac{1}{2}}(p\epsilon) + s_{\alpha} J_{-c-\frac{1}{2}}(p\epsilon)}. \quad (2.14)$$

After using the bulk EOMs, the bulk action reduces to a pure boundary term on the UV brane:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \left(\frac{R}{\epsilon} \right)^4 \chi(\epsilon)\psi(\epsilon) + h.c. = \frac{1}{2} \left(\frac{R}{\epsilon} \right)^4 f(p, \epsilon)g(p, \epsilon)\chi_4\psi_4 + h.c. \\ &= \left(\frac{R}{\epsilon} \right)^4 \frac{c_{\alpha} J_{c-\frac{1}{2}}(p\epsilon) - s_{\alpha} J_{-c+\frac{1}{2}}(p\epsilon)}{c_{\alpha} J_{c+\frac{1}{2}}(p\epsilon) + s_{\alpha} J_{-c-\frac{1}{2}}(p\epsilon)} \frac{\bar{\chi}_0 \bar{\sigma}^{\mu} p_{\mu} \chi_0}{p}, \end{aligned} \quad (2.15)$$

where we have used Eqs. (2.10). Since χ_0 is the source for the CFT operator, we can identify the 2 point correlator of the CFT operator with the kinetic term of χ_0 :

$$\langle \Theta_R \Theta_R \rangle = \Delta_R(p, c) \sim \left(\frac{R}{\epsilon} \right)^4 \frac{c_\alpha J_{c-\frac{1}{2}}(p\epsilon) - s_\alpha J_{-c+\frac{1}{2}}(p\epsilon)}{c_\alpha J_{c+\frac{1}{2}}(p\epsilon) + s_\alpha J_{-c-\frac{1}{2}}(p\epsilon)} \frac{\bar{\sigma}^\mu p_\mu}{p}. \quad (2.16)$$

In order to study the properties of the CFT operator, we need to recover exact conformal invariance by removing the UV cutoff. We can now expand the Bessel functions for large UV cutoff, $p\epsilon \ll 1$:

$$J_\nu(p\epsilon) = (p\epsilon)^\nu \sum_{n=0}^{\infty} (p\epsilon)^{2n} \frac{(-1)^n}{2^{2n+\nu} \Gamma(n+1) \Gamma(n+\nu+1)} \equiv (p\epsilon)^\nu \Sigma_\nu(p\epsilon), \quad (2.17)$$

where Σ_ν only contains positive and integer powers of p^2 .

In the case $c > -1/2$, we find that the propagator splits into a local and a non-local term

$$\Delta_R = \Delta_{R,(local)} + \Delta_{R,(non-local)}. \quad (2.18)$$

After rescaling χ_0 by $R^{-2}\epsilon^{2-c}$, we find ²:

$$\Delta_{R,(local)} = -\epsilon^{1-2c} (\bar{\sigma} \cdot p) \frac{\Sigma_{-c+1/2}(p\epsilon)}{\Sigma_{-c-1/2}(p\epsilon)}, \quad (2.19)$$

$$\Delta_{R,(non-local)} = \frac{c_\alpha \bar{\sigma} \cdot p}{s_\alpha p^{1-2c}} \left[\left(\frac{\Sigma_{c-1/2}(p\epsilon)}{\Sigma_{-c-1/2}(p\epsilon)} + (p\epsilon)^2 \frac{\Sigma_{c+1/2}(p\epsilon) \Sigma_{-c+1/2}(p\epsilon)}{\Sigma_{-c-1/2}^2(p\epsilon)} \right) + \mathcal{O}(\epsilon^{2c+1}) \right]. \quad (2.20)$$

From Eq. (2.20) we can deduce that the dimension of the CFT operator is

$$d_{fR} = 2 + c \quad \text{for } c > -1/2, \quad (2.21)$$

in agreement with Refs [10, 11]. In the limit $\epsilon \rightarrow 0$, the non-local part of the propagator is equal to the unfermion propagator (2.1), up to a normalization which depends on the boundary conditions in the IR.

Now consider the local terms. For $-1/2 < c < 1/2$, the local terms are subleading, i.e. they will vanish in the limit $\epsilon \rightarrow 0$. However, for $c > 1/2$, when the dimension of the CFT operator is $d_{fR} > 5/2$, some of the local terms are enhanced by negative powers of ϵ and they diverge in the limit $\epsilon \rightarrow 0$. Such terms are equivalent to the UV sensitivity of the unfermion propagator (2.1) for $d_f > 5/2$: indeed, one has to add counterterms on the UV brane to cancel them³. Those counterterms only affect the propagator of the unparticle, but not

²An expansion of ratios of Σ 's for small arguments is implicitly assumed in this formulas (and in the following). In this sense $\Delta_{(local)}$ contains (infinite) local terms.

³Note that for a given d_f , only a finite number of counterterms is necessary.

the phase space. In fact, when the unparticle propagator mediates an interaction between ordinary particles, those local terms will generate effective higher dimension operators. Note also that the coefficient of the local term is independent of the IR physics and it is completely determined in terms of the AdS₅ geometry.

In the region $c < -1/2$, the propagator can be expanded as

$$\Delta_R(p) \sim \frac{\bar{\sigma} \cdot p}{p^2} \frac{\Sigma_{c-1/2}(p\epsilon)}{\Sigma_{c+1/2}(p\epsilon)} + \mathcal{O}(\epsilon^{-1-2c}). \quad (2.22)$$

In the conformal limit, $\epsilon \rightarrow 0$, we recover the propagator of a massless particle: in this region, therefore, the theory describes a free massless fermion [11]. Note that this interpretation differs from the one in Ref. [10] where the authors are considering a theory with a fixed UV cutoff rather than taking the limit $\epsilon \rightarrow 0$ to remove the cutoff and recover pure CFT behavior.

We can repeat the same analysis for a left-handed CFT operator by changing the UV BCs: this corresponds to a different CFT. The physics is again different from Ref. [10] where the authors interpreted the two BCs to correspond to equivalent CFT theories up to UV localized degrees of freedom: when the UV brane is removed, this interpretation is not possible. The relevant BC now is

$$\psi(p, \epsilon) = \psi_0 = f_0(p) \psi_4, \quad (2.23)$$

where ψ_0 is the right-handed source field. In this case, the CFT propagator is

$$\langle \Theta_L \Theta_L \rangle = \Delta_L(p, c) \sim \left(\frac{R}{\epsilon} \right)^4 \frac{c_\alpha J_{c+\frac{1}{2}}(p\epsilon) + s_\alpha J_{-c-\frac{1}{2}}(p\epsilon)}{c_\alpha J_{c-\frac{1}{2}}(p\epsilon) - s_\alpha J_{-c+\frac{1}{2}}(p\epsilon)} \frac{\sigma_\mu p^\mu}{p}. \quad (2.24)$$

For $c < 1/2$ we find, after rescaling ψ_0 :

$$\Delta_{L,(local)} = \epsilon^{1+2c} (\sigma \cdot p) \frac{\Sigma_{c+1/2}(p\epsilon)}{\Sigma_{c-1/2}(p\epsilon)}, \quad (2.25)$$

$$\Delta_{L,(non-local)} = \frac{s_\alpha \bar{\sigma} \cdot p}{c_\alpha p^{1+2c}} \left[\left(\frac{\Sigma_{-c-1/2}(p\epsilon)}{\Sigma_{c-1/2}(p\epsilon)} + (p\epsilon)^2 \frac{\Sigma_{c+1/2}(p\epsilon) \Sigma_{-c+1/2}(p\epsilon)}{\Sigma_{c-1/2}^2(p\epsilon)} \right) + \mathcal{O}(\epsilon^{1-2c}) \right]. \quad (2.26)$$

Therefore, the dimension of the left-handed operator is

$$d_{fL} = 2 - c, \quad \text{for } c < 1/2, \quad (2.27)$$

and the local terms are relevant when $c < -1/2$ (and $d_{fL} > 5/2$). Again, for $c > 1/2$ we get a free massless fermion. Note finally the relation:

$$\Delta_R(p, -c) = -\Delta_L(p, c). \quad (2.28)$$

3 Holographic Boundary Actions and Unparticle Actions

In the previous section we showed how to relate boundary actions to unparticle propagators. Boundary actions can also be used as effective unparticle actions in some cases. Consider for instance a real bulk scalar in AdS₅:

$$S_{bulk} = \frac{1}{2} \int d^4x dz \left(\frac{R}{z} \right)^3 \left(\partial_M \phi \partial^M \phi + \frac{m^2 R^2}{z^2} \phi^2 \right). \quad (3.1)$$

We know from the AdS/CFT correspondence that this field corresponds to a CFT operator \mathcal{O} with scaling dimension

$$d_s = 2 \pm \nu = 2 \pm \sqrt{4 + m^2 R^2}. \quad (3.2)$$

For $z \sim \epsilon$ the bulk solution of the equation of motion takes the form:

$$\phi(p, z) \approx \phi_0(p)(pz)^{2+\nu} + \beta(p)(pz)^{2-\nu}. \quad (3.3)$$

In the AdS/CFT correspondence $\phi_0(p)$ is associated with the source of the operator \mathcal{O} while $\beta(p)$ is associated with the VEV of \mathcal{O} . Plugging the solution into the bulk action, and integrating by parts one finds a holographic boundary action which depends on ϕ_0 :

$$S_{holo} = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \phi_0(p) p^{2\nu} \phi_0(p). \quad (3.4)$$

For $d_s > 2$ the two-point function of the corresponding operator \mathcal{O} is

$$\langle \mathcal{O}(p') \mathcal{O}(p) \rangle \propto \frac{\delta^2 S_{holo}}{\delta \phi_0(p') \delta \phi_0(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4} (p^2)^{d_s-2}. \quad (3.5)$$

For $d_s < 2$ one must perform a Legendre transformation [14] which interchanges the source and the field:

$$S'_{holo}[\beta] = S_{holo}[\phi_0] + \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \phi(p) \beta(p) \quad (3.6)$$

and the two-point function of the corresponding operator \mathcal{O} is

$$\langle \mathcal{O}(p') \mathcal{O}(p) \rangle \propto \frac{\delta^2 S'_{holo}}{\delta \beta(p') \delta \beta(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4} (p^2)^{d_s-2}. \quad (3.7)$$

In both cases we find the correct scaling for the two-point function of an operator of dimension d_s in a 4D CFT. The Legendre transformation interchanges the roles of ϕ_0 and β , β is now the source and ϕ_0 corresponds to the VEV. Thus for $d_s < 2$ we have that the propagator of \mathcal{O} and the propagator $\Delta_s(p)$ of the boundary field ϕ_0 are proportional:

$$\langle \mathcal{O}(p') \mathcal{O}(p) \rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4} (p^2)^{d_s-2} = \Delta_s(p), \quad (3.8)$$

whereas they are inversely related for $d_s > 2$.

As with fermions, there is a direct correspondence between the AdS/CFT description given above and the unparticle/CFT description of Georgi [1]. For a scaling dimension d_s where $1 \leq d_s < 2$ we have

$$\begin{aligned} \Delta_s(p, d_s) &\equiv \int d^4x e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle \\ &= \frac{A_{d_s}}{2\pi} \int_0^\infty (M^2)^{d_s-2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2 \\ &= \frac{A_{d_s}}{2 \sin d_s \pi} \frac{i}{(-p^2 - i\epsilon)^{2-d_s}} . \end{aligned} \quad (3.9)$$

Thus, up to a normalization, we can identify the the holographic boundary action (3.4) with an effective action for a scalar unparticle for $d_s < 2$. There is a similar effective action for fermions with scaling dimensions $d_f < 5/2$. Consider a left-handed CFT Weyl fermion operator Θ with dimension d_f which couples to a right-handed fermion source, $\bar{\psi}_0$. In the AdS/CFT correspondence we consider a bulk Dirac fermion made out of two Weyl fermions: χ which is left-handed and $\bar{\psi}$ which is right-handed with a bulk mass c . As described in section 2, after using the equations of motion and imposing the boundary condition

$$\chi(p, \epsilon) = 0 \quad \text{and} \quad \psi(p, \epsilon) = \psi_0(p) , \quad (3.10)$$

we find a boundary holographic action

$$S_{holo,f} = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \psi_0 [i\sigma^\mu p_\mu (p^2)^{d_f-5/2}] \bar{\psi}_0 . \quad (3.11)$$

This of course implies that ψ_0 has dimension $4 - d_f$. The two-point function of the CFT fermion is given by

$$\langle \Theta(p')\Theta(p) \rangle \propto \frac{\delta^2 S_{holo,f}}{\delta\psi_0(p') \delta\bar{\psi}_0(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4} i\sigma^\mu p_\mu (p^2)^{d_f-5/2} . \quad (3.12)$$

Now we can rewrite the boundary holographic action in terms of a the left-handed fermion χ_0 with dimension d_f by performing a Legendre transformation in order to interchange the source and the field:

$$S'_{holo,f} = S_{holo,f} + \int \frac{d^4p}{(2\pi)^4} [\chi_0 \psi_0 + \bar{\chi}_0 \bar{\psi}_0] . \quad (3.13)$$

Solving for ψ_0 we find

$$\sigma^\mu p_\mu (p^2)^{d_f-5/2} \bar{\psi}_0 + \chi_0 = 0 , \quad (3.14)$$

$$\psi_0 \sigma^\mu p_\mu (p^2)^{d_f-5/2} + \bar{\chi}_0 = 0 . \quad (3.15)$$

This gives the action in terms of a left-handed fermion

$$S'_{holo,f} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \bar{\chi}_0 [i\bar{\sigma}^\mu p_\mu (p^2)^{3/2-d_f}] \chi_0 . \quad (3.16)$$

As in the scalar case, the two point function of the left-handed CFT fermion is proportional to the propagator, $\Delta_f(p)$, of the field χ_0

$$\langle \Theta(p') \Theta(p) \rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4} \frac{i}{\bar{\sigma}^\mu p_\mu (p^2)^{3/2-d_f}} = \Delta_f(p) . \quad (3.17)$$

So we see that the boundary holographic action $S'_{holo,f}$ approaches the free fermion action as $d_f \rightarrow 3/2$ and can be used as an effective action for the left-handed CFT fermion when $d_f < 5/2$.

4 The IR Cutoff: Soft Breaking of Conformal Invariance

In order to make unparticles with Standard Model (SM) gauge interactions phenomenologically realistic, it is important to break the conformal symmetry so that the unparticle does not propagate at low invariant mass squared. Another reason is that the SM has a natural source of conformal symmetry breaking in the Higgs VEV [8, 19, 20]. Some phenomenological models of conformal breaking that do preserve the continuum at high energies have already been proposed in the literature for scalar unparticles [4, 8]: the idea is to modify the propagator so that the continuum spectrum is cut off below an IR scale μ

$$\begin{aligned} \Delta_s(p, \mu, d_s) &= i \frac{A_{d_s}}{2\pi} \int_{\mu^2}^{\infty} (M^2 - \mu^2)^{d_s-2} \frac{1}{p^2 - M^2 + i\epsilon} dM^2 \\ &= i \frac{A_{d_s}}{2 \sin d_s \pi} (\mu^2 - p^2 - i\epsilon)^{d_s-2} + \dots \end{aligned} \quad (4.1)$$

This model can be easily extended to fermion unparticles: the modified propagator is

$$\begin{aligned} \Delta_f(p, \mu, d_f) &= \frac{A_{d_f-1/2}}{2\pi i} \int_{\mu^2}^{\infty} (M^2 - \mu^2)^{d_f-5/2} \frac{\not{p} + \mu}{p^2 - M^2 + i\epsilon} dM^2 \\ &= \frac{A_{d_f-1/2}}{2i \cos d_f \pi} (\not{p} + \mu) (\mu^2 - p^2 - i\epsilon)^{d_f-5/2} + \dots , \end{aligned} \quad (4.2)$$

where $\not{p} = \gamma^\alpha p_\alpha$. The fermion propagator has the form found for a massive fermion after resumming collinear emissions of a massless gauge boson [12].

In AdS₅, the simplest way to introduce an IR cutoff is to cut off the space at large z with an IR brane [21]. However, this cutoff would turn the CFT into a confining theory and the spectrum would be a tower of massive states whose spacing in mass is given by the IR cutoff itself. Can we implement a simple IR cutoff in AdS that preserves the continuum above

the scale μ ? This would be easily achieved if the propagator were a function of $\sqrt{p^2 - \mu^2}$ instead of $\sqrt{p^2}$: this happens in flat space, given a bulk mass μ . In AdS space, in order to generate a shift of the p^2 dependence, we need to add a mass term with a profile along the extra dimension to compensate for the warp factors, for instance generated by the VEV of a bulk scalar. In the case of a scalar unparticle this is simple to do. We can add a bulk interaction with a new field H : $H\phi\phi$. If H has scaling dimension 2 then its profile will scale like z^2 with an appropriate choice of boundary conditions for H . With the background field solution $H = \mu^2 z^2$ the equation of motion for ϕ (with 4-momentum p) is:

$$z^5 \partial_z \left(\frac{1}{z^3} \partial_z \phi \right) - z^2 (p^2 - \mu^2) \phi - m^2 R^2 \phi = 0 . \quad (4.3)$$

With the boundary condition $\phi(p, \epsilon) = \phi_0(p)$ the solution is

$$\phi(p, z) = \phi_0(p) \left(\frac{z}{\epsilon} \right)^2 \frac{a J_\nu(Ez) + b J_{-\nu}(Ez)}{a J_\nu(E\epsilon) + b J_{-\nu}(E\epsilon)} , \quad (4.4)$$

where $E = \sqrt{p^2 - \mu^2}$. It is now clear that if we had chosen a different power for the IR growth of the H VEV we could implement a different type of IR cutoff, however other choices would lead to much more complicated bulk equations of motion, which would not, in general, have such simple solutions. Adding a mass term to a CFT is a soft breaking of conformal symmetry that does not change the UV behavior and this calculation seems to show that in the AdS description this soft breaking corresponds to a spontaneous breaking of the symmetry. With the simple choice we have made the effective mass term scales with the same power of z as the kinetic term. Thus in the absence of a VEV for H , the term $H\phi^2$ scales like the kinetic term for ϕ and preserves the $SO(4, 2)$ isometry that corresponds to the conformal symmetry of the 4D theory.

For unfermions things are more complicated. A mass term for a single bulk fermion is still not enough: we need to have two bulk fields, Ψ_L with bulk mass c_L and Ψ_R with bulk mass c_R , and add a cross term like

$$\delta \mathcal{L}_{bulk} = \int_\epsilon^\infty dz \left(\frac{R}{z} \right)^5 \lambda H (\chi_L \psi_R + \chi_R \psi_L + h.c.) . \quad (4.5)$$

After the scalar H develops a VEV, $\lambda R \langle H \rangle = v(z)$, this term will generate a z -dependent mass term mixing the two bulk fermions. If $v(z) = \mu z$, the bulk wave functions depend on $E = \sqrt{p^2 - \mu^2}$, provided that $c_L = -c_R \equiv c$. The interpretation is clear: the two bulk fields are two CFT operators $\Theta_L(\Psi_L)$ and $\Theta_R(\Psi_R)$ with opposite chirality but the same anomalous dimension $2 - c$ thanks to the relation between c_L and c_R . H , on the other hand, is a scalar field with dimension 1 whose VEV breaks the conformal invariance and generates a mass term $\mu \Theta_L \Theta_R$. After imposing the two boundary conditions

$$\psi_L(\epsilon) = \psi_0 = f_0 \psi_4 , \quad \text{and} \quad \chi_R(\epsilon) = \chi_0 = g_0 \chi_4 , \quad (4.6)$$

we can calculate the boundary action (see Appendix A for details)

$$\begin{aligned} \mathcal{L}_{boundary} = & \left(\frac{R}{\epsilon}\right)^4 \left\{ \frac{c_{\alpha L} J_{c+1/2}(E\epsilon) - s_{\alpha L} J_{-c-1/2}(E\epsilon)}{c_{\alpha L} J_{c-1/2}(E\epsilon) + s_{\alpha L} J_{-c+1/2}(E\epsilon)} \frac{\psi_0 \sigma \cdot p \bar{\psi}_0}{E} \right. \\ & - \frac{s_{\alpha R} J_{c+1/2}(E\epsilon) - c_{\alpha R} J_{-c-1/2}(E\epsilon)}{s_{\alpha R} J_{c-1/2}(E\epsilon) + c_{\alpha R} J_{-c+1/2}(E\epsilon)} \frac{\bar{\chi}_0 \bar{\sigma} \cdot p \chi_0}{E} \\ & \left. + \frac{1}{2} \frac{\mu}{E} M(E\epsilon) (\psi_0 \chi_0 + h.c.) \right\}, \end{aligned} \quad (4.7)$$

where

$$M(E\epsilon) = \frac{c_{\alpha L} J_{c+1/2}(E\epsilon) - s_{\alpha L} J_{-c-1/2}(E\epsilon)}{c_{\alpha L} J_{c-1/2}(E\epsilon) + s_{\alpha L} J_{-c+1/2}(E\epsilon)} - \frac{s_{\alpha R} J_{c+1/2}(E\epsilon) - c_{\alpha R} J_{-c-1/2}(E\epsilon)}{s_{\alpha R} J_{c-1/2}(E\epsilon) + c_{\alpha R} J_{-c+1/2}(E\epsilon)}. \quad (4.8)$$

For $c < 1/2$, we can expand the Bessel functions for small argument, $E\epsilon \ll 1$, as in section 2. The local terms, dominant for $c < -1/2$ ($d_f > 5/2$) are:

$$\mathcal{L}_{local} = \epsilon^{1+2c} \frac{\Sigma_{c+1/2}(E\epsilon)}{\Sigma_{c-1/2}(E\epsilon)} (\psi_0 \sigma \cdot p \bar{\psi}_0 - \bar{\chi}_0 \bar{\sigma} \cdot p \chi_0), \quad (4.9)$$

where we have rescaled ψ_0 and χ_0 by a factor $R^{-2}\epsilon^{2+c}$. Note that in the local terms the mass μ cancels out: the conformal breaking is generated in the IR, and it does not affect the UV physics. The non-local part is proportional to

$$E^{2c-1} \left(\frac{s_{\alpha L}}{c_{\alpha L}} (\psi_0 \sigma \cdot p \bar{\psi}_0) - \frac{c_{\alpha R}}{s_{\alpha R}} (\bar{\chi}_0 \bar{\sigma} \cdot p \chi_0) + \frac{1}{2} \left(\frac{s_{\alpha L}}{c_{\alpha L}} - \frac{c_{\alpha R}}{s_{\alpha R}} \right) \mu (\psi_0 \chi_0 + h.c.) \right), \quad (4.10)$$

and, if we normalize the two kinetic terms properly, i.e. $s_{\alpha L}/c_{\alpha L} = -c_{\alpha R}/s_{\alpha R}$, we can rewrite it as

$$\mathcal{L}_{non-local} \sim E^{2c-1} \bar{\Psi}_0 (\not{p} + \mu) \Psi_0, \quad \text{where} \quad \Psi_0 = \begin{pmatrix} \chi_0 \\ \bar{\psi}_0 \end{pmatrix}. \quad (4.11)$$

The corresponding propagator coincides with Eq. (4.2) with $d_f = 2 - c$.

4.1 Stability of the AdS geometry in the IR

A bulk scalar field with a VEV that grows with z can provide an effective IR cutoff for unparticles in AdS₅, preserving the continuum spectrum, typical of an unparticle, for momenta larger than the IR cutoff. This behavior has to be compared to the traditional Randall-Sundrum approach [21], where a brane is used to cut-off the space at large z : in the latter case a mass gap is introduced and the spectrum reduces to a series of KK resonances spaced by a mass scale $\propto 1/z_{IR}$. As we have seen above, a simple model of an unparticle IR cutoff is achieved with a linearly growing VEV for fermions and a quadratic one for scalars. However, the growing VEV will, at large z , destabilize the AdS geometry and its back-reaction will

eventually cut off the space and possibly reintroduce a mass gap. Our description of the model is therefore safe only if such effects appear at a scale much lower than the IR cutoff μ , i.e. $z_{IR} \ll 1/\mu$.

In order to address this issue, we will cutoff the space at z_{IR} by hand and push this brane to the largest possible z . In 5D the strength of gravitational interactions are given by the inverse of the third power of the 5D Planck scale, M_*^{-3} . The curvature of the AdS₅ bulk is [21]

$$\Lambda = -24 \frac{M_*^3}{R^2} \quad (4.12)$$

and classical solutions for the metric should be a good approximation as long as

$$R \gg \frac{1}{M_*} . \quad (4.13)$$

With a scalar VEV for H growing in the IR like a power, i.e. $\mu z^2/R^3$ and its size controlled by a potential on the IR brane at z_{IR} , there are contributions from brane terms $\sim \mu^2 H^2$ to the stress-energy tensor that grow in the IR like

$$T^{MN} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{MN}} \sim \eta^{MN} \left(\frac{z_{IR}}{R} \right)^2 \frac{\mu^4}{R^6} z_{IR}^5 . \quad (4.14)$$

For stability we need this contribution to be small compared to the contribution from the bulk curvature:

$$g^{MN} \Lambda = \eta^{MN} \left(\frac{z}{R} \right)^2 \Lambda , \quad (4.15)$$

thus we require

$$\begin{aligned} \frac{\mu^4}{R^6} z_{IR}^5 &\ll \frac{24 M_*^3}{R^2} , \\ z_{IR} &\ll \frac{1}{\mu} (24 R^4 \mu M_*^3)^{1/5} . \end{aligned} \quad (4.16)$$

As long as μ is sufficiently small compared to $1/R$ then Eq. (4.16) is consistent with

$$\mu \gg \frac{1}{z_{IR}} . \quad (4.17)$$

In this case the spacing between KK modes introduced by the IR brane is tiny compared to the mass gap μ . For example with $M_* = 10^{19}$ GeV, $R^{-1} = 10^9$ GeV and $\mu = 100$ GeV we have

$$\frac{1}{z_{IR}} \gg 10^{-23} \text{ GeV} , \quad (4.18)$$

so that the minimum spacing between the modes can be 10^{25} times smaller than the threshold μ . This would be a very good approximation to a continuum.

5 Gauge Interactions

A method for adding gauge interactions for unparticle fields was proposed in Ref. [4] (see also Refs. [22] for a discussion on gauging unparticles). In this section we will study how to reproduce the same results in the context of an infinite AdS space: the natural choice would be to gauge a global symmetry by adding a bulk gauge field. It is well known that such a gauge field corresponds to a vector operator of dimension 3 (i.e a conserved current) in the CFT theory. However, due to the conformal invariance, the associated external gauge coupling runs to zero in the IR due to a logarithmic dependence on the momentum in the propagator. Moreover in the 5D description the zero mode gauge boson is non-normalizable as its normalization is proportional to the volume of the space: in order to have a realistic description of ordinary gauge interactions we need a way to effectively cut off the space without introducing an IR brane.

One possibility is to add a dilaton factor $\Phi(z)$ in front of the kinetic term:

$$-\frac{1}{4g_5^2} \int_{\epsilon}^{\infty} d^4x dz \left(\frac{R}{z}\right) \Phi(z) F^{aMN} F_{MN}^a. \quad (5.1)$$

The spectrum still contains a flat zero mode, however the relation between 4D and 5D couplings changes. For a dilaton background that falls sufficiently fast for large z , the 4D gauge coupling can be finite. As a particular example consider

$$\Phi(z) = e^{-mz}. \quad (5.2)$$

In this case we have

$$\frac{1}{g_4^2} = \frac{R}{g_5^2} \int_{\epsilon}^{\infty} \frac{e^{-mz}}{z} = \frac{R}{g_5^2} [\text{Ei}(-m\infty) - \text{Ei}(-m\epsilon)] \approx \frac{R}{g_5^2} [-\gamma_E - \log(m\epsilon)]. \quad (5.3)$$

Remembering that the UV cutoff scale is $\Lambda_{UV} = 1/\epsilon$ we see that the approximate CFT corresponding to this dilaton background contributes to the running of the gauge coupling from the cutoff Λ down to a scale $m e^{\gamma_E}$. This is just what we expect if the CFT unparticles that couple to the external gauge boson have a threshold near $\mu = m e^{\gamma_E}$. In other words the space is effectively cutoff near $z = 1/\mu = e^{-\gamma_E}/m$ and the 4D gauge coupling is

$$\frac{1}{g_4^2} \approx \frac{R}{g_5^2} \log\left(\frac{\Lambda_{UV}}{\mu}\right). \quad (5.4)$$

If we took $\mu \rightarrow 0$ we would find that the 4D gauge coupling ran to zero and the gauge boson zero mode would be non-normalizable.

The equation of motion for the gauge field in this dilaton background is

$$f''(z) - \left(m + \frac{1}{z}\right) f'(z) + p^2 f(z) = 0. \quad (5.5)$$

This equation can be solved in terms of confluent hypergeometric functions (more details on the solution can be found in Appendix B). In the large z region it is easy to see that the

approximate solution is exponential for $0 < p < m/2$ and oscillatory for $p > m/2$. Therefore we expect the two point function of the corresponding CFT operator to have a massless pole and a cut for $p > m/2$ corresponding to the logarithmic running of the gauge coupling for large momenta where the conformal invariance is recovered. In order to see this, we can study the spectrum of the gauge field in presence of an IR brane at $z = z_{IR} \gg 1/m$. The spacing between the zero mode and the first KK mode is controlled by the value of m ; the first KK mode appears near $p = m/2$ while the spacing of the remaining higher KK modes is determined by the infrared cutoff $z = z_{IR}$. The low-lying KK modes should correspond to the spin one bound states of the massive degrees of freedom. The spacing of subsequent modes is, as usual, $\Delta p \sim 1/z_{IR}$. Since the 4D gauge coupling (5.4) is independent of z_{IR} if $z_{IR} \gg 1/m$ it is clear that we can take z_{IR} to be arbitrarily large and recover the continuum above $m/2$.

We can now consider the interactions of the gauge zero mode with the fields in the holographic boundary action. 4D gauge invariance requires that we replace the 4D derivative in the EOMs with

$$\partial_\mu \rightarrow \partial_\mu - ig_4 T^a A_\mu , \quad (5.6)$$

where g_4 is the 4D coupling defined in Eq. (5.3). This corresponds to minimal gauging of the holographic Lagrangians, and allows one to calculate vertices with an arbitrary number of gauge bosons. To calculate the effective vertices, one should solve the bulk EOMs of the matter field including the bulk interactions with the gauge zero mode, which can be considered as a background field because its wave-function is constant in the extra dimension. To find the single gauge boson vertex, however, we can use the solution for the free bulk EOM: since the bulk gauge interaction is already first order term in the gauge field. So we can simply propagate these boundary fields into the bulk and integrate over bulk gauge interaction. This integral will give the exact gauge vertex for the holographic action. More explicitly, in the case of a bulk scalar, we must calculate the integral

$$\int dz \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \left(\frac{R}{z}\right)^3 (\partial_M \phi^\dagger(p+q, z) T^a A^{aM}(q, z) \phi(p, z) + h.c.) . \quad (5.7)$$

This gives the interaction term with the gauge zero mode to be:

$$S_{\text{int}} = \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} A^{a\alpha}(q) \int_\epsilon^\infty dz \left(\frac{R}{z}\right)^3 (\partial_\alpha \phi^\dagger(p+q, z) T^a \phi(p, z) + h.c.) \quad (5.8)$$

which just yields the usual unparticle vertex [4] for the canonically normalized gauge field after we expand for small momenta ($p\epsilon, q\epsilon \ll 1$):

$$\begin{aligned} ig\Gamma^{a\alpha}(p, q) &\equiv \frac{i\delta^3 S_{\text{int}}}{\delta A^{a\mu}(q) \delta \phi_0^\dagger(p+q) \delta \phi_0(p)} \\ &= ig_4 T^a \frac{2 \sin d_s \pi}{A_{d_s}} \frac{(2p+q)^\alpha}{2p \cdot q + q^2} \left[(\mu^2 - (p+q)^2)^{2-d_s} - (\mu^2 - p^2)^{2-d_s} \right] . \end{aligned} \quad (5.9)$$

6 Unparticle Production Cross-sections

In this section we want to discuss the unparticle production cross-section via gauge interactions: this would be especially relevant for the LHC in the case of colored unparticles [4]. A simple way to do the calculation is to gauge the holographic boundary action and calculate the imaginary part of unparticle loop contributions to the $qq \rightarrow qq$ scattering: this would give the inclusive cross-section of $qq \rightarrow$ unparticles (a calculation along similar lines can give the gluon fusion contribution). If we neglect the contribution of the logarithmic running of the gauge boson coupling (the IR cutoff m can be arbitrary large), the result for scalar unparticles of dimension d_s is simply given by [4]

$$\sigma_{d_s} = (2 - d_s)\sigma_1, \quad (6.1)$$

where σ_1 is the ordinary cross-section into a pair of scalar particles (limit $d_s \rightarrow 1$). This result makes sense only for $d_s < 2$: if we naively extended it to larger dimensions we would find a negative cross-section.

This paradox is strictly related to the fact that if we wish to extend the unparticle analysis to $d_s \geq 2$ we find that the spectral density integration in Eq. (3.9) is divergent. In order to get the finite result in Eq. (3.9) with the proper scaling, we need to subtract some terms from the integral. For instance, for $2 < d_s < 3$ we must subtract a constant term:

$$\begin{aligned} \Delta(p, 2 < d_s < 3) &= \frac{A_{d_s}}{2\pi} \int_0^\infty (M^2)^{d_s-2} \left(\frac{i}{p^2 - M^2 + i\epsilon} - \frac{i}{-M^2} \right) dM^2 \\ &= \frac{A_{d_s}}{2\pi} (p^2 + i\epsilon) \int_0^\infty (M^2)^{d_s-3} \frac{i}{p^2 - M^2 + i\epsilon} dM^2 \\ &= \frac{A_{d_s}}{2 \sin(d_s - 1)\pi} (p^2 + i\epsilon) \frac{i}{(-p^2 - i\epsilon)^{2-(d_s-1)}} \\ &= \frac{A_{d_s}}{2 \sin d_s \pi} \frac{i}{(-p^2 - i\epsilon)^{2-d_s}}. \end{aligned} \quad (6.2)$$

In general, for larger dimensions we need to subtract a series of terms with increasing powers of p^2 :

$$\frac{A_{d_s}}{2\pi} \int_0^\infty (M^2)^{d_s-2} \left(\frac{i}{p^2 - M^2 + i\epsilon} - \frac{i}{-M^2} - \frac{-ip^2}{(-M^2)^2} - \dots \right) dM^2. \quad (6.3)$$

Thus, in order to get the unparticle propagator with the proper scaling for $d_s > 2$, we need to subtract a local (divergent) piece from the integral in Eq. (3.9):

$$\begin{aligned} \Delta_{\text{local}}(p) &= \frac{A_{d_s}}{2\pi} \int_0^{\Lambda^2} (M^2)^{d_s-2} \left(\frac{i}{-M^2} + \frac{-ip^2}{(-M^2)^2} + \dots \right) dM^2 \\ &= \frac{iA_{d_s}}{2\pi} \Lambda^{2d_s-4} \left(\frac{1}{d_s - 2} - \frac{p^2 \Lambda^{-2}}{d_s - 3} + \dots \right), \end{aligned} \quad (6.4)$$

where we have imposed a UV cutoff on the momentum integration.

This corresponds directly with what happens in the 5D picture: the scalar 2 point function can be split into a local and non-local term

$$\Delta_s(p) = \Delta_{\text{local}}(p) + \Delta_{\text{non-local}}(p), \quad (6.5)$$

in analogy with Eq. (2.18) for fermions. For $d_s > 2$ the local terms are divergent as the UV brane is removed, $\epsilon \sim 1/\Lambda \rightarrow 0$. Thus for $1 < d_s < 2$ we can write an effective action for scalar unparticles, which is insensitive to cutoff effects, using the propagator (3.9). The fact that conformal symmetry is broken in the UV is unimportant, and all the local terms associated with such breaking can be removed by sending the UV cutoff to infinity. For $d_s > 2$ however, it is not possible to consider the conformal sector alone, without including the local terms in the propagator, which are sensitive to cutoff behavior. In this case the theory consists of a continuum of states (unparticles), peaked around a mass scale $1/\epsilon$, plus some local contact terms induced by the local part of the propagator in Eq.(6.5). If one removes the UV cutoff, i.e. $\epsilon \rightarrow 0$, the ($d_s > 2$) unparticles decouple: this explains why for $d_s \rightarrow 2$ the cross-section goes to zero (or to a small number suppressed by the UV cutoff). For $d_s > 2$, we expect the cross-section to remain small and suppressed by powers of ϵ . In order to see this, we need to consider the full propagator in Eq. (6.5) (while in Ref. [4] the non-local term only was considered). The interaction vertex with one gauge boson would be

$$\begin{aligned} \Gamma^\mu(p, q) \propto & \left(\Delta_s^{-1}(p+q) - \Delta_s^{-1}(p) \right) \simeq \\ & \left(\Delta_{\text{local}}^{-1}(p+q) - \Delta_{\text{local}}^{-1}(p) \right) - \left(\frac{\Delta_{\text{non-local}}(p+q)}{\Delta_{\text{local}}^2(p+q)} - \frac{\Delta_{\text{non-local}}(p)}{\Delta_{\text{local}}^2(p)} \right) + \dots, \end{aligned} \quad (6.6)$$

where in the expansion we have used the fact the the local terms are dominant. If we want to calculate the cross-section for the process $q\bar{q} \rightarrow$ unparticles we need to calculate the imaginary part of the gauge boson propagator dressed with a loop of unparticles. Such imaginary parts can come from taking either local or non-local terms in the propagator and the vertex: however, at leading order when taking the local terms only, there is no imaginary part because the propagators do not have poles and the vertices are real. Thus, the dominant term will come from taking the local part of the propagator and the non local part of the vertex or vice-versa. The dominant contribution to the imaginary part comes from terms like

$$\left(\frac{\Delta_{\text{non-local}}}{\Delta_{\text{local}}} \right)^2 \sim (\epsilon^2)^{2d_s-4}. \quad (6.7)$$

This is telling us that, for d_s close to 2 and $d_s > 2$, Eq. (6.1) should be replaced by a more complicated function of the cutoff, which however is very suppressed if the cutoff is much larger than the energy of the experiment. Thus, in first approximation, it is correct to take Eq. (6.1) for $d_s < 2$ and zero for $d_s > 2$. Trying to extrapolate Eq. (6.1) to $d_s > 2$ is, as we have shown, meaningless⁴.

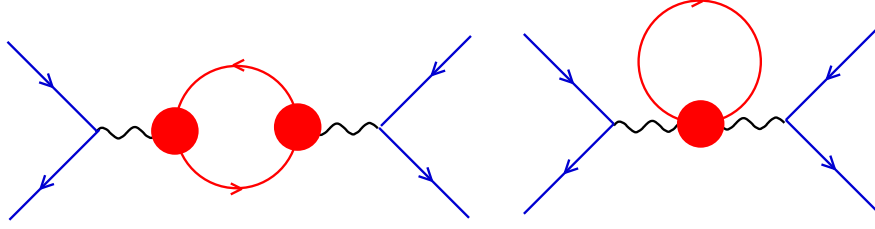


Figure 1: Forward scattering of $q\bar{q}$ through a gluon with a fermion unparticle vacuum polarization loop. The imaginary parts of those diagrams contribute to the inclusive unfermion production cross-section.

6.1 Fermion unparticle cross-section

The calculation of the inclusive $q\bar{q} \rightarrow$ fermion unparticle cross-section is very similar to the scalar case in [4], the only difference being that the vertices are more complicated. The same two types of diagrams in Fig. 1 contribute due to the fact that the non-local action contains couplings with an arbitrary number of gauge bosons.

The fermion effective action can be written as (see Eq. (3.16)):

$$S = \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \int \frac{d^4 p}{(2\pi)^4} \bar{\psi}(\not{p} - \mu) [\mu^2 - p^2]^{3/2-d_f} \psi . \quad (6.8)$$

After gauging such non-local action, we find the following vertex with one gauge boson

$$\begin{aligned} ig \Gamma_f^{a\nu}(p, q) &= ig T^a \gamma^\nu \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \frac{1}{2} \left[(\mu^2 - (p+q)^2)^{3/2-d_f} + (\mu^2 - p^2)^{3/2-d_f} \right] + \\ &ig T^a \left(\not{p} + \frac{\not{q}}{2} - \mu \right) (2p^\nu + q^\nu) \mathcal{F}_f(p, q) , \end{aligned} \quad (6.9)$$

where

$$\mathcal{F}_f(p, q) = \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \frac{(\mu^2 - (p+q)^2)^{3/2-d_f} - (\mu^2 - p^2)^{3/2-d_f}}{2p \cdot q + q^2} . \quad (6.10)$$

The vertex with two gauge bosons is more complicated, and we relegated it to the Appendix C, where a more general discussion of unparticle gauge vertices can also be found.

It is easy to check that such vertices do respect the (generalized) Ward-Takahashi identities [24] and they reduce to the usual vertices in the $d_f \rightarrow 3/2$ limit (in particular the two gauge boson vertex vanishes - see Appendix C).

In the massless limit, $\mu \rightarrow 0$, the imaginary part of the two diagrams give the following contributions to the cross-section:

$$\left. \frac{\sigma_{d_f}}{\sigma_F} \right|_{\text{diag1}} = \frac{1}{3} d_f^4 - \frac{8}{3} d_f^3 + \frac{31}{6} d_f^2 - \frac{5}{6} d_f - \frac{33}{16} , \quad (6.11)$$

$$\left. \frac{\sigma_{d_f}}{\sigma_F} \right|_{\text{diag2}} = -\frac{1}{3} d_f^4 + \frac{8}{3} d_f^3 - \frac{20}{3} d_f^2 + \frac{19}{3} d_f - \frac{29}{16} , \quad (6.12)$$

⁴This contradicts what is advocated in Ref. [23].

where we have normalized the results with the cross-section into two massless fermions, σ_F . In the limit $d_f \rightarrow 3/2$, the first diagram will reproduce the usual result, while the second one vanishes. Summing the two contributions we obtain

$$\sigma_{d_f}(q\bar{q} \rightarrow \text{unfermions}) = \left(-\frac{3}{2}d_f^2 + \frac{11}{2}d_f - \frac{31}{8} \right) \times \sigma_F(q\bar{q} \rightarrow f\bar{f}). \quad (6.13)$$

As in the scalar case, a non-trivial cancellation between the two diagrams removes higher powers in d_f . However, unlike in the scalar case, the cross-section does not vanish when the dimension reaches the critical value $d_f \rightarrow 5/2$, but it is actually reduced to a half of the particle one. In the scalar case the cross-section vanishes for $d_s = 2$ because the scalar unparticle becomes non-dynamical and all the couplings with gauge bosons vanish individually: this is not the case for fermions due to the non trivial spinorial structure of the couplings and of the kinetic term. However, when d_f approaches the value $5/2$, the propagator becomes more and more UV sensitive and the action in Eq. (6.8) cannot be used anymore: the local terms in the propagator suppress the cross-section more than Eq. (6.13), as we explained in the scalar case.

7 Conclusions

Unparticles were originally proposed as a candidate for new physics at the LHC that couples to the SM only through irrelevant operators. The possibility that unparticles carry SM gauge quantum numbers is particularly interesting, as it can enhance the production cross-section via strong or weak interactions (compared to higher dimension operators) and give rise to new interesting signals in the ATLAS and CMS detectors (compared to just missing energy). However, the early work on unparticle physics left many theoretical issues that had not been sufficiently clarified in the literature. For large scaling dimensions ($d_s > 2$ for scalars and $d_f > 5/2$ for fermions) the unparticle propagator is UV sensitive. Even though the resulting phase space is regular, other calculations that make use of such a propagator can give nonsensical results unless care is taken.

Comparison between the AdS description of a CFT and the unparticle description allows for a simple way to deal with complicated issues like gauge interactions for unparticles. In this paper we showed that AdS theories without an IR cutoff are equivalent to unparticles, in the sense that CFT operators corresponding to bulk fields have the same scaling properties and propagators as unparticles. It could not have been otherwise. Moreover, we showed that holographic boundary actions can be used as effective actions for unparticles, and that they reproduce the proposal in [4] for scalars with $1 < d_s < 2$ (and, in the case of fermions, $3/2 < d_f < 5/2$). For larger scaling dimensions, local terms in the boundary action appear that are enhanced by powers of the UV cutoff Λ (which is inversely proportional to the position ϵ of the UV brane in the extra dimension). Such terms correspond to the UV sensitivity of unparticle propagators. These terms do not contribute to the final state phase space, however they cannot be neglected in the propagator and they give rise to effective contact interactions. We also showed that, taking them into account, the calculation of

the production cross-section via gauge interactions can be extended to $d_s > 2$ and that the cross-section is cutoff suppressed in this regime. We also presented the result for fermion unparticle production, which shows a very similar behavior.

Unparticles can also shed light on AdS theories. We used unparticles as a guide to develop new results in AdS field theories. There is a very simple way to introduce an IR cutoff for unparticles which preserves continuum behavior above a threshold. We showed how to reproduce this behavior in AdS with a bulk VEV which grows in the IR. This provides an alternative to the usual Randall-Sundrum constructions with an IR brane producing a hard IR cutoff. The new IR cutoff introduces a mass gap, but without the discrete KK tower which is associated with confining theories. We studied two simple cases of a bulk VEV with a profile growing towards the IR, one with a quadratic profile which produces a threshold for a scalar field and a linear profile for a fermion threshold. The back-reaction of such a growing VEV will eventually modify the geometry and effectively cut off the space, however this can happen at much lower scales so that the continuum approximation is still valid.

Finally we addressed the issue of gauge interactions for unparticles. In the AdS description without an IR cutoff, there is no normalizable zero mode, corresponding to the fact that the gauge coupling for a gauge boson which gauges a global symmetry of a large N CFT runs to zero in the IR. This effect can be thought of as the result of a mixing with CFT states with the same quantum numbers as the gauge boson and with arbitrarily small masses. However, if we consider an AdS theory with a dilaton factor multiplying the bulk gauge action then if the dilaton profile falls sufficiently fast in the IR the running is effectively cut off at some threshold scale and only the ordinary massless gauge boson is left at low energies. Moreover we showed that the couplings of the massless gauge boson to the unparticles (boundary values of bulk fields) will respect the minimal coupling prescription used in [4]: we therefore calculated the inclusive fermion unparticle production cross-section (relevant for the LHC) which shows similar properties as the scalar one.

Note Added: While finishing the write-up of this paper we noticed the paper in Ref. [25] which attempts to consider gauge interactions of unfermions, however the propagators used in that work are not consistent with those found in the AdS/CFT correspondence. Moreover, the effective action the authors use is different and, in particular, it does not have a d -dependent power in the momentum.

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A Appendix: IR cutoff for fermion fields in AdS

The bulk action for the two bulk fermions is:

$$\begin{aligned}
S_{bulk} = \int d^4x dz \left(\frac{R}{z}\right)^4 \left\{ -i\bar{\chi}_{L/R}\bar{\sigma}^\mu\partial_\mu\chi_{L/R} - i\psi_{L/R}\sigma^\mu\partial_\mu\bar{\psi}_{L/R} \right. \\
+ \frac{1}{2}(\psi_{L/R}\overleftrightarrow{\partial}_z\chi_{L/R} - \bar{\chi}_{L/R}\overleftrightarrow{\partial}_z\bar{\psi}_{L/R}) + \frac{c_{L/R}}{z}(\psi_{L/R}\chi_{L/R} + \bar{\chi}_{L/R}\bar{\psi}_{L/R}) \\
\left. + \frac{v(z)}{z}(\psi_L\chi_R + \psi_R\chi_L + \bar{\chi}_L\bar{\psi}_R + \bar{\chi}_R\bar{\psi}_L) \right\}. \tag{A.1}
\end{aligned}$$

The EOMs can be solved by

$$\chi_{L/R} = g_{L/R}(z)\chi_4 \quad \text{and} \quad \bar{\psi}_{L/R} = f_{L/R}(z)\bar{\psi}_4, \tag{A.2}$$

where the wave functions $g_{L/R}$ and $f_{L/R}$ are the solutions of

$$p \begin{pmatrix} g_L \\ g_R \end{pmatrix} + \left[\partial_z - \frac{1}{z} \begin{pmatrix} c_L + 2 & v \\ v & c_R + 2 \end{pmatrix} \right] \begin{pmatrix} f_L \\ f_R \end{pmatrix} = 0, \tag{A.3}$$

$$p \begin{pmatrix} f_L \\ f_R \end{pmatrix} - \left[\partial_z + \frac{1}{z} \begin{pmatrix} c_L - 2 & v \\ v & c_R - 2 \end{pmatrix} \right] \begin{pmatrix} g_L \\ g_R \end{pmatrix} = 0. \tag{A.4}$$

We can now determine the f 's from Eq. (A.4) and plug it back in Eq. (A.3) to find second order equations for the g 's:

$$\left[\partial_z^2 - \frac{4}{z}\partial_z + p^2 - \frac{1}{z^2} \begin{pmatrix} v^2 + c_L^2 - c_L - 6 & v(c_L + c_R) + v - v'z \\ v(c_L + c_R) + v - v'z & v^2 + c_R^2 - c_R - 6 \end{pmatrix} \right] \begin{pmatrix} g_L \\ g_R \end{pmatrix} = 0. \tag{A.5}$$

If $c_L = -c_R \equiv c$ and if the VEV is linear in z , $v(z) = \mu z$, then the two equations decouple:

$$\left[\partial_z^2 - \frac{4}{z}\partial_z + p^2 - \mu^2 - \frac{c^2 \mp c - 6}{z^2} \right] g_{L/R} = 0. \tag{A.6}$$

A similar equation can be derived for the f 's:

$$\left[\partial_z^2 - \frac{4}{z}\partial_z + p^2 - \mu^2 - \frac{c^2 \pm c - 6}{z^2} \right] f_{L/R} = 0. \tag{A.7}$$

Defining $E = \sqrt{p^2 - \mu^2}$, the bulk solutions are:

$$g_{L,R} = z^{5/2} (a_{L/R} J_{\pm c+1/2}(Ez) + b_{L/R} J_{\mp c-1/2}(Ez)), \tag{A.8}$$

$$f_{L,R} = z^{5/2} (a'_{L/R} J_{\pm c-1/2}(Ez) + b'_{L/R} J_{\mp c+1/2}(Ez)). \tag{A.9}$$

Imposing the first order Eqs. (A.3) (or equivalently Eqs. (A.4)), we can determine half of the parameters in the solutions:

$$\begin{cases} pb_L + Eb'_L - \mu a'_R = 0 \\ pa_L - Ea'_L - \mu b'_R = 0 \end{cases} \Rightarrow \begin{cases} a_L = \frac{pa'_L - \mu b_R}{E} \\ b_L = -\frac{pb'_L - \mu a_R}{E} \end{cases} \quad (\text{A.10})$$

$$\begin{cases} pb_R + Eb'_L - \mu a'_L = 0 \\ pa_R - Ea'_R - \mu b'_L = 0 \end{cases} \Rightarrow \begin{cases} a'_R = \frac{pa_R - \mu b'_L}{E} \\ b'_R = -\frac{pb_R - \mu a'_L}{E} \end{cases} \quad (\text{A.11})$$

The BCs on the UV brane

$$\psi_L(\epsilon) = \psi_0 = f_0 \psi_4, \quad \text{and} \quad \chi_R(\epsilon) = \chi_0 = g_0 \chi_4,$$

will determine the overall normalizations

$$\begin{pmatrix} a_R \\ b_R \end{pmatrix} = \frac{\epsilon^{-5/2} g_0}{s_{\alpha R} J_{c-1/2}(E\epsilon) + c_{\alpha R} J_{-c+1/2}(E\epsilon)} \begin{pmatrix} c_{\alpha R} \\ s_{\alpha R} \end{pmatrix} \quad (\text{A.12})$$

$$\begin{pmatrix} a'_L \\ b'_L \end{pmatrix} = \frac{\epsilon^{-5/2} f_0}{c_{\alpha L} J_{c-1/2}(E\epsilon) + s_{\alpha L} J_{-c+1/2}(E\epsilon)} \begin{pmatrix} s_{\alpha L} \\ c_{\alpha L} \end{pmatrix} \quad (\text{A.13})$$

with $c_{\alpha L}^2 + s_{\alpha L}^2 = 1 = c_{\alpha R}^2 + s_{\alpha R}^2$.

After imposing the EOMs in the bulk, we are left with a boundary action

$$S_{\text{boundary}} = \int d^4x \left(\frac{R}{\epsilon} \right)^4 \frac{1}{2} (g_L(\epsilon) f_L(\epsilon) + g_R(\epsilon) f_R(\epsilon)) [\psi_4 \chi_4 + \bar{\chi}_4 \bar{\psi}_4]. \quad (\text{A.14})$$

Plugging in the solutions, we obtain Eq. (4.7).

B Appendix: IR threshold in the gauge sector

In presence of the dilaton factor in Eq. (5.1), the EOM for the gauge boson becomes

$$f''(z) - \left(\frac{1}{z} - \frac{\Phi'}{\Phi} \right) f'(z) + p^2 f(z) = 0. \quad (\text{B.1})$$

If the dilaton decreases with a power of z , the solutions will still be Bessel functions and generate a continuum all the way down to zero momentum. Therefore we need a faster decrease, an exponential, in order to get a completely different spectrum. If $\Phi = e^{-mz}$, the EOM is

$$f''(z) - \left(\frac{1}{z} + m \right) f'(z) + p^2 f(z) = 0; \quad (\text{B.2})$$

for $z \ll 1/m$ the solutions are exponential functions, while for $z \gg 1/m$ we obtain the usual combination of Bessel functions. The equation can be solved analytically: first we can rescale the solution

$$f(z) = z^2 e^{\frac{z}{2}(m - \sqrt{m^2 - 4p^2})} g(z), \quad (\text{B.3})$$

such that g satisfies the following equation:

$$\frac{z}{\sqrt{m^2 - 4p^2}} g''(z) + \left(\frac{3}{\sqrt{m^2 - 4p^2}} - z \right) g'(z) - \left(\frac{3}{2} + \frac{m}{2\sqrt{m^2 - 4p^2}} \right) g(z) = 0 . \quad (\text{B.4})$$

After a change of variables $y = \sqrt{m^2 - 4p^2} z$, we recognize this as a confluent hypergeometric (or Kummer) differential equation:

$$y g'' + (3 - y) g' - \alpha g = 0 , \quad \text{where} \quad \alpha = \frac{3}{2} + \frac{m}{2\sqrt{m^2 - 4p^2}} , \quad (\text{B.5})$$

whose solution is a combination of confluent hypergeometric functions.

The form of the spectrum can be inferred by the exponential prefactor in Eq. (B.3): for $p < m/2$ it is a real exponential, therefore only the zero mode is present at low energies; for $p > m/2$, the square root develops an imaginary part and the solution becomes oscillatory, therefore producing, above $m/2$, a tower of KK states with spacing given by the IR brane.

C Appendix: summary of Feynman rules for unparticle gauge interactions

In this Appendix we will summarize the Feynman rules for unparticle gauge interactions. We will start with the scalar case, already discussed in [4]. The effective action for $1 \leq d_s < 2$ is

$$S_{\text{scalar}} = \frac{2 \sin d_s \pi}{A_{d_s}} \int \frac{d^4 p}{(2\pi)^4} \phi^\dagger(-p) [\mu^2 - p^2]^{2-d_s} \phi(p) . \quad (\text{C.1})$$

Minimal gauging applied to this non-local action [5, 7] gives the following vertex with one gauge boson:

$$i g \Gamma_s^{a\nu}(p, q) = i g T^a (2p^\nu + q^\nu) \mathcal{F}_s(p, q) , \quad (\text{C.2})$$

where the dependence on the unparticle propagator is embedded in the function \mathcal{F} , defined for scalars as:

$$\mathcal{F}_s(p, q) = \frac{2 \sin d_s \pi}{A_{d_s}} \frac{(\mu^2 - (p+q)^2)^{2-d_s} - (\mu^2 - p^2)^{2-d_s}}{2p \cdot q + q^2} . \quad (\text{C.3})$$

The vertex with two gauge bosons is $i g^2 \Gamma_s^{a\mu, b\nu}(p, q_1, q_2)$, with

$$\begin{aligned} \Gamma_s^{a\mu, b\nu} &= (T^a T^b + T^b T^a) g^{\mu\nu} \mathcal{F}_f(p, q_1 + q_2) + \\ & T^a T^b \frac{(2p^\nu + q_2^\nu)(2p^\mu + q_1^\mu + 2q_2^\mu)}{q_1^2 + 2p \cdot q_1 + 2q_1 \cdot q_2} (\mathcal{F}_f(p, q_1 + q_2) - \mathcal{F}_f(p, q_2)) + \\ & T^b T^a \frac{(2p^\mu + q_1^\mu)(2p^\nu + q_2^\nu + 2q_1^\nu)}{q_2^2 + 2p \cdot q_2 + 2q_1 \cdot q_2} (\mathcal{F}_f(p, q_1 + q_2) - \mathcal{F}_f(p, q_1)) . \end{aligned} \quad (\text{C.4})$$

In the $d_s \rightarrow 1$ limit, $\mathcal{F}_s \rightarrow 1$, and we recover the usual scalar vertices

$$ig \Gamma_{s(d_s=1)}^{a\mu} = ig T^a (2p^\mu + q^\mu) , \quad \text{and} \quad ig^2 \Gamma_{s(d_s=1)}^{a\mu,b\nu} = ig^2 (T^a T^b + T^b T^a) g^{\mu\nu} . \quad (\text{C.5})$$

In the fermion case, the effective action for $3/2 \leq d_f < 5/2$ is

$$S_{\text{fermion}} = \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \int \frac{d^4 p}{(2\pi)^4} \bar{\psi}(-p) (\not{p} - m) [m^2 - p^2]^{3/2-d_f} \psi(p) . \quad (\text{C.6})$$

The gauge vertices are more complicated, however they can be written in terms of the scalar ones. The correct dependence on the fermion dimension is recovered replacing \mathcal{F} with the following function

$$\mathcal{F}_f(p, q) = \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \frac{(\mu^2 - (p+q)^2)^{3/2-d_f} - (\mu^2 - p^2)^{3/2-d_f}}{2p \cdot q + q^2} . \quad (\text{C.7})$$

For the one gauge boson vertex the result is

$$\begin{aligned} ig \Gamma_f^{a\nu}(p, q) &= ig T^a \gamma^\nu \frac{2 \cos d_f \pi}{A_{d_f-1/2}} \frac{1}{2} \left[(\mu^2 - (p+q)^2)^{3/2-d_f} + (\mu^2 - p^2)^{3/2-d_f} \right] \\ &\quad + \left(\not{p} + \frac{\not{q}}{2} - \mu \right) ig \Gamma_s^{a\nu}(p, q) . \end{aligned} \quad (\text{C.8})$$

The two gauge boson vertex can also be written in terms of scalar vertices

$$\begin{aligned} ig^2 \Gamma_f^{a\rho,b\nu}(p, q_1, q_2) &= \left(\not{p} + \frac{\not{q}_1 + \not{q}_2}{2} - \mu \right) ig^2 \Gamma_s^{a\rho,b\nu}(p, q_1, q_2) \\ &\quad + \frac{1}{2} \gamma^\rho ig^2 \Gamma^{b\nu,a}(p, q_2, q_1) + \frac{1}{2} \gamma^\nu ig^2 \Gamma^{a\rho,b}(p, q_1, q_2) , \end{aligned} \quad (\text{C.9})$$

where $\Gamma^{a\nu,b}$ is given by:

$$\begin{aligned} \Gamma^{a\nu,b}(p, k_1, k_2) &= T^a T^b (2p^\nu + k_1^\nu) \mathcal{F}_f(p, k_1) + \\ &\quad T^b T^a (2(p^\nu + k_2^\nu) + k_1^\nu) \mathcal{F}_f(p + k_2, k_1) . \end{aligned} \quad (\text{C.10})$$

In the particle limit, $d_f \rightarrow 3/2$, we find that $\mathcal{F}_f \rightarrow 0$ and we easily recover the usual fermion couplings

$$ig \Gamma_{f(d_f=3/2)}^{a\nu} = ig T^a \gamma^\nu , \quad \text{and} \quad ig^2 \Gamma_{f(d_f=3/2)}^{a\mu,b\nu} = 0 . \quad (\text{C.11})$$

As a non-trivial check of our results, we can verify that the vertices do respect the Ward-Takahashi identities [24] in the abelian case:

$$ig q_\mu \Gamma_{s/f}^\mu(p, q) = g \left(\Delta_{s/f}^{-1}(p) - \Delta_{s/f}^{-1}(p+q) \right) , \quad (\text{C.12})$$

$$ig^2 q_{1\mu} \Gamma_{s/f}^{\mu\nu}(p, q_1, q_2) = g \left(ig \Gamma_{s/f}^\nu(p + q_1, q_2) - ig \Gamma_{s/f}^\nu(p, q_2) \right) , \quad (\text{C.13})$$

where we have suppressed the group indices and set $T^a = 1$. In the non-abelian case, the vertices respect more complicated relations:

$$ig q_\mu \Gamma_{s/f}^{a\mu}(p, q) = g T^a \left(\Delta_{s/f}^{-1}(p) - \Delta_{s/f}^{-1}(p + q) \right) , \quad (\text{C.14})$$

$$ig^2 q_{1\mu} \Gamma_{s/f}^{a\mu, b\nu}(p, q_1, q_2) = g \left(ig \Gamma_{s/f}^{b\nu}(p + q_1, q_2) T^a - ig T^a \Gamma_{s/f}^{b\nu}(p, q_2) \right) + gf^{abc} ig \Gamma_{s/f}^{c\nu}(p, q_1 + q_2) . \quad (\text{C.15})$$

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