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Duals for $SU(N)$ SUSY gauge theories with an antisymmetric tensor: five easy flavors

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Abstract

I consider $\mathcal{N}=1$ supersymmetric $SU(N_c)$ gauge theories with matter fields consisting of one antisymmetric representation, five flavors, and enough anti-fundamental representations to cancel the gauge anomaly. Previous analyses are extended to the case of even N_c with no superpotential. Using holomorphy I show that the theory has an interacting infrared fixed point for sufficiently large N_c. These theories are interesting due to the fact that in going from five to four flavors the theory goes from a non-trivial infrared fixed point to confinement, in contradistinction to SUSY QCD, but in analogy to the behavior expected in non-SUSY QCD.

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1 Introduction

In recent years our understanding of the infrared behavior of vector-like $\mathcal{N} = 1$ supersymmetric (SUSY) gauge theories has increased dramatically, primarily due to the work of Seiberg [1, 2]. In particular it is now known for SUSY QCD with a given number of flavors whether the theory has: an unstable vacuum, a confined description, a weakly coupled (infrared free) dual gauge description, a non-trivial infrared fixed point, or a trivial infrared fixed point. Some work has also been done on chiral SUSY gauge theories, but our understanding of these more complex theories is far from complete. Chiral SUSY gauge theories are of special interest since they can dynamically break SUSY, unlike most theories with vector matter[1]. Among the simplest chiral theories are those with an antisymmetric tensor. Consider $SU(N_c)$ with one antisymmetric tensor, $(N_c - 4)$ $\overline{\mathbf{N_c}}$'s and F flavors (a flavor is one $\mathbf{N_c}$ and one $\overline{\mathbf{N_c}}$); it is known that this theory is confining [4, 5, 6] for $F = 3$ or 4. Thus the simplest example of this type of chiral SUSY theory which admits a dual gauge description is $F = 5$. What is unknown is whether the theory has an infrared free dual gauge description or an interacting infrared fixed point. In this paper I show how to use the "deconfinement" method introduced by Berkooz in ref. [7] and elaborated in refs. [5, 8, 9] to construct simple duals for the case $F = 5$ (pointing out why this case is special) and N_c even and compare with the previously know dual for odd N_c. Using holomorphy I show that the dual (and hence the original theory) has an interacting infrared fixed point at the origin of moduli space for sufficiently large N_c. Finally I present my conclusions, and discuss the analogous behavior in non-SUSY QCD.

2 Duality for $SU(2N)$

The theory I wish to study has gauge group $SU(N_c)$ with 5 chiral superfields q in the (defining) $\mathbf{N_c}$ representation, one matter field A in the antisymmetric tensor representation, and $N_c + 1$ matter fields $\overline{q}$ in the $\overline{\mathbf{N_c}}$ representation. This theory has the anomaly-free global symmetry $SU(5) \times SU(N_c + 1) \times U(1)_R \times U(1)_X \times U(1)_Y$. The field content (with global charges) is given in Table 1.

This theory has been considered previously by Berkooz [7] for even $N_c (= 2N)$, with the addition of a superpotential $W = \mathrm{Pf}(A)$. I will consider this theory with

[1] For a vector-like theory that dynamically breaks SUSY see ref. [3].

	$SU(N_c)$	$SU(5)$	$SU(N_c+1)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q	$\Box$	$\Box$	$\mathbf{1}$	$\frac{4}{N_c+6}$	1	-1
$\bar{q}$	$\overline{\Box}$	$\mathbf{1}$	$\Box$	$\frac{4}{N_c+6}$	-1	-1
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N_c-4}{N_c-2}$	$\frac{N_c+6}{N_c-2}$

Table 1: Field content of the theory.

no superpotential. The case N_c odd with no superpotential has been discussed by Pouliot [5] (see also ref. [10]).

I can replace the antisymmetric tensor by a composite "meson" operator of a confining $Sp(2N-2)$ group:

$$A^{ab} \rightarrow x^{aa'} x^{bb'} J_{a'b'}, \tag{2.1}$$

where a, b are $SU(2N)$ indices and a', b' are $Sp(2N-2)$ indices and $J_{a'b'}$ is the invariant tensor. I must also introduce additional fields that transform under $Sp(2N-2)$ and add terms to the superpotential in the deconfined description. The matter content of the model that accomplishes this is displayed in Table 2.

	$SU(N_c)$	$Sp(D)$	$SU(2)_f$	$SU(5)$	$SU(C)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q	$\Box$	$\mathbf{1}$	$\mathbf{1}$	$\Box$	$\mathbf{1}$	$\frac{2}{N+3}$	1	-1
$\bar{q}$	$\overline{\Box}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\Box$	$\frac{2}{N+3}$	-1	-1
x	$\Box$	$\Box$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N-2}{2N-2}$	$\frac{N+3}{2N-2}$
p	$\overline{\Box}$	$\mathbf{1}$	$\Box$	$\mathbf{1}$	$\mathbf{1}$	1	$\frac{N-2}{2}$	$\frac{N+3}{2}$
r	$\mathbf{1}$	$\Box$	$\Box$	$\mathbf{1}$	$\mathbf{1}$	1	$\frac{-N(N-2)}{2N-2}$	$\frac{-N(N+3)}{2N-2}$
s	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N(N-2)}{N-1}$	$\frac{N(N+3)}{N-1}$

Table 2: Field content of the "deconfined" theory, where $D = 2N-2$, and $C = 2N+1$.

The superpotential in the "deconfined" description is

$$W = xrp + rrs. \tag{2.2}$$

(I have set the coefficients of the superpotential to 1 by rescaling the fields.) The purpose of the superpotential is to remove the unwanted "meson" states (xr) and (rr) that appear when the $Sp(2N-2)$ group confines. These two "mesons" get masses with p and s respectively. Note that, as discussed in ref. [9], gauge anomaly cancellation for $Sp(2N-2)$ forces the fields r and p to have a fictitious global $SU(2)_f$ symmetry. This symmetry is fictitious in the sense that none of the physical low energy degrees of freedom transform under it: r is confined, and p is massive. This symmetry will be useful later in determining which of several dual descriptions might be useful.

I can now use the known dual description of $SU(2N)$ gauge theory with fundamentals [1] to write a dual description of this theory in terms of a theory with gauge group $SU(3) \times Sp(2N-2)$. The field content of this dual is given in Table 3. This

	$SU(3)$	$Sp(D)$	$SU(2)_f$	$SU(5)$	$SU(C)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q_1	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\frac{4}{3(N+3)}$	$\frac{N}{3}$	$\frac{N+1}{3}$
$\bar{q}_1$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\frac{6N+2}{3(N+3)}$	$-\frac{N}{3}$	$\frac{5-N}{3}$
x_1	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{10}{3(N+3)}$	$\frac{N(2N+1)}{3(2N-2)}$	$\frac{(2N+1)(N-5)}{3(2N-2)}$
p_1	$\overline{\square}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3N-1}{3(N+3)}$	$\frac{-5N(N-1)}{3(2N-2)}$	$\frac{-5(N+1}{6}$
r	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	1	$\frac{-N(N-2)}{2N-2}$	$\frac{-N(N+3)}{2N-2}$
s	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N(N-2)}{N-1}$	$\frac{N(N+3)}{N-1}$
$(\bar{q}q)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{4}{N+3}$	0	-2
(xp)	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	1	$\frac{N(N-2)}{2N-2}$	$\frac{N(N+3)}{2N-2}$
(qp)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\frac{N+5}{N+3}$	$\frac{N}{2}$	$\frac{N+1}{2}$
$(\bar{q}x)$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{2}{N+3}$	$\frac{-N}{2N-2}$	$\frac{5-N}{2N-2}$

Table 3: Field content of the first dual description, where $D = 2N-2$, $C = 2N+1$.

dual has a superpotential

$$W = (xp)r + rrs + (\bar{q}q)q_1\bar{q}_1 + (xp)x_1p_1 + (qp)q_1p_1 + (\bar{q}x)\bar{q}_1x_1 \ . \tag{2.3}$$

I have introduced some notation here to simplify the later exposition: q_i refers to the field which is the "dual" of q_{i-1}, where $q_0 \equiv q$, and I will denote a "meson" which is the mapping of q_ip_j (and couples to q_{i+1} and p_{j+1} in the dual superpotential) by (q_ip_j). For later convenience I will relabel the "meson" $(\overline{q}x)$ by y. The massive fields (xp) and r can be integrated out, leaving the superpotential:

$$W = x_1^2p_1^2s + (\bar{q}q)q_1\bar{q}_1 + (qp)q_1p_1 + y\bar{q}_1x_1 \ . \tag{2.4}$$

The anomaly matching is guaranteed to work by the anomaly matching of the SU duality used in its construction.

The dual description obtained above is, unfortunately, almost completely useless, since it possesses a fictitious global $SU(2)_f$ symmetry: any field which transforms under this fictitious symmetry must either be massive or strongly coupled since it cannot appear in the physical spectrum. It is obvious that additional dual descriptions can be obtained by alternating the gauge group that duality is applied to [7, 5, 9]; what is surprising is that such an exercise turns out to be useful. Going through a repeated application of alternating dualities produces (for the case of five flavors) a dual with no fields transforming under $SU(2)_f$. The remainder of this section is devoted to detailing this procedure, the reader who is interested in results rather than techniques is urged to skip ahead to Table 6 where the final dual is presented.

The next step is to apply duality to the $Sp(2N-2)$ gauge group. The field content of the resulting dual is given in Table 4, and the superpotential is

$$\begin{aligned} W &= zp_1^2 s + (\bar{q}q)q_1\bar{q}_1 + (qp)q_1p_1 + (yx_1)\bar{q}_1 \\ &\quad + zx_2x_2 + (yy)y_1y_1 + (x_1y)y_1x_2 \ , \end{aligned} \tag{2.5}$$

where I have renamed (x_1x_1) to be z.

	$SU(3)$	$SU(2)$	$SU(2)_f$	$SU(5)$	$SU(C)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q_1	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\frac{4}{3(N+3)}$	$\frac{N}{3}$	$\frac{N+1}{3}$
$\bar{q}_1$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\frac{6N+2}{3(N+3)}$	$-\frac{N}{3}$	$\frac{5-N}{3}$
x_2	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3N-1}{3(N+3)}$	$\frac{-N(2N+1)}{3(2N-2)}$	$\frac{-(2N+1)(N-5)}{3(2N-2)}$
p_1	$\overline{\square}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3N-1}{3(N+3)}$	$\frac{-5N(N-1)}{3(2N-2)}$	$\frac{-5(N+1)}{6}$
s	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N(N-2)}{N-1}$	$\frac{N(N+3)}{N-1}$
$(\bar{q}q)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{4}{N+3}$	0	-2
(qp)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\frac{N+5}{N+3}$	$\frac{N}{2}$	$\frac{N+1}{2}$
y_1	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\frac{N+1}{N+3}$	$\frac{N}{2N-2}$	$\frac{N-5}{2N-2}$
z	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{20}{3(N+3)}$	$\frac{N(2N+1)}{3(N-1)}$	$\frac{(2N+1)(N-5)}{3(N-1)}$
(yy)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{4}{N+3}$	$\frac{-N}{N-1}$	$\frac{5-N}{N-1}$
(x_1y)	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{16}{3(N+3)}$	$\frac{N}{3}$	$\frac{N-5}{3}$

Table 4: Field content of the second dual description, where $C = 2N+1$.

After integrating out $\overline{q_1}$ and (x_1y), the superpotential is:

$$W = zp_1^2 s - (\bar{q}q)q_1y_1x_2 + (qp)q_1p_1 + zx_2x_2 + (yy)y_1y_1 \ . \tag{2.6}$$

At this point it can be seen why the case of 5 flavors is so special. If the analysis so far had been done for F flavors, then the gauge group $SU(3)$ would instead be $SU(F-2)$, and the field z would be an antisymmetric tensor[2]. Then to further dualize the $SU(F-2)$ would require the introduction of an additional "deconfinement" module, and hence an even more complicated description of the theory.

I can now use the known dual of an $SU(3)$ gauge theory with fundamental representations to find another dual, with the field content given in Table 5; the superpotential is

$$\begin{aligned} W &= x_3^2 s - (\bar{q}q)(q_1x_2)y_1 + (qp)(q_1p_1) + p_2^2 + (yy)y_1y_1 + (q_1x_2)q_2x_3 \\ &\quad +(q_1p_1)q_2p_2 + (q_1z)q_2z_1 \ . \end{aligned} \tag{2.7}$$

	$SU(2)$	$SU(2)$	$SU(2)_f$	$SU(5)$	$SU(C)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q_2	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\frac{2}{N+3}$	$\frac{N}{2}$	$\frac{N+1}{2}$
x_3	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1	$\frac{-N(N-2)}{2N-2}$	$\frac{-N(N+3)}{2N-2}$
p_2	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	1	0	0
s	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N(N-2)}{N-1}$	$\frac{N(N+3)}{N-1}$
$(\bar{q}q)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{4}{N+3}$	0	-2
(qp)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\frac{N+5}{N+3}$	$\frac{N}{2}$	$\frac{N+1}{2}$
y_1	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\square}$	$\frac{N+1}{N+3}$	$\frac{N}{2N-2}$	$\frac{N-5}{2N-2}$
z_1	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{2N-4}{N+3}$	$\frac{-N(3N-1)}{2N-2}$	$\frac{5+6N-3N^2}{2N-2}$
(yy)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{4}{N+3}$	$\frac{-N}{N-1}$	$\frac{5-N}{N-1}$
(q_1x_2)	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\bar{\square}$	$\mathbf{1}$	$\frac{N+1}{N+3}$	$\frac{-N}{2N-2}$	$\frac{3N+1}{2N-2}$
(q_1p_1)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\bar{\square}$	$\mathbf{1}$	$\frac{N+1}{N+3}$	$\frac{-N}{2}$	$\frac{-N-1}{2}$
(q_1z)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\square}$	$\mathbf{1}$	$\frac{8}{N+3}$	$\frac{N^2}{N-1}$	$\frac{N^2-3N-2}{N-1}$

Table 5: Field content of the third dual description, where $C = 2N + 1$.

Note the "baryonic" operator mapping:

$$\begin{aligned} zp_1p_1 &\rightarrow x_3^2, \\ zx_2x_2 &\rightarrow p_2^2, \\ x_2p_1p_1 &\rightarrow z_1x_3, \\ x_2x_2p_1 &\rightarrow z_1p_2 \ . \end{aligned} \tag{2.8}$$

[2]The antisymmetric tensor for $SU(3)$ is just a $\bar{\mathbf{3}}$.

After integrating out p_2, (qp), and (q_1p_1) there are no longer any fields that transform under the fictitious global $SU(2)_f$ symmetry, although the singlet field s still remains. The superpotential is given by

$$\begin{aligned} W &= x_3^2 s - (\bar{q}q)(q_1x_2)y_1 + (yy)y_1y_1 + (q_1x_2)q_2x_3 \\ &\quad +(q_1p_1)q_2p_2 + (q_1z)q_2z_1 \ . \end{aligned} \tag{2.9}$$

To obtain a slightly simpler dual description, I can now apply duality one more time to the first $SU(2)$ gauge group. After integrating out a number of massive fields I find the field content given in Table 6 with a superpotential given by:

$$W = (\bar{q}q)q_3x_4y_1 + (yy)y_1y_1 + (q_2q_2)q_3q_3 + (x_3z_1)x_4z_2 \ . \tag{2.10}$$

	$SU(2)_1$	$SU(2)_2$	$SU(5)$	$SU(2N+1)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q_3	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\frac{N+1}{N+3}$	$\frac{-N}{2}$	$\frac{-N-1}{2}$
x_4	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{N(N-2)}{2N-2}$	$\frac{N(N+3)}{2N-2}$
y_1	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\frac{N+1}{N+3}$	$\frac{N}{2N-2}$	$\frac{N-5}{2N-2}$
z_2	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{7-N}{N+3}$	$\frac{N(3N-1)}{2N-2}$	$\frac{3N^2-6N-5}{2N-2}$
(x_3z_1)	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3N-1}{N+3}$	$\frac{-N(4N-3)}{2N-2}$	$\frac{5+3N-4N^2}{2N-2}$
$(\bar{q}q)$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{4}{N+3}$	0	-2
(q_2q_2)	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\frac{4}{N+3}$	N	$N+1$
(yy)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{4}{N+3}$	$\frac{-N}{N-1}$	$\frac{5-N}{N-1}$

Table 6: Field content of the "final" dual description.

The operator mapping of the chiral ring is:

$$\begin{array}{ccc} \overline{q}q & \rightarrow & (\overline{q}q), \\ \overline{q}A\overline{q} & \rightarrow & (yy), \\ qA^{N-1}q & \rightarrow & (q_2q_2), \\ q^4A^{N-2} & \rightarrow & q_3z_2, \\ A^N & \rightarrow & x_4^2, \\ \overline{q}^{2N} & \rightarrow & y_1(x_3z_1) \ , \end{array} \tag{2.11}$$

where $y \equiv (\overline{q}x)$ and $z \equiv (x_1x_1)$.

This dual has a simple relation to the spectrum of the confined description found by Pouliot [5] for the case of four flavors. Adding a mass term for one flavor in the original theory gives breaks the flavor symmetry to $SU(4) \times SU(2N)$. In the dual description the mass term maps to a linear term for $(\overline{q}q)$, which induces a vev for for the product $q_3 x_4 y_1$. D-flatness ensures that, in an appropriate basis, each of these three fields has only one non-zero component. These vevs break the gauge symmetries completely and produce mass terms for extra components of $(\overline{q}q)$, (yy), and $(q_2 q_2)$ with uneaten pieces of q_3 and y_1. The vev of x_4 gives a mass to one component each of $(x_3 z_1)$ and z_2, leaving two massless fields, and one component of x_4 remains uneaten. The operator mapping for four flavors is:

$$\begin{array}{ccc} \overline{q}q & \rightarrow & (\overline{q}q), \\ \overline{q}A\overline{q} & \rightarrow & (yy), \\ qA^{N-1}q & \rightarrow & (q_2q_2), \\ q^4A^{N-2} & \rightarrow & \widehat{z_2}, \\ A^N & \rightarrow & \widehat{x_4}, \\ \overline{q}^{2N} & \rightarrow & \widehat{(x_3z_1)}\ , \end{array} \tag{2.12}$$

where the $\widehat{}$ superscript indicates the remaining massless (singlet) component. Poppitz and Trivedi [4] showed that these theories can break SUSY with the addition of some singlet fields, some superpotential terms, and the gauging of a chiral $U(1)$ symmetry. With corresponding manipulations of the dual, it can provide a weakly coupled description of their SUSY breaking models.

3 Comparison with Duality for $SU(2M-1)$

The case of odd N_c has been studied previously by Pouliot [5]. In this section I will briefly review his dual in order make comparisons with the even N_c case discussed above. The field content (with global charges) is given in Table 1, with $N_c = 2M - 1$. Pouliot deconfined the antisymmetric tensor with $Sp(2M-4)$ by introducing fields x, r, and p (as discussed earlier for the even case) with a superpotential $W = xrp$. The odd N_c case is much simpler than the even case because no fictitious global symmetry is needed nor is a singlet field required. Pouliot then dualized $SU(2M-1)$ to $SU(2)$ in the usual fashion, and further dualized $Sp(2M-4)$ to $SU(2)$. After integrating out massive fields, he arrived at a dual with a superpotential given by:

$$W \ = (\bar{q}q)q_1y_1x_2 + (yy)y_1y_1 + (qp)q_1p_1 + (x_1x_1)x_2x_2\ , \tag{3.1}$$

and the field content shown in Table 7 (using the notation $y \equiv (\overline{q}x)$).

	$SU(2)$	$SU(2)$	$SU(5)$	$SU(2M)$	$U(1)_R$	$U(1)_X$	$U(1)_Y$
q_1	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\frac{6}{2M+5}$	$\frac{(M-1)(2M-1)}{4M-6}$	$\frac{2M^2+-5M-1}{4M-6}$
x_2	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\frac{2M-5}{2M+5}$	$\frac{-M(2M-1)}{4M-6}$	$\frac{-M(2M-11)}{4M-6}$
y_1	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\frac{2M+1}{2M+5}$	$\frac{2M-1}{4M-6}$	$\frac{2M-11}{4M-6}$
p_1	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{4M}{2M+5}$	$\frac{-6M^2+13M-5}{4M-6}$	$\frac{-6M^2+3M+5}{4M-6}$
$(x_1 x_1)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{20}{2M+5}$	$\frac{2M(2M-1)}{4M-6}$	$\frac{2M(2M-11)}{4M-6}$
$(\bar{q}q)$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{8}{2M+5}$	0	-2
(qp)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\frac{4}{2M+5}$	$\frac{4M^2-10M+4}{4M-6}$	$\frac{4M^2+2M-4}{4M-6}$
(yy)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{8}{2M+5}$	$\frac{2-4M}{4M-6}$	$\frac{22-4M}{4M-6}$

Table 7: Field content of the dual description for the case $N_c = 2M-1$.

The operator mapping is:

$$\begin{array}{rcl} \overline{q}q & \rightarrow & (\overline{q}q), \\ \overline{q}A\overline{q} & \rightarrow & (yy), \\ qA^{M-1} & \rightarrow & (qp), \\ q^3A^{M-2} & \rightarrow & q_1q_1, \\ \overline{q}^{2M-1} & \rightarrow & y_1x_2p_1, \end{array} \tag{3.2}$$

and for $M \geq 3$

$$q^5 A^{M-3} \rightarrow (x_1 x_1) \ . \tag{3.3}$$

Even though only three dualities were required in the derivation of the odd N_c case, as opposed to five dualities for even N_c, the resulting dual descriptions are quite similar. The five or six flavor models are special for odd N_c as well, since for a larger number of flavors the dual contains tensor representations.

4 The Infrared Fixed Point

I would now like to demonstrate that the dual of the $SU(2N)$ theory described above (and the original theory itself) has a non-trivial infrared fixed point at the origin of moduli space. The situation is more difficult than in SUSY QCD since there are two gauge groups in the dual. The analysis can be simplified by using the fact that the ratio of the two intrinsic scales, Λ_1 and Λ_2 (corresponding to the $SU(2)_1$ and $SU(2)_2$ gauge groups in the final dual description given in Table 6), can be varied arbitrarily. Holomorphy [11] requires that, aside from singular points, there can be no phase transitions as this ratio is varied. There are two cases[3] to consider: for $N > 4$ the $SU(2)_2$ gauge group is infrared free and $\Lambda_1 \ll \Lambda_2$ corresponds to the $SU(2)_2$ gauge coupling g (renormalized at a scale near Λ_1) becoming arbitrarily small, for $N < 4$ the $SU(2)_2$ gauge group is asymptotically free and the limit $\Lambda_1 \gg \Lambda_2$ also corresponds to weak coupling for $SU(2)_2$. In both cases the gauge coupling $g \to 0$ as $\Lambda_1/\Lambda_2 \to 0$ or ∞. Of course g cannot be simply set to zero for at least two reasons. Firstly, the massless spectrum is discontinuous in this limit, since setting $g = 0$ causes D-terms to vanish, thus enlarging the moduli space. More importantly non-perturbative effects from the $SU(2)_1$ gauge interactions can affect the running of g in the infrared. Thus a careful study is required.

Before proceeding with the details of the calculation I will sketch an outline of the analysis. I will analyze the dual at a renormalization scale somewhat below the interaction scale of $SU(2)_1$ (i.e. $\mu < \Lambda_1$) with an arbitrarily small (but non-zero) value for $g(\mu)$. At this scale the theory can be studied with perturbation theory in $g(\mu)$, and at lowest order in $g(\mu)$ I will show that the $SU(2)_1$ has a non-trivial infrared fixed point. I will then proceed to show that for sufficiently large N the $SU(2)_2$ interactions are infrared free at this scale, i.e. that coupling g has a trivial infrared stable fixed point. This is sufficient to prove that the theory with an arbitrary ratio Λ_1/Λ_2 has the same infrared fixed point, since there cannot be a phase transition in the space of holomorphic couplings [11]. Thus the infrared limit can be understood simply through a perturbative analysis in the coupling g.

It is instructive to first consider the theory at zero-th order in g (i.e. with g set to zero). With $SU(2)_2$ turned off, the fields y_1 and $(x_3 z_1)$ as well as the products $q_3 x_4$ and $x_4 z_2$ become gauge invariant operators, so their scaling dimensions satisfy the bounds

$$\begin{aligned}
D(y_1) &= 1 + \gamma_{y_1}(g=0) \geq 1 && (4.1)\\
D((x_3 z_1)) &= 1 + \gamma_{(x_3 z_1)}(g=0) \geq 1 && (4.2)\\
D(q_3 x_4) &= 2 + \gamma_{q_3}(g=0) + \gamma_{x_4}(g=0) \geq 1 && (4.3)\\
D(x_4 z_2) &= 2 + \gamma_{x_4}(g=0) + \gamma_{z_2}(g=0) \geq 1 && (4.4)
\end{aligned}$$

(where γ_ϕ is the anomalous dimension of the field ϕ) with equality holding if the operators are free. Thus for $g = 0$ the first two interaction terms in the superpotential (2.10) are the products of three gauge invariant operators, and are thus irrelevant

[3]For $N = 4$, the holomorphic gauge coupling does not run, and can be set to be arbitrarily small.

operators[4]. In other words, if the coefficients of the first two terms are labeled λ_1/μ_0 and λ_2 then for $g = 0$, λ_1 and λ_2 run to zero in the infrared. Since the fields $(\bar{q}q)$, (yy), and y_1 only interact through these irrelevant terms, they are free fields and their anomalous dimensions vanish. The effective theory containing the remaining two operators and the $SU(2)_1$ gauge interactions is in an interacting non-Abelian Coulomb phase (i.e. it has a non-trivial infrared fixed point at the origin of moduli space). This can be seen by noting that this is a special case of an $SU(2)$ theory with N_F flavors (here $N_F = 4$) and trilinear superpotential terms which is dual to an $Sp(2N_F - 6)$ theory with N_F flavors and trilinear superpotential terms. The $SU(2)$ theory is asymptotically free for $N_F < 6$, while the $Sp(2N_F - 6)$ theory is asymptotically free for $N_F > 18/5$, so (assuming a la SUSY QCD that there is a conformal range of fixed point theories between the two Banks-Zaks [12] fixed points[5]) the theory is at an infrared fixed point for $18/5 < N_F < 6$. Thus for $g = 0$ the bounds in (4.2)-(4.4) are not saturated. Recall that the case of five flavors in the original theory was special because it led to a simple dual without tensor gauge representations, and it is the absence of tensor representations that allows for a simple demonstration of an infrared fixed point for $g = 0$.

A few more relations between anomalous dimensions are required to reach some definite conclusions for non-zero g. Recall that the exact β function for the $SU(2)_1$ coupling is [13]:

$$\beta(g_1) \;=\; -\frac{g_1^3}{16\pi^2}\,\frac{2 + 5\gamma_{q_3}(g=0) + 2\gamma_{x_4}(g=0) + \gamma_{z_2}(g=0)}{1 - \frac{g_1^2}{4\pi^2}} \;, \qquad (4.5)$$

thus at the fixed point:

$$0 = 2 + 5\gamma_{q_3}(g=0) + 2\gamma_{x_4}(g=0) + \gamma_{z_2}(g=0) \;. \qquad (4.6)$$

Since the last term in the superpotential (2.10) is a relevant operator (for $g = 0$) with R-charge 2 the anomalous dimensions satisfy

$$\gamma_{(x_3 z_1)}(g=0) + \gamma_{x_4}(g=0) + \gamma_{z_1}(g=0) = 0 \;, \qquad (4.7)$$

[4]They can only be relevant if all three operators are dimension 1, in which case the operators are free, a contradiction.

[5]If the superpotential couplings are taken to be arbitrarily small, a Banks-Zaks fixed point can be established in perturbation theory at the point where asymptotic freedom is almost lost, then by holomorphy [11] there is a fixed point for arbitrary superpotential couplings.

which, with the bound (4.4), implies

$$\gamma_{(x_3 z_1)}(g=0) < 1 \ . \tag{4.8}$$

Furthermore q_3^2, x_4^2, and $q_3 z_1$ are gauge invariant operators so

$$\gamma_{q_3}, \gamma_{x_4} \geq -\frac{1}{2} \ , \tag{4.9}$$
$$\gamma_{q_3} + \gamma_{z_1} \geq -1 \ , \tag{4.10}$$

independent of g. Combining the fixed point condition (4.6) with the bound (4.10) gives

$$2\gamma_{x_4}(g=0) < -1 - 4\gamma_{q_3}(g=0) \ . \tag{4.11}$$

This inequality with the bound (4.9) implies

$$\gamma_{x_4}(g=0) < \frac{1}{2} \ . \tag{4.12}$$

Returning to the theory with $g(\mu)$ arbitrarily small, but non-zero, I note that the anomalous dimensions of $(\bar{q}q)$, (yy), and y_1 as well as the couplings λ_1 and λ_2 vanish at $g=0$. I proceed by making the plausible assumption that the anomalous dimensions and β functions of the theory with $g(\mu)$ arbitrarily small can be reliably analyzed near the scale μ with a perturbative expansion in $g(\mu)$. Thus I am assuming that the anomalous dimensions of the fields with $SU(2)_1$ interactions have reliable perturbative expansions in $g(\mu)$, although I do not know the value first ($g(\mu)$ independent) terms in these expansions since they are determined by the dynamics of the pure $SU(2)_1$ fixed point discussed above.[6] Now consider the running of $g(\mu)$ which is determined by [13]:

$$\begin{aligned} \beta(g) &= -\frac{g^3}{16\pi^2} \frac{4 - N + (2N+1)\gamma_{y_1} + 2\gamma_{x_4} + \gamma_{(x_3 z_1)}}{1 - \frac{g^2}{4\pi^2}} \ , \\ &= -\frac{g^3}{16\pi^2} \Big(4 - N + 2\gamma_{x_4}(g=0) + \gamma_{(x_3 z_1)}(g=0)\Big) + \mathcal{O}(g^5) \ . \end{aligned} \tag{4.13}$$

The bounds (4.8) and(4.12) imply that the $SU(2)_2$ interactions are infrared free ($\beta(g) > 0$) for $N > 6$.

[6] Alternatively they are determined by the (unknown) superconformal R-charge [1, 2].

Turning to the superpotential interactions, the β functions of the the first two terms in the superpotential (expanded to leading order in λ_1, λ_2 and $g(\mu)$) are:

$$\begin{aligned}\beta_1 &= 1+\gamma_{(\bar{q}q)}+\gamma_{q_3}+\gamma_{x_4}+\gamma_{y_1}\\ &= 1+\gamma_{q_3}(g=0)+\gamma_{x_4}(g=0)+a\lambda_1^2+b\lambda_2^2-cg(\mu)^2\ , \qquad (4.14)\\ \beta_2 &= \gamma_{(yy)}+2\gamma_{y_1}\\ &= d\lambda_1^2+e\lambda_2^2-fg(\mu)^2\ , \qquad (4.15)\end{aligned}$$

where $a...f$ are positive numbers. For the first two terms in the superpotential to be relevant these two β functions must vanish. The discussion above indicates that λ_1 and λ_2 must vanish with $g(\mu)$. Therefore for sufficiently small $g(\mu)$, β_1 cannot vanish since the bound in equation (4.3) is not saturated. These considerations suggest that it is however possible for β_2 to vanish. However the solution of $\beta_2=0$ with $\lambda_1=0$ is that $\lambda_2 \propto g(\mu)$. Thus in the infrared limit $\mu \to 0$ the three couplings λ_1, λ_2, and g all run to zero.

The conclusion of this analysis is that for $N>6$ the chiral operators $\overline{q}q \to (\overline{q}q)$ and $\overline{q}A\overline{q} \to (yy)$ (as well as the $SU(2)_2$ gauge (vector) multiplet and the dual quark y_1), correspond to free fields in the infrared, while the remaining fields are at an interacting fixed point. Thus these theories provide explicit examples of the phenomena suggested in refs. [9, 14] of a gauge theory splitting into a free sector and an interacting fixed point sector in the infrared. A similar analysis can be applied to the odd case, $N_c = 2M-1$, which can also be shown to have an interacting infrared fixed point for $M>6$. Although I have only proven that the theory with $F=5$ flavors has an interacting infrared fixed point, I expect that the fixed point will persist up to the point where asymptotic freedom is lost: $F = 2N_c+3$. It should be noted that this analysis does not preclude a fixed point for $N \leq 6$; to obtain information about these theories would require more information about the anomalous dimensions $\gamma_{x_4}(g=0)$ and $\gamma_{(x_3 z_1)}(g=0)$. It is suggestive however that for $N=1$ and $M=2$ the original theory reduces to vector-like $SU(2)$ and $SU(3)$ theories both of which do indeed have a non-trivial infrared fixed points.

5 Conclusions

I have displayed a new dual for $SU(2N)$ with an antisymmetric tensor, five flavors, and no superpotential. Using holomorphy to adjust the ratio of the scales of the two gauge groups in the dual description I have been able to show that in the five flavor case two composite "mesons" become free fields in the infrared, while other degrees of freedom are at an interacting infrared fixed point for $N > 6$. Thus in going from five to four flavors (for sufficiently large N) the theory goes from a fixed point to confinement[7] without passing through an infrared free phase. Such behavior was seen previously [1, 2] in the isolated case of vector-like $SU(2)$, whereas in the generic case of vector-like $SU(N)$ theories there is confinement for $F = N + 1$ flavors and an infrared free gauge description for $N + 1 < F < 3N/2$. (The two bounds coalesce for $N = 2$.) The transition from a fixed point phase directly to a confining phase as the number of flavors is reduced has been argued to occur[8] in (non-SUSY) QCD [15]. Given that there is currently no non-perturbative understanding of non-SUSY $SU(N)$ gauge theories with an arbitrary number of flavors, it is somewhat reassuring to find that the expected behavior of the confinement transition is actually realized in a large class of theories that *are* under non-perturbative control. However, there is no evidence to suggest that in QCD there are free, massless composites on the fixed point side of the transition. On the contrary the scalar and pseudoscalar (pion) mesonic states are expected to be massive (and broad) resonances on the fixed point side of the transition [15]. Thus while some of the qualitative behavior of the confinement transitions in QCD and the chiral SUSY theories discussed here is similar, the detailed physics of the two confinement transitions appears to be quite different.

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[7] Without chiral symmetry breaking.

[8] However non-SUSY QCD has confinement *with* chiral symmetry breaking.

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